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Properties of Parabolic Linear Bound Potential and Coulomb Bound Potential Quantum Dot Qubit*

WANG Zi-Wu,[1,†] LI Wei-Ping[1], YIN Ji-Wen,[1] and XIAO Jing-Lin[2]

[1]Department of Physics and Electron Information Engineering, Inner Mongolia Chifeng College, Chifeng 024000, China

[2]Department of Physics and Electromechanics, Inner Mongolia Nationality University, Tongliao 028043, China

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Abstract *On the condition of electric-LO phonon strong-coupling in a parabolic quantum dot, we obtain the eigenenergy of the ground-state and the first-excited state, the eigenfunctions of the ground-state and the first- excited state by using variational method of Pekar type. This system in quantum dot may be employed as a two-level quantum system-qubit. When the electron is in the superposition state of the ground- and the first-excited state, we obtain the time evolution of the electron density. The relation of the probability density of electron on the Coulomb binding parameter and the relations of the period of oscillation on the Coulomb binding parameter, the electron-LO-phonon coupling constant and the confinement length are derived.*

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1 Introduction

Quantum computer has attracted considerable attention in information science field. The two-level system is usually employed as the elementary unit for storing information. Quantum computation will be realized by the laws of quantum mechanics. Several schemes have been proposed for realizing quantum computer in recent years.[1−3] In order to show the advantage of quantum computer over the most classical computer, quantum computer needs to be composed of at least thousands of qubits to be feasible. Consequently, it is clear that quantum computer with a large significant number of qubits would be more realizable in solid-state systems. However, self-assembled quantum dots (QDs) have been attracted substantial attention due to their perfect crystal structures. Therefore, it is one of the most popular solid-state quantum information research fields that qubits can be realized by solid-state devices. Many schemes have been proposed for researching quantum dot and have many kinds of contents, but they are in the initial research stage at present.

In this article, the eigenenergies and their relevant eigenwavefunctions of the ground and the first-excited state of an electron have been obtained in a parabolic quantum dot (QD) using the Pekar variational method considering the parabolic potential and the Coulomb bound potential in the electron-LO-phonon strong-coupling region. A single qubit can be envisaged as this kind of two-level quantum system in a QD. For this single-electron QD qubit, Li *et al.* presented a kind of parameter-phase diagram schemes and defined the parameters region for the use of the an InAs/GaAs as a two-level quantum system.[4,5] We have obtained the probability density of electron oscillating with a period when electron is in a superposition state of the ground and first-excited state. The relation of the probability density of electron on the Coulomb binding parameter and the relations of the period of oscillation on the Coulomb binding parameter, the electron-LO-phonon coupling constant and the confinement length are derived. Our results should be meaningful for designing the solid-state implementation of quantum computing both theoretically and experimentally.

2 Theoretical Model

We consider the system in the electrons is bounded by the parabolic potential and the Coulomb potential. The electrons are much more confined in one direction (taken as the Z-direction) than in other two directions. Therefore, we shall confine ourselves to considering only taking into account the effect of electron and LO-phonon and electron only moving on the X-Y plane. We assume that the confining potential in a single QD is parabolic

$$V_{(\rho)} = \frac{1}{2} m^* \omega_0^2 \rho^2 \,, \tag{1}$$

where m^* is the band mass of electron, ρ is the coordinate vector of a two-dimensional one and ω_0 is the confinement strength. The Hamiltonian of electron-phonon system is given by

$$H = -\frac{\hbar^2}{2m^*}\nabla_\rho^2 + \frac{1}{2} m^* \omega_0^2 \rho^2 + \sum_q \hbar\omega_{L0} b_q^\dagger b_q + \sum_q (V_q \, \mathrm{e}^{\mathrm{i}q\cdot r} b_q + \mathrm{h.c}) - \frac{e^2}{\varepsilon_\infty r} \,. \tag{2}$$

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†Corresponding author, E-mail: wangziwu211@126.com

