

## Formalism of Helicity Coupling Amplitudes for $J/\psi \rightarrow \pi^+\pi^-\pi^0$ \*

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**Abstract** The formalisms of helicity coupling amplitudes for  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  are presented. A detailed discussion is also given on the barrier factor, Breit–Wigner, and density matrix. A Monte Carlo simulation of  $J/\psi \rightarrow \rho(770)\pi \rightarrow \pi^+\pi^-\pi^0$  is carried out. The results show that the  $\rho(770)$  resonance is well reproduced compared with experimental data.

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**Key words:** helicity coupling amplitude,  $J/\psi$  decays,  $\rho\pi$  puzzle

### 1 Introduction

From perturbative QCD (pQCD), it is expected that both  $J/\psi$  and  $\psi'$  decaying into light hadrons are dominated by the annihilation of  $c\bar{c}$  into three gluons or one virtual photon, with a width proportional to the square of the wave function at the origin.<sup>[1]</sup> This yields the pQCD “12% rule”,

$$Q_h = \frac{\mathcal{B}_{\psi' \rightarrow h}}{\mathcal{B}_{J/\psi \rightarrow h}} = \frac{\mathcal{B}_{\psi' \rightarrow e^+e^-}}{\mathcal{B}_{J/\psi \rightarrow e^+e^-}} \approx 12\%.$$

A remarkable violation of this rule was first observed by Mark II,<sup>[2]</sup> which was historically called  $\rho\pi$  puzzle. Many theoretical attempts have been devoted to solving  $\rho\pi$  puzzle for a long time, for instance, intermediate vector glueballs, hadronic form factors, final state interactions, S-D wave mixture of  $\psi'$ , etc., which are investigated by many authors.<sup>[3,4]</sup> By far, to solve the  $\rho\pi$  puzzle is still a big challenge both to theoretical and experimental experts. This is mainly due to the fact that these theories did not provide further predictions for experimental physicists to test for and to distinguish them.

On the other hand, further experimental information is desired by theoretical physicists to solve this puzzle. Recently, a more accurate measurement of  $\psi' \rightarrow \rho\pi$  was released from BES Collaboration,<sup>[5]</sup> which gave  $B(\psi' \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0) = (18.1 \pm 1.8 \pm 1.9) \times 10^{-5}$ .

For test of  $\rho\pi$  violation, higher statistical measurement of  $J/\psi \rightarrow \rho\pi$  is desirable. From the BES measurement of  $J/\psi \rightarrow \psi^+\pi^-\pi^0$ ,<sup>[6]</sup> the  $\rho(770)$  signal is clearly seen in the Dalitz plot. However the partial wave analysis (PWA) is still not performed on this decay. In this paper, we present the formalism of helicity coupling amplitudes for  $J/\psi \rightarrow \pi^+\pi^-\pi^0$ , which serves as the reference of PWA on this decay by BES Collaboration in the near future.

### 2 Formalism

Consider a decay  $a(J, \eta_J) \rightarrow b(s, \eta_s) + c(\sigma, \eta_\sigma)$ , where the quantum number  $(J, \eta_J)$  denotes (spin, parity), and the decay amplitudes are given by

$$A_{\lambda_1, \lambda_2}^J(\theta, \phi; M) \propto D_{M, \lambda_1 - \lambda_2}^{J*}(\phi, \theta, 0) F_{\lambda_1, \lambda_2}^J, \quad (1)$$

where  $\lambda_1$  and  $\lambda_2$  are the helicities of the two daughter particles, and  $F_{\lambda_1, \lambda_2}^J$  is the helicity-coupling amplitude given by  $F_{\lambda_1, \lambda_2}^J \propto \langle JM\lambda_1\lambda_2 | T | JM \rangle$ , which is constrained by the parity conservation and satisfies

$$F_{\lambda\nu}^J = \eta_J \eta_s \eta_\sigma (-)^{J-s-\sigma} F_{-\lambda-\nu}^J. \quad (2)$$

