

The Mass Bounds on Exotic Particles in an $SU(3)_L \times U(1)_X$ Model

CHEN Feng-Zhi

Department of Physics, Zhejiang University, Hangzhou 310027, China

(Received December 29, 2000)

Abstract We study an $SU(3)_L \times U(1)_X$ electroweak model. By requiring $M_{Z'}^2 - M_W^2 / \cos^2 \theta_w$ to be less than experimental value we obtain a lower bound on Z' boson mass, $M_{Z'} > 600$ GeV. The relation between $M_{Z'}$ and M_U (M_V) then gives a lower bound on M_U (M_V): M_U (M_V) > 490 GeV. From the constraint $\sin^2 \theta_w(M_{Z'}) < 0.3$, the upper bounds on $M_{Z'}$ and M_U (M_V) are computed to be $M_{Z'} < 5.8 \times 10^4$ TeV and M_U (M_V) $< 4.6 \times 10^4$ TeV. We estimate further the $K_L - K_S$ mass difference due to Z' exchange and try to use the result to obtain stronger lower bounds. On cosmological grounds we find that $M_N < 390$ eV for $T_f = 2.3$ GeV and $M_N < 740$ eV for $T_f = 300$ GeV.

PACS numbers: 12.60.-i, 12.60.Fr

Key words: exotic gauge boson, N -particle, mass bound

1 Introduction

Ever since the advent of the standard model (SM) of electroweak interactions various extensions of it have been considered. An extension called $SU(3)_L \times U(1)_X$ model was introduced for the first time in Ref. [1], and several new versions of the model were suggested^[2] in early 1990s. This sort of model has an interesting feature: an anomaly-free model can be obtained only if the number of families is a multiple of three. It provides a possible answer to the family question. Here we will focus on an $SU(3)_L \times U(1)_X$ model we proposed recently.^[3] The original motivation leading to the proposal was to provide an economic accommodation of the dark matter particle. Hence we introduced two neutral leptons in each family. And we found that the extra neutral lepton (N -particle) can be chosen as a candidate of dark matter particle. Our aim in this paper is to place mass bounds on the exotic gauge bosons and the N -particle.

This paper is organized as follows. In Sec. 2 we briefly

review this model. The exotic gauge bosons and their mass bounds are discussed in Sec. 3. In Sec. 4 the flavor-changing neutral current processes are described. Also the $K_L - K_S$ mass difference due to Z' exchange is estimated and used to discuss possible lower mass bounds on the exotic gauge bosons. Section 5 is devoted to deriving cosmological bound on N -particle mass. Finally, the conclusions are presented in Sec. 6.

2 Review of the Model

We first give a brief review of the model. The three families of leptons transform under $SU(3)_L \times U(1)_X$ as

$$L_i = \begin{bmatrix} \nu_i \\ l_i \\ N_i^c \end{bmatrix}_L \sim \left(3, -\frac{2}{3} \right),$$

$$\nu_{iR} \sim (1, 0) \quad l_{iR}^- \sim (1, -2), \quad N_{iR}^c \sim (1, 0)$$

with $i = e, \mu, \tau$. The quarks are given by the following representations under $SU(3)_L \times U(1)_X$,

$$\begin{aligned} Q_{1L} &= \begin{pmatrix} d^\alpha \\ u^\alpha \\ P_1^\alpha \end{pmatrix}_L \sim (\bar{3}, 0), & d_R^\alpha &\sim (1, -2/3), & U_R^\alpha &\sim (1, 4/3), & P_{1R}^\alpha &\sim (1, -2/3), \\ Q_{2L} &= \begin{pmatrix} S^\alpha \\ C^\alpha \\ P_2^\alpha \end{pmatrix}_L \sim (\bar{3}, 0), & S_R^\alpha &\sim (1, -2/3), & C_R^\alpha &\sim (1, 4/3), & P_{2R}^\alpha &\sim (1, -2/3), \\ Q_{3L} &= \begin{pmatrix} t^\alpha \\ b^\alpha \\ P_3^\alpha \end{pmatrix}_L \sim (3, 2/3), & t_R^\alpha &\sim (1, 4/3), & b_R^\alpha &\sim (1, -2/3), & P_{3R}^\alpha &\sim (1, 4/3), \end{aligned}$$

where α is $SU(3)_c$ color index. It is useful to note that there exists a mixing between neutrinos and N -particles. For simplicity we will work with only the first lepton family. Then the mass eigenstates $(\tilde{\nu}, \tilde{N})$ are related to the weak eigenstates (ν, N) by

$$\tilde{\nu} = \nu \cos \varphi - N \sin \varphi, \quad \tilde{N} = \nu \sin \varphi + N \cos \varphi,$$

where φ is a mixing angle.