To compute more easily, we choose the usual polaron units,[6] $\hbar = 2m^* = \omega_{\rm LO} = 1$ and $\beta = e^2/\varepsilon_\infty$ is the Coulomb binding parameter. The Hamiltonian is written as follows:

$$H = -\nabla_\rho^2 + \frac{1}{4}\omega_0^2\rho^2 + \sum_q b_q^\dagger b_q + \sum_q (V_q\,\mathrm{e}^{\mathrm{i}q\cdot r}b_q + \mathrm{h.c}) - \frac{\beta}{r}\,, \tag{3}$$

where $b_q^\dagger(b_q)$ is the creation (annihilation) operator of bulk LO-phonon with the wave vector q ($q = q_\parallel, q_\perp$), $r = (\rho, z)$ is the coordinate of the electron, $-\beta/r$ is the Coulomb bound potential, and

$$q = \frac{\mathrm{i}}{q}\Big(\frac{4\pi\alpha}{V}\Big)^{1/2}, \tag{4}$$

$$\alpha = \frac{e^2}{2}\Big(\frac{1}{\varepsilon_\infty} - \frac{1}{\varepsilon_0}\Big). \tag{5}$$

Using the Fourier expansion to the Coulomb bound potential in Eq. (2),

$$-\frac{\beta}{r} = -\sum_q \frac{4\pi\beta}{Vq^2}\exp(-\mathrm{i}q\cdot r)\,. \tag{6}$$

Using the LLP transformation to Eq. (2),

$$U = \exp\Big[\sum_q (f_q b_q^\dagger - f_q^* b_q)\Big], \tag{7}$$

where f_q will be treated as a variational function, we have

$$H' = U^{-1}HU\,. \tag{8}$$

Supposing that the Gaussian function approximation is valid in the ground-state of electron-phonon system by variational method of the Pekar type to be

$$|\varphi_{e-p}\rangle = \frac{\lambda}{\sqrt{\pi}}\exp\Big(-\frac{\lambda^2\rho^2}{2}\Big)|\xi(z)\rangle|0_{\rm ph}\rangle\,, \tag{9}$$

where λ is the variational parameter, since the electrons are much more strongly confined in Z-direction than in other two directions and are considered to be confined in an infinitesimally narrow layer, so $\langle\xi(z)|\xi(z)\rangle = \delta(z)$. And $|0_{\rm ph}\rangle$ is unperturbed zero phonon state which satisfies $b_q|0_q\rangle = 0$. We then obtain the electron ground-state energy as the following form

$$E_0(\lambda) = \langle\varphi_{e-p}|H'|\varphi_{e-p}\rangle = \lambda^2 + \frac{1}{\lambda^2 l_0^4} - \frac{1}{2}(2\pi)^{1/2}\alpha\lambda - \beta\sqrt{\pi}\lambda\,, \tag{10}$$

where $l_0 = (2/\omega_0)^{1/2}$. We have the electron ground-state energy,

$$E_0 = \lambda_0^2 + \frac{1}{\lambda_0^2 l_0^4} - \frac{1}{2}(2\pi)^{1/2}\alpha\lambda_0 - \beta\sqrt{\pi}\lambda_0\,. \tag{11}$$

Similarly, the trial wave-function of electron-phonon system in the first-excited state may be chosen as

$$|\varphi_{e-p}\rangle' = \frac{\lambda^2}{\sqrt{\pi}}\rho\exp\Big(-\frac{\lambda^2\rho^2}{2}\Big)\exp(\pm\mathrm{i}\phi)|\xi(z)\rangle|0_{\rm ph}\rangle\,. \tag{12}$$

This satisfies the following relations

$$\langle\varphi_{e-p}|\varphi_{e-p}\rangle' = 0\,, \quad '\langle\varphi_{e-p}|\varphi_{e-p}\rangle' = 1\,. \tag{13}$$