In  $LS$ -coupling scheme, the helicity-coupling amplitudes  $F_{\lambda\nu}^J$  can be built out of the particles' wave functions and momenta contracted with the modified metric  $\tilde{g}_{\alpha\beta}(W) = -g_{\alpha\beta} + p_a p_b / W$ . Here we cite Chung's formula<sup>[7]</sup> as follows:

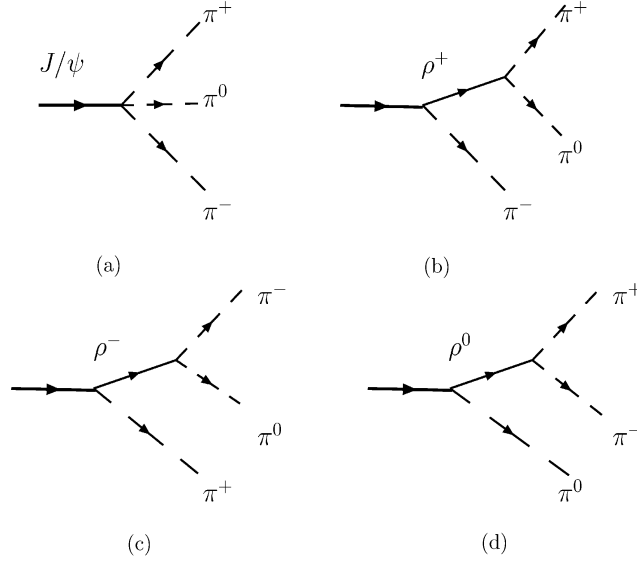
$$F_{\lambda\nu}^J = \sum_{ls} g_{ls} \left( \frac{2l+1}{2J+1} \right)^{1/2} \langle l0S\delta | J\delta \rangle \langle s\lambda\sigma - \nu | S\delta \rangle W^n r^l f_\lambda^s(\gamma_s) f_\nu^\sigma(\gamma_\sigma),$$

$$f_m^j(\gamma) = \frac{(j+m)!(j-m)!}{(2j)!} \sum_{m_0} \frac{j!(2\gamma)^{m_0}}{[(j+m-m_0)/2]! [(j-m-m_0)/2]! m_0!}, \quad (3)$$

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where  $g_{ls}$  is a coupling constant, and  $\gamma_{s/\sigma} = E_{a/b}/m_{a/b}$  and  $r = p_b - p_c$ , which is always replaced with the barrier factor in data analysis, and  $W$  is the mass of the mother particle, and  $n = 1$  for  $s + \sigma + l - J$  odd and  $n = 0$  otherwise.



**Fig. 1** The schematic diagram for  $J/\psi \rightarrow \pi^+ \pi^0 \pi^-$ .

For the  $J/\psi(M) \rightarrow \pi^+(p_1)\pi^0(p_2)\pi^-(p_3)$  decay (see Fig. 1), the possible intermediate resonances include  $\rho(1^-)$ ,  $\rho(3^-)$ , and  $\rho(5^-)$ . Here we ignore isospin breaking effects from  $\rho$ - $\omega$  mixing, since the contribution is negligible as reported by KLOE and ALEPH Collaborations.<sup>[8,9]</sup> We list the helicity-coupling amplitudes as follows:

$$A_0(M) = D_{M,0}^{1*}(\alpha, \beta, 0)F_0, \quad (4)$$

$$A_j^{\rho^+}(M) = \sum_{\lambda=-1,1} F_{\lambda,0}^1(r_3)D_{M,\lambda}^{1*}(\theta_3, \phi_3)BW_j(s_{12})R_{0,0}^j(r_1^*)D_{\lambda,0}^{1*}(\theta_{12}, \phi_{12}), \quad (5)$$

$$A_j^{\rho^-}(M) = \sum_{\lambda=-1,1} F_{\lambda,0}^1(r_1)D_{M,\lambda}^{1*}(\theta_1, \phi_1)BW_j(s_{23})R_{0,0}^j(r_2^*)D_{\lambda,0}^{1*}(\theta_{23}, \phi_{23}), \quad (6)$$

