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# Statistical Separability of the World and Consistency Between Quantum Theory, Relativity, and Causality*

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***Abstract*** *We show that the quantum world with non-local states and original statistics is statistically separable. According to relativistic dynamics, the super-luminal signal transmission is impossible. The present quantum theory is therefore consistent with the relativity and the causality.*

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**Key words:** statistical separability

In 1935, Einstein *et al.*[1] assumed that the world is separable to prove the incompleteness of quantum theory. The separability there means one can always separate two systems by a space, so that a disturbance acting on one of them cannot influence another system immediately. This separability seemed to be self-evident, since otherwise the prompt signal transmission would be possible. About thirty years later, Bell[2] changed the thought experiment used in Ref. [1] to be a kind of experiment which can be really done. After that, many experimental results on the problem of separability have been reported. They showed that Einstein's assumption of separability is false. One may see this conclusion in Refs. [3] and [4] for examples. The world is proven to be non-separable in Einstein's sense. This result is almost magic, and causes diverse controversy. d'Espagnat[5] was interested in its philosophical meaning and oppugned the objectivity of the world. Stapp[6] considered its possible meaning on mind-brain interaction. More people tried their reformulations or reinterpretations of quantum theory. Examples are Refs. [7] ∼ [10]. But from the observational point of view, the real question it raises is on the consistency between the quantum theory, relativity, and the causality, and on the possibility of the super-luminal transmission of signals.

The quantum state of a system is usually defined at a given time but in the whole space, therefore is non-local and non-separable, in contrast to the classical state. Two parts of a system even-though separated by empty space may be correlated. Their states may be entangled if they had contacted and therefore interacted each other previously. People tried to use this entanglement for signal transmission, and have got experimental successes.[11] It is called quantum communication, and is much more secure[12] than the classical communication. The state entanglement is also used to design various quantum computers,[12,13] which would be much more effective and rapid.[14] However the state entanglement between space-likely separated objects itself is not a signal transmission, but a correlation between quantum states only. It manifests itself in the correlation between statistical results of measurements. The super-luminal signal transmission is still not realized, and has been proven to be impossible[15] by quantum theory, though the proof is not complete enough, especially it ignored the role played by dynamics. Some early examples may help us to distinguish the space-like correlation and the signal transmission. The phase velocity of a plane de Broglie wave for a massive particle is larger than light velocity. So do some modes of electromagnetic waves in wave guides. It is well known that they mean the correlation of phases at space-likely separated points only. No signal transmits with these waves. One may load signals onto these waves by disturbing them at one point and measure the result of disturbance at another point. This is the signal transmission. However, it means the modulation of wave amplitude or frequency. The result will propagate with group velocity, which is less than the light velocity in vacuum. The relativity and causality are satisfied. This situation makes people feel that the nonlocal quantum theory may be consistent with relativity and causality. In the following we show that this feeling is reliable, and put it on a principle of statistical separability of the world. The idea has developed from a talk given on a conference held in September 2003, and collected in its proceedings.[16]

To make the theory be Lorentz-invariant, people define the quantum state on a space-like super-surface of space-time in general. It is non-local. However, there are already various forms of relativistic quantum dynamics, such as QED, QCD, and other relativistic quantum field theories, in which the state evolution along a time-like direction is local and governed by differential equations. According to their multi-time formulation,[17−19] the state $|\sigma\rangle$ on a

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space-like super-surface $\sigma$ and the state $|\sigma'\rangle$ on a nearby space-like super-surface $\sigma'$ are related by the generalized Schrödinger equation,

$$\mathrm{i}\hbar\frac{\partial|\sigma\rangle}{\partial\sigma} = -T_{\mu\nu}n_\mu n_\nu|\sigma\rangle\,. \quad (1)$$