We can obtain the energy in the first-excited state by using the $E_1 = '\langle\varphi_{e-p}|H'|\varphi_{e-p}\rangle'$,

$$E_1(\lambda) = 2\lambda^2 + \frac{2}{\lambda^2 l_0^4} - \frac{11}{32}(2\pi)^{1/2}\alpha\lambda - \frac{\beta\sqrt{\pi}\lambda}{2}\,. \tag{14}$$

The first-excited state energy of electron in a parabolic QD can be written as

$$E_1 = 2\lambda_0^2 + \frac{2}{\lambda_0^2 l_0^4} - \frac{11}{32}(2\pi)^{1/2}\alpha\lambda_0 - \frac{\beta\sqrt{\pi}\lambda_0}{2}\,. \tag{15}$$

We can obtain λ_0 by the variational method. Then we can get the eigenlevel and the eigenwavefunction. Then, we obtain the two-level system needed by a single qubit. The superposition state of electron can be expressed as

$$|\psi_{01}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,, \tag{16}$$

where

$$|0\rangle = \varphi_0(\rho) = \frac{\lambda_0}{\sqrt{\pi}}\exp\Big(-\frac{\lambda_0^2\rho^2}{2}\Big)\,, \tag{17}$$

$$|1\rangle = \varphi_1(\rho) = \frac{\lambda_0^2}{\sqrt{\pi}}\rho\exp\Big(-\frac{\lambda_0^2\rho^2}{2}\Big)\exp(\pm\mathrm{i}\phi)\,. \tag{18}$$

The time evolution of the quantum state of the electron can be written as

$$\psi_{01}(t,\rho) = \frac{1}{\sqrt{2}}\varphi_0(\rho)\exp\Big(-\frac{\mathrm{i}E_0 t}{\hbar}\Big) + \frac{1}{\sqrt{2}}\varphi_1(\rho)\exp\Big(-\frac{\mathrm{i}E_1 t}{\hbar}\Big)\,. \tag{19}$$

The probability density is in the following form,

$$Q(\rho,t) = |\psi_{01}(t,\rho)|^2 = \frac{1}{2}[|\varphi_0(\rho)|^2 + |\varphi_1(\rho)|^2 + \varphi_0^*(\rho)\varphi_1(\rho)\exp(\mathrm{i}\omega_{01}t) + \varphi_0(\rho)\varphi_1^*(\rho)\exp(-\mathrm{i}\omega_{01}t)]\,, \tag{20}$$

where

$$\omega_{01} = \frac{E_1 - E_0}{\hbar}\,. \tag{21}$$

3 Results and Discussions

The numerical results of the probability density of electron and the period of oscillation versus the Coulomb binding parameter, the electron-LO-phonon coupling constant and the confinement length in a parabolic QD are presented in Figs. 1.

The left peaks of Figs. 1(a) $\sim$ 1(e) show the time evolution of the electron probability density $|\psi_{01}(t,x,y,0)|^2$ when electrons exist in the superposition state of $(1/\sqrt{2})(|0\rangle + |1\rangle)$ for electron-LO-phonon coupling constant $\alpha = 6$, the confinement length $l_0 = 0.5$, phase difference $\phi = 2\pi$, the Coulomb binding parameter $\beta = 0$ (that is, without Coulomb bound potential). The probability density of electron oscillated with the period of

oscillation $T_0 = h/(E_1 - E_0)$, the time t in the left peaks of Figs. 1(a) $\sim$ 1(e) is 0, 0.25, 0.5, 0.75, and $1T_0$, respectively. The right peaks of Figs. 1(a) $\sim$ 1(e) show the time evolution of the electron probability density $|\psi_{01}(t, x, y, 0)|^2$ when electrons exist in the superposition state of $(1/\sqrt{2})(|0\rangle + |1\rangle)$ for electron-LO-phonon coupling constant $\alpha = 6$, the confinement length $l_0 = 0.5$, phase difference $\phi = 2\pi$, the Coulomb binding parameter $\beta = 0.5$ (that is, with Coulomb bound potential). The probability density of electron oscillates with the period of oscillation $T_0 = h/(E_1 - E_0)$, the time t in the right peaks of Figs. 1(a) $\sim$ 1(e) is 0, 0.25, 0.5, 0.75, and $1T_0$, respectively. From Fig. 2, we find the probability density of electron in the superposition state is enhanced because of the Coulomb bound potential.