$$A_j^{\rho^0}(M) = \sum_{\lambda=-1,1} F_{\lambda,0}^1(r_2)D_{M,\lambda}^{1*}(\theta_2, \phi_2)BW_j(s_{12})R_{0,0}^j(r_3^*)D_{\lambda,0}^{1*}(\theta_{13}, \phi_{13}), \quad (7)$$

where  $r_1^*$ ,  $r_2^*$ , and  $r_3^*$  are respectively the momentum differences of the two pions' in the CM system of their mother particles  $\rho$  with quantum number  $J^P = j^-$ , and  $\theta_{12}(\phi_{12})$ ,  $\theta_{23}(\phi_{23})$ , and  $\theta_{13}(\phi_{13})$  are the polar (azimuthal) angles of the momentum vector of  $\pi^+$ ,  $\pi^0$ , and  $\pi^-$  in their mother particle CM system.  $BW_j(s)$  denotes the Breit-Wigner of the  $\rho(j^-)$  resonance with a CM energy  $\sqrt{s}$ .  $\alpha$  and  $\beta$  are the polar and azimuthal angle of the vector  $\vec{n} = \vec{p}_1 \times \vec{p}_2$  in the  $J/\psi$  rest frame. To consider the parity conservation for  $J/\psi(1^-) \rightarrow \rho(j^-)\pi(0^-)$ , it leads to  $F_{00}^1 = 0$ , thus the sum in the above equations runs over  $\lambda = \pm 1$ .

The helicity-coupling amplitudes  $F_{\mu,\nu}^1$  and  $R_{0,0}^j$  can be constructed in terms of Eq. (3), which are explicitly given by

$$\text{Fig. 1(a)} \quad F_0 = g_0(\vec{p}_1 \times \vec{p}_2) \cdot \vec{\epsilon}_\psi(M=0), \quad (8)$$

$$\rho(1^-): \quad F_{1,0}^1 = -F_{-1,0}^1 = -\frac{1}{\sqrt{2}}g_{11}m_\psi r B_1(r), \quad R_{0,0}^1 = g_{10}r^* B_1(r^*), \quad (9)$$

$$\rho(3^-): \quad F_{1,0}^1 = -F_{-1,0}^1 = -\frac{1}{12\sqrt{2}}g_{33}m_\psi(12\gamma_\rho^2 + 3)r^3 B_3(r), \quad R_{0,0}^3 = g_{30}r^{*3} B_3(r^*), \quad (10)$$

$$\rho(5^-): \quad F_{1,0}^1 = -F_{-1,0}^1 = -\frac{1}{210\sqrt{2}}g_{55}m_\psi(80\gamma_\rho^4 + 120\gamma_\rho^2 + 10)r^5 B_5(r), \quad R_{0,0}^5 = g_{50}r^{*5} B_5(r^*), \quad (11)$$

where  $g_{ls}$  is the coupling constant,  $m_\psi$  is the mass of  $J/\psi$ ,  $B_L(r)$  denotes the barrier factor, and  $\gamma_\rho = E_\rho/m_\rho$ .

The amplitudes including the contact term and resonances  $\rho_i$  is expressed by

$$A(M) = A_0(M) + \sum_{j=1,3,5} [A_j^{\rho^+}(M) + A_j^{\rho^-}(M) + A_j^{\rho^0}(M)]. \quad (12)$$

The differential cross-section is given by

$$d\Gamma = \left(\frac{3}{8\pi^2}\right) \sum_{MM'} \omega_{MM'} A(M) A^*(M') d\phi_3, \quad (13)$$

where  $\omega_{MM'}$  is the density matrix of  $J/\psi$  production, and  $d\phi_3$  is the element of standard 3-body phase space.