Here, the differential $\mathrm{d}\sigma$ denotes the infinitesimal 4-dimensional space-time volume between super-surfaces $\sigma'$ and $\sigma$ around a space-time point $x$ (see Fig. 1). In relativistic quantum field theories, the energy-momentum tensors are local functions of space-time point. In Eq. (1), $T_{\mu\nu}$ is the $\mu\nu$ element of the energy-momentum tensor operator acting on the state $|\sigma\rangle$, and $n_\mu(n_\nu)$ is the $\mu(\nu)$ component of a unit time-like 4-vector normal to the super-surface $\sigma$, both at the point $x$. The solution of Eq. (1) may be written in the form

$$|\sigma\rangle = U(\sigma,\sigma_0)|\sigma_0\rangle\,, \quad (2)$$

with an evolution operator $U(\sigma,\sigma_0)$ satisfying

$$\mathrm{i}\hbar\frac{\partial U(\sigma,\sigma_0)}{\partial\sigma} = -T_{\mu\nu}n_\mu n_\nu U(\sigma,\sigma_0)\,, \quad (3)$$

where $\sigma_0$ is an arbitrarily fixed space-like super-surface. The differential equation shows the evolution of the state from point to point. The relativity of dynamics ensures that a disturbance of the state $|\sigma_0\rangle$ at the point $(\boldsymbol{r}_0,t_0)$ on $\sigma_0$ may influence the properties of state $|\sigma\rangle$ at those points on $\sigma$ only, which are inside the light cone with its vertex at $(\boldsymbol{r}_0,t_0)$. Properties of the state $|\sigma\rangle$ at the point $(\boldsymbol{r},t)$ on $\sigma$ outside this light cone do not respond to this disturbance. $U(\sigma,\sigma_0)$ is therefore a matrix (or an integral operator), only those parts have non-zero non-diagonal elements, which relate points inside the light cone of each other. The operation of the matrix on remaining degrees of freedom equals that of a unit matrix.

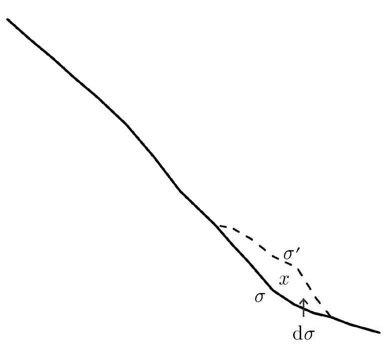


**Fig. 1** Infinitesimal 4-dimensional volume $\mathrm{d}\sigma$ between super-surfaces $\sigma'$ and $\sigma$.

A state of the system is determined by a complete measurement. For a macroscopic system, the measurement is usually incomplete. In this case, a density operator, instead of a state, is determined. The density operator is also defined on a space-like super-surface. Its time evolution is governed by the generalized von Neumann equation

$$\frac{\mathrm{d}\rho}{\mathrm{d}\sigma} = -c[T_{\mu\nu},\rho]n_\mu n_\nu\,, \quad (4)$$

in which $[A,B]\equiv(AB-BA)/\mathrm{i}\hbar$ is the quantum Poisson bracket of $A$ and $B$. From Eq. (3) one may directly verify that the solution of Eq. (4) is

$$\rho(\sigma) = U(\sigma,\sigma_0)\rho(\sigma_0)U(\sigma_0,\sigma)\,. \quad (5)$$

Since the super-surface $\sigma$ is space-like, dynamical variables on different points of it commute with each other. They belong to different degrees of freedom. Taking the direct products of the eigenstates of commuting dynamical variables at different points on $\sigma$ as bases, one may expand the state of the system. Although the bases in the state expansion are entangled in general, one can still measure the local properties of the system. The local properties of the system at the point $(\boldsymbol{r},t)$ on $\sigma$ are described by the reduced density operator

$$\rho(\boldsymbol{r}) \equiv \rho(\boldsymbol{r},t) = \mathrm{tr}_{\bar{\boldsymbol{r}}}\,\rho(\sigma)\,. \quad (6)$$

The subscript $\bar{\boldsymbol{r}}$ in Eq. (6) denotes that the trace is a sum of matrix elements which are diagonal with respect to the degrees of freedom other than those at the point $\boldsymbol{r}$. We show in the following that if $(\boldsymbol{r},t)$ is outside the light cone of the point $(\boldsymbol{r}_0,t_0)$, the disturbance at $(\boldsymbol{r}_0,t_0)$ cannot influence the reduced density matrix $\rho(\boldsymbol{r},t)$, therefore cannot influence the result of the local measurement at point $(\boldsymbol{r},t)$. We call this property the statistical separability.