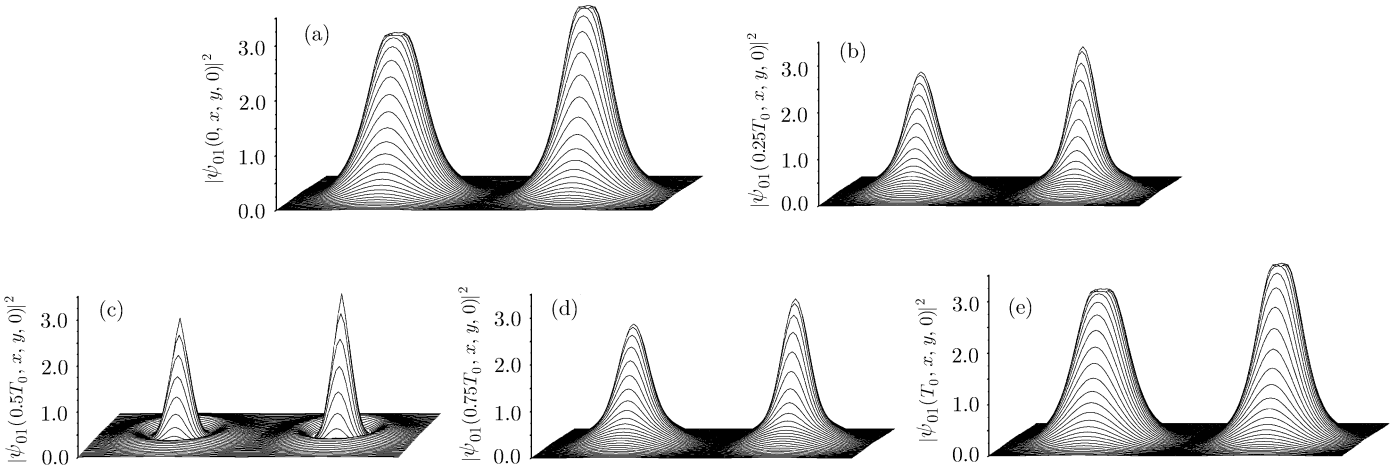

Fig. 1 The time evolution of the electron probability density in superposition of $|0\rangle$ and $|1\rangle$.

Figure 2 presents that the period of oscillation as a function of the electron-LO-phonon coupling constant for the confinement length $l_0 = 0.5$, the Coulomb binding parameter $\beta = 0$. and the confinement length $l_0 = 0.5$, the Coulomb binding parameter $\beta = 0.5$.

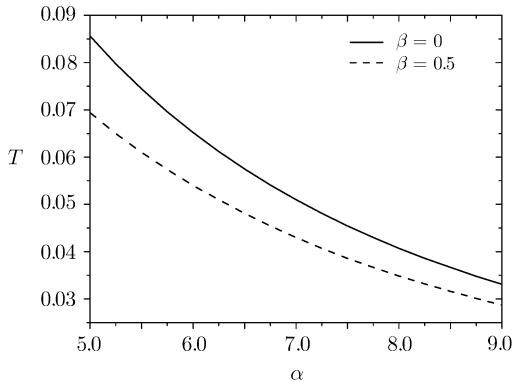

Fig. 2 The period of oscillation in a parabolic quantum dot as a function of the electron-LC-phonon coupling constant.