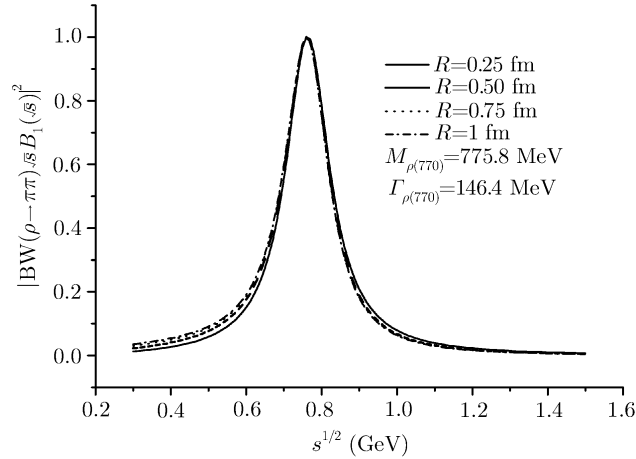
### 3 Discussion

#### 3.1 Barrier Factor

Near the threshold of a resonance production, the resonance shape and width are changed by an action of the fugitive barrier. This effect cannot be well described by the amplitudes built out of the kinematic tensors in partial wave analysis (PWA). Experimentally, in the PWA, the Blatt–Weisskopf barrier factor  $B_L$  is introduced to the orbit momentum wavefunction. Here we list three of them possibly involved in our calculation,

$$\begin{aligned} B_1(q) &= \sqrt{\frac{2}{q^2 + q_0^2}}, & B_3(q) &= \sqrt{\frac{277}{q^6 + 6q^4q_0^2 + 45q^2q_0^4 + 225q_0^6}}, \\ B_5(q) &= \sqrt{\frac{998\,881}{q^{10} + 15q^8q_0^2 + 315q^6q_0^4 + 6300q^4q_0^6 + 99\,225q^2q_0^8 + 893\,025q_0^{10}}}, \end{aligned} \quad (14)$$

where  $q_0 = 1/R$  is a constant associated with the radius of centrifugal barrier  $R$ . Generally,  $R = 0.25 \sim 0.75$  fm is for meson decays and  $R = 0.5 \sim 1.0$  fm is for baryon decays. Actually, the barrier factor is not sensitive to the  $R$  as shown in Fig. 2.



**Fig. 2** The distribution of  $|\text{BW}(\rho \rightarrow \pi\pi)\sqrt{s}B_1(\sqrt{s})|^2$  versus  $\sqrt{s}$  with the different set of the radius of the centrifugal barrier  $R$ .

#### 3.2 Breit–Wigner

In our calculation, the Breit–Wigner of  $\rho$  resonance is taken as

$$\text{BW}_j(s_{\pi\pi}) = \frac{1}{s_{\pi\pi} - m_\rho^2 + i\sqrt{s_{\pi\pi}} \Gamma_\rho(s_{\pi\pi})}, \quad (15)$$

where  $\Gamma_\rho(s)$  is the resonance width dependent on the energy  $\sqrt{s}$ . Experimentally, there are different ways of parametrization of the  $\Gamma_\rho(s)$  in data analyses. For example, the Kühn–Santamaria (KS) parametrization<sup>[10]</sup> was used by KLOE group in the  $\phi \rightarrow \pi^+\pi^-\pi^0$  analysis,<sup>[9]</sup> SND group in the  $e^+e^- \rightarrow \pi^+\pi^-\pi^0$  analysis,<sup>[11]</sup> and ALEPH group in  $\tau \rightarrow 2\pi, 4\pi$  analysis,<sup>[8]</sup>

$$\Gamma_a(s) = \Gamma_a(m_a^2) \left(\frac{m_a^2}{s}\right) \left(\frac{p(s)}{p(m_a^2)}\right)^{2n+1}. \quad (16)$$

Hipple–Quigg (HQ) parametrization was adopted by SDN group in the  $\rho, \omega \rightarrow \pi^0 \pi^0 \gamma$  analysis,<sup>[12]</sup>

$$\Gamma_a(s) = \Gamma_a(m_a^2) \left( \frac{m_a^2}{s} \right) \left( \frac{p(s)}{p(m_a^2)} \right)^{2n+1} \frac{B_n^2(p(s))}{B_n^2(p(m_a^2))}. \quad (17)$$