Denote the degrees of freedom in a macroscopically infinitesimal neighborhood of the point $\boldsymbol{r}$ by $a$, and all the other degrees of freedom by $b$. The bases to be used in expanding the state of the system are $[|n_a,n_b\rangle\equiv|n_a\rangle|n_b\rangle]$. $n_a$ is a complete set of quantum numbers for degrees $a$ of freedom, while $n_b$ is that for degrees $b$ of freedom. In this representation, the density operator on super-surface $\sigma_0$ may be written in the form

$$\rho(\sigma_0) \equiv \sum_{n_a,n_b,n'_a,n'_b}|n'_a\rangle n'_b\rangle\rho_{n'_a n'_b;n_a n_b}\langle n_a|\langle n_b|\,, \quad (7)$$

in which $[\,\rho_{n'_a n'_b;n_a n_b}]$ are the matrix elements of $\rho(\sigma_0)$. According to the argument after Eq. (3), if $(\boldsymbol{r},t)$ is outside the light cone of the point $(\boldsymbol{r}_0,t_0)$, we may write

$$U(\sigma,\sigma_0) \equiv \sum_{n_a,n_b,n'_b}|n_a\rangle|n'_b\rangle U_{n'_b,n_b}\langle n_a|\langle n_b|\,, \quad (8)$$

$$U(\sigma_0,\sigma) \equiv \sum_{n_a,n_b,n'_b}|n_a\rangle|n'_b\rangle U^*_{n_b,n'_b}\langle n_a|\langle n_b|\,. \quad (9)$$

The relation $U(\sigma_0,\sigma)U(\sigma,\sigma_0)=1$ requires

$$\sum_{n'_b}U^*_{n'_b n''_b}U_{n'_b n_b} = \delta_{n''_b,n_b}\,. \quad (10)$$

Substituting Eqs. (7) ~ (9) into Eq. (5), then substituting the result into Eq. (6), by use of Eq. (10) and the orthonormality of base states $[|n_a\rangle|n_b\rangle]$ we obtain

$$\rho(\boldsymbol{r}) = \sum_{n_a n'_a n_b n'_b n''_b}|n'_a\rangle U_{n_b,n''_b}\rho_{n'_a n''_b;n_a n'_b}U^*_{n_b,n'_b}\langle n_a|$$
$$= \sum_{n_a n'_a n'_b}|n'_a\rangle\rho_{n'_a n'_b;n_a n'_b}\langle n_a| = \rho_0(\boldsymbol{r})\,, \quad (11)$$

in which

$$\rho_0(\boldsymbol{r}) \equiv \sum_{n_a n'_a n_b} |n'_a\rangle \rho_{n'_a n_b; n_a n_b} \langle n_a| \equiv \mathrm{tr}_{\bar{\boldsymbol{r}}} \rho(\sigma_0) \qquad (12)$$

is the reduced density operator at point $\boldsymbol{r}$ on the super-surface $\sigma_0$. Since the local reduced density operator is the complete description of the local measurement in quantum theory, equation (11) shows that the result of local measurement at a point does not change unless the point lies in the light cone of the disturbance. This is the statistical separability of the world. One may immediately conclude that although the quantum states are non-local and local states at two points separated by space-like distance may be entangled (non-separable), the communication between two space-likely separated points (super-luminal communication) is still impossible by disturbance and measurement on a quantum system. The causality in relativity is therefore ensured by the present quantum theory.

A deterministic correlation between the disturbance and the measurement at two space-likely separated points may be used for the instantaneous signal transmission. The deterministic theory with non-local state is therefore in contradiction with the relativity and the causality. Fortunately, in quantum theory, one works not only with non-local state but also with original statistics. Indeed, local disturbance may change the non-local quantum state, and therefore changes the correlation between statistical results of measurements at different points, even though they are separated by a space-like distance. However, the above proven statistical separability ensures that the statistical result of a local measurement itself is not changed by the local disturbance at another point space-likely separated from it. The non-locality of the states and the original statistics are two wings, which make the quantum theory be consistent with relativity and causality.

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