From Fig. 2, it can be seen that the period of oscillation increases with decreasing the electron-LO-phonon coupling constant. This is because the electron-LO-phonon coupling strength in the first-excited state is weaker than that in the ground-state and the increase in the first-excited state is smaller than that in the ground-state with increasing the coupling strength.[7,8] So the increasing of the energy spacing between the first-excited and ground-state causes the reduction of the period of oscillation. Meanwhile, we also see that the period of oscillation reduces because of the Coulomb bound potential. As a result of the Coulomb bound potential, the ground- and the first-excited state energies are reduced and the influence is greater to the ground-state energy. For this reason, the energy spacing between the ground- and the first-excited state increases and the period of oscillation decreases.

Figure 3 plots the period of oscillation T_0 as a function of the confinement length for $\alpha = 6$, Coulomb binding parameter $\beta = 0$ and $\alpha = 6$, Coulomb binding parameter $\beta = 0.5$. From Fig. 3, one finds that the period of oscillation increases with increasing the confinement length. For the same reason as in Fig. 2, the period of oscillation T_0 decreases because of the Coulomb bound potential.

Figure 4 shows the period of oscillation T_0 as a function of the Coulomb binding parameter for the electron-LO-phonon coupling constant $\alpha = 6$ and the confinement length $l_0 = 0.5$. It is seen that the period of oscillation decreases with increasing the Coulomb binding parameter.

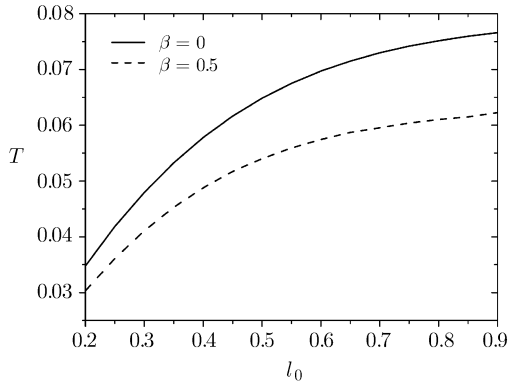

Fig. 3 The period of oscillation in a parabolic quantum dot as a function of the confinement length.

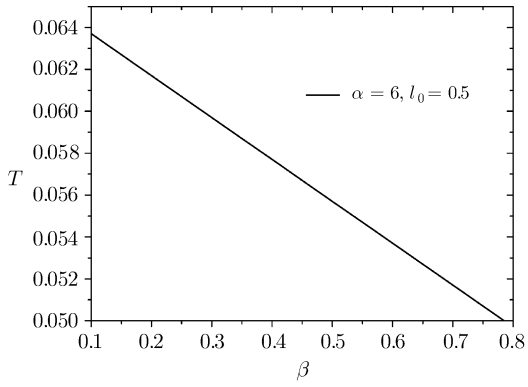

Fig. 4 The period of oscillation in a parabolic quantum dot as a function of the confinement length.

Coulomb bound potential usually exists in real crystals and it is believed that it is because of the hydrogenlike impurities in them. The above discussions indicate the probability density of electron increases and the period of oscillation decreases in superposition state of electron a QD due to the existence of the Coulomb bound potential. However, a qubit cannot be independent of environment and must be interacted with the heat bath. As a result, the interaction destroys the superposition state of a qubit, which is decoherence.[9] The period of oscillation T_0 decreases, that is the life time of a qubit reduces, so the process of decoherence is quicken. It is very harmful to store information which makes the QD as its elementary unit. For this reason, we should try our best to make the crystal not contain impurities when making the QD as elementary unit of storing information using one kind of crystal material. This is in agreement with discussions in the literature.[10]

4 Summary

The energies and the relevant eigenwavefunctions of the ground- and the first-excited state of electron have been obtained in a parabolic QD using the Pekar variational method considering the parabolic potential and the Coulomb bound potential in the electron-LO-phonon strong-coupling region. The single qubit can be envisaged as this kind of two-level quantum system in a QD. The probability density of electron oscillates with a period when the electron is in the superposition state of the ground and the first-excited state. The probability density of electron increases in the superposition because of the Coulomb bound potential. The results indicate that the period of oscillation increases with decreasing the electron-LO-phonon coupling constant, decreasing the Coulomb binding parameter and increasing the confinement length.

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