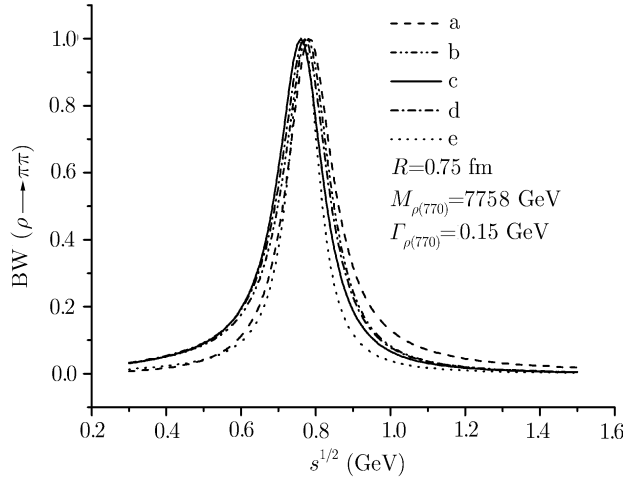
There is also another complex form of Breit–Wigner, Gounaris–Sakurai (GS) parametrization, which was adopted by experimental groups such as ALEPH, CMD2, and BES,<sup>[13]</sup> i.e.

$$\text{BW}(a \rightarrow bc) = \frac{m_a^2 + dm_a \Gamma_a(m_a^2)}{s_{bc} - m_a^2 - H(s_{bc}) + i\sqrt{s_{bc}} \Gamma_a(s_{bc})}, \quad (18)$$

where  $\Gamma(s)$  is defined to be KS form, which is given by

$$\begin{aligned} H(s) &= \Gamma_a(m_a^2) \frac{m_a^2}{p(m_a^2)^3} \left\{ p(s)^2 [h(s) - h(m_a^2)] - (s - m_a^2) p(m_a^2)^2 \frac{dh(s)}{ds} \Big|_{s=m_a^2} \right\}, \\ \text{with } h(s) &= \frac{2}{\pi} \frac{p(s)}{\sqrt{s}} \ln \frac{\sqrt{s} + 2p(s)}{2m_{\pi\pm}}, \\ \frac{dh(s)}{ds} \Big|_{s=m_a^2} &= h(m_a^2) \left[ \frac{1}{2p(m_a^2)^2} - \frac{1}{2m_a^2} \right] + \frac{1}{2\pi m_a^2}, \\ d &= \frac{3m_{\pi\pm}^2}{\pi p(m_a^2)^2} \ln \frac{m_a + 2p(m_a^2)}{2m_{\pi\pm}} + \frac{m_a}{2\pi p(m_a^2)} - \frac{m_a m_{\pi\pm}^2}{\pi p(m_a^2)^3}. \end{aligned} \quad (19)$$

For the description of  $\rho(770)$  resonance, numerical result shows that the differences among the KS and HQ parametrization are quite small as shown in Fig. 3, while differences in GS form and other Breit–Wigner forms can be seen in the figure. As reported by ALEPH Collaboration,<sup>[8]</sup> using these two forms yields respectively about 0.2% and 4.0% differences in the extraction of the mass and width of  $\rho(770)$  resonance.



**Fig. 3** The comparison of the different parametrizations of Breit–Wigner  $\text{BW}(\rho \rightarrow \pi\pi)$ . (a) The distribution of  $|\text{BW}\sqrt{s}|^2$  versus  $\sqrt{s}$ , where the constant width is used, (b) the distribution of  $|\text{BW}\sqrt{s}B_1(\sqrt{s})|^2$  versus  $\sqrt{s}$  and the constant width is used, (c)  $|\text{BW}\sqrt{s}B_1(\sqrt{s})|^2$  versus  $\sqrt{s}$  and the KS parametrization is adopted, (d)  $|\text{BW}\sqrt{s}B_1(\sqrt{s})|^2$  versus  $\sqrt{s}$  and the HQ parametrization is adopted, and (e) GS form of Breit–Wigner.

### 3.3 Density Matrix

For the  $J/\psi$  produced from an  $e^+e^-$  pair annihilation, the density matrix is taken as

$$\omega_{MM'} = \frac{1}{3} \delta_{MM'} - \frac{1}{2} \left( \hat{k}_{+M} \hat{k}_{+M'} - \frac{1}{3} \delta_{MM'} \right), \quad (20)$$

where  $\hat{k}_+$  denotes the moving direction of  $e^+$ , which leads to the explicit form  $\omega = \text{diag}(1/2, 0, 1/2)$ . Here the normalization  $\sum_M \omega_{MM} = 1$  has been adopted.

For the  $J/\psi$  produced from the decay  $\psi'(\Lambda) \rightarrow \pi^+ \pi^- J/\psi(M)$ , the density matrix can be constructed from the transitional amplitudes

$$\omega_{MM'} = \frac{\sum_{\Lambda} A_{\Lambda, M} A_{\Lambda, M'}^*}{3 \int \sum_{\Lambda} A_{\Lambda, M} A_{\Lambda, M'}^* d\phi_3}, \quad (21)$$

where  $d\phi_3$  denotes the Lorentz-invariant phase space of three-body decays, and  $A_{\Lambda,M}$  is the transitional amplitude for  $\psi'(\Lambda) \rightarrow \pi^+\pi^-J/\psi(M)$ , which is explicitly given by the effective chiral Lagrangian,<sup>[14,15]</sup>

$$A_{\Lambda,M} = -\frac{4}{F_\pi^2} \left[ \frac{g}{2}(m_{\pi\pi}^2 - 2m_\pi^2) + g_1 E_{\pi^+} E_{\pi^-} \right] \epsilon_\psi^*(M) \epsilon_{\psi'}(\Lambda), \quad (22)$$

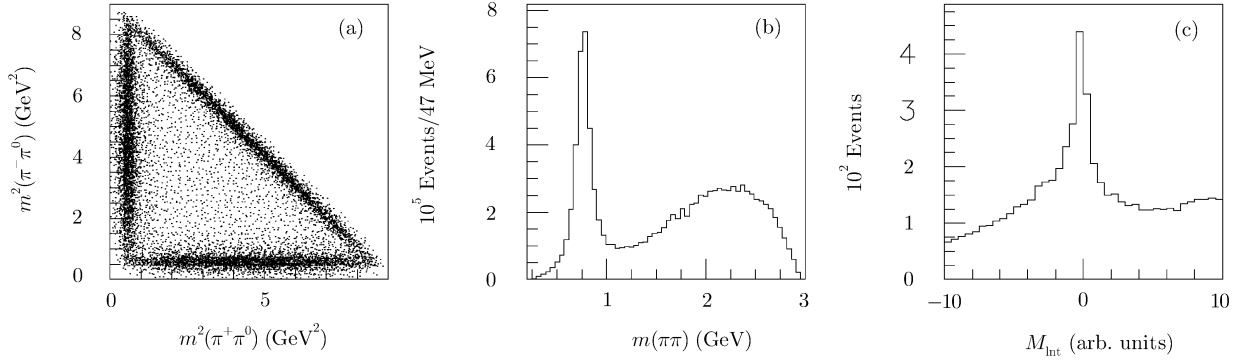
where  $F_\pi \simeq 0.093$  GeV. From fit to the experimental mass distribution of  $m_{\pi\pi}$ , one gets parameters  $g = 0.3 \pm 0.02$  and  $g_1 = -0.11 \pm 0.01$ . It should be pointed out that partial  $D$ -wave contributions of the pion system have been included in the amplitude.<sup>[15]</sup> Thus a numerical evaluation of Eq. (21) gives the density matrix:

$$\omega = \text{diag} \quad (1/3, 1/3, 1/3). \quad (23)$$

This result shows that the component of  $J/\psi$  transverse polarization has contribution to the density matrix in the  $\psi'$  decays.

### 3.4 Contribution of $\rho(770)$

From experimental data of  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  in the  $e^+e^-$  annihilation and  $\psi'$  decays, it is clear to see the dominant signal of  $\rho(770)$  in the Dalitz plot.<sup>[6]</sup> In the distribution of the invariant mass of two pions, the  $\rho(770)$  is clearly seen around 0.77 MeV, and a wide bump around 2.3 GeV is from contributions of  $\rho(770)$  isobars. In the Monte-Carlo simulation including only  $\rho(770)$  resonances, we will show that the Dalitz plot and invariant mass distributions of two pions are well reproduced.



**Fig. 4** The Monte-Carlo simulation including only  $\rho(770)$  resonance. (a) Dalitz plot, (b) the invariant mass distribution of two pions, (c) the distribution of the interference terms.

We naively assume that SU(3) symmetry holds in  $J/\psi \rightarrow \rho\pi$  decays, thus couplings of  $J/\psi$  to  $\rho^\pm\pi^\mp, \rho^0\pi^0$  are equal. A Monte-Carlo simulation is carried out. Figure 4(a) shows Dalitz plot of  $m^2(\pi^+\pi^0)$  versus  $m^2(\pi^-\pi^0)$ , where three black bands correspond to  $\rho(770)$  resonances. The distribution of  $m_{\pi\pi}$  is given in Fig. 4(b), in which the  $\rho(770)$  signal is clearly seen. Figure 4(c) shows contributions from the interference term, whose negative and positive parts are not equal. Numerical results show that the contribution from interference terms is less than 5% of the total width. If the interference effects are neglected, then the partial decay widths of  $J/\psi \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$  are explicitly given by

$$\frac{d\Gamma}{d\phi_3} \propto \frac{|F_{00}^{\rho \rightarrow \pi\pi}(s_{12})|^2 |F_{10}^{\psi \rightarrow \rho\pi}(s_{12})|^2 \sin^2 \theta_2^*}{|s_{12} - m_\rho^2 + i\sqrt{s_{12}}\Gamma(s_{12})|^2} + (s_{12} \rightarrow s_{13}) + (s_{12} \rightarrow s_{23}), \quad (24)$$

for  $J/\psi$  production from  $\psi'$  decays, while for  $e^+e^- \rightarrow J/\psi$ , it reads

$$\frac{d\Gamma}{d\phi_3} \propto \frac{|F_{00}^{\rho \rightarrow \pi\pi}(s_{12})|^2 |F_{10}^{\psi \rightarrow \rho\pi}(s_{12})|^2 \sin^2 \theta_2^* [1 + \cos^2 \theta_3 + \sin^2 \theta_3 \cos 2(\phi_3 + \phi_2^*)]}{|s_{12} - m_\rho^2 + i\sqrt{s_{12}}\Gamma(s_{12})|^2} + (s_{12} \rightarrow s_{13}) + (s_{12} \rightarrow s_{23}), \quad (25)$$

where  $d\phi_3$  is an element of the standard three-body phase space,  $\phi_2^*$  and  $\theta_2^*$  are the azimuthal and polar angles of a pion in the  $\rho$  helicity system.  $F_{00}$  and  $F_{10}$  are the helicity amplitudes given by

$$\begin{aligned} F_{00}^{\rho \rightarrow \pi\pi}(s) &= g_{10} \sqrt{s - 4m_\pi^2} / \sqrt{s}, \\ F_{10}^{\psi \rightarrow \rho\pi}(s) &= g_{11} \sqrt{[m_\psi^2 - (m_\pi + \sqrt{s})^2][m_\psi^2 - (\sqrt{s} - m_\pi)^2]}, \end{aligned} \quad (26)$$

where  $g_{ls}$  denotes coupling constant.

## 4 Summary

The formalisms of helicity coupling amplitudes for  $J/\psi \rightarrow \pi^+\pi^-\pi^0$  are presented. A detailed discussion are also given on the barrier factor, Breit–Wigner, and density matrix. A Monte–Carlo simulation of  $J/\psi \rightarrow \rho(770)\pi \rightarrow \pi^+\pi^-\pi^0$  is carried out. The results show that the  $\rho(770)$  resonance is well reproduced, which is in agreement with experimental data.

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