In the Higgs sector we first introduce a Higgs triplet ϕ_3 which transforms as $\phi_3 \sim (3, -2/3)$. When ϕ_3 gets the vacuum expectation value (VEV),

$$\langle \phi_3 \rangle_0 = \begin{bmatrix} 0 \\ 0 \\ v_3/\sqrt{2} \end{bmatrix},$$

the exotic quarks ($P_{1,2,3}$) acquire masses and the gauge symmetry is broken down to $SU(2)_L \times U(1)_Y$, where $Y = -(1/\sqrt{3})\lambda^8 + X$ is the hypercharge.

We then introduce two more Higgs triplets ϕ_1 and ϕ_2 , which transform as $\phi_1 \sim (3, -2/3)$ and $\phi_2 \sim (3, 4/3)$, respectively. When ϕ_1 and ϕ_2 get VEVs,

$$\langle \phi_1 \rangle_0 = \begin{bmatrix} v_1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}, \quad \langle \phi_2 \rangle_0 = \begin{bmatrix} 0 \\ v_2/\sqrt{2} \\ 0 \end{bmatrix},$$

all quarks acquire masses. However not all of the leptons acquire masses. To remedy this defect we introduce the fourth Higgs scalar S , which transforms as $(6, -4/3)$ and has VEV of the form

$$\langle S \rangle_0 = \begin{bmatrix} \frac{1}{2}v_4 \\ \frac{1}{2}v_4 \end{bmatrix}.$$

This VEV together with $\langle \phi_1 \rangle_0$ and $\langle \phi_2 \rangle_0$ gives leptons' acceptable masses and breaks the electroweak gauge sym-

metry down to $U(1)_Q$.

3 Gauge Bosons

In this section we discuss gauge bosons and their masses. To start with, we have an octet W_μ^a ($a = 1, 2, \dots, 8$) of massless bosons associated with $SU(3)_L$ and a massless singlet X_μ associated with $U(1)_X$. The covariant derivative for a triplet is defined as

$$D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} W_\mu^a - ig_x \frac{X}{2} X_\mu,$$

where λ^a are the Gell-Mann matrices, g and g_x are the $SU(3)_L$ and $U(1)_X$ coupling constants respectively. By matching the gauge coupling constants at the $SU(3)_L \times U(1)_X$ breaking we get the matching condition

$$\frac{1}{g'^2} = \frac{1}{3g^2} + \frac{1}{g_x^2},$$

where g' is $U(1)_Y$ coupling constant with $g'/g = \text{tg } \theta_w$.

After electroweak breaking the gauge bosons $W_\mu^\pm = (1/\sqrt{2})(W_\mu^1 \mp iW_\mu^2)$, $U_\mu^0 = (1/\sqrt{2})(W_\mu^4 - iW_\mu^5)$, $\bar{U}_\mu^0 = (1/\sqrt{2})(W_\mu^4 + iW_\mu^5)$ and $V_\mu^\pm = (1/\sqrt{2})(W_\mu^6 \pm iW_\mu^7)$ acquire the masses

$$M_W^2 = \frac{1}{4}g^2(v_1^2 + v_2^2 + v_4^2), \quad M_V^2 = \frac{1}{4}g^2(v_2^2 + v_3^2 + v_4^2), \quad M_U^2 = \frac{1}{4}g^2(v_1^2 + v_3^2 + 4v_4^2).$$

On the other hand, the neutral gauge bosons have the following mass-squared matrix in the (W^3, W^8, X) basis

$$M^2 = \frac{1}{4}g^2 \begin{bmatrix} v_1^2 + v_2^2 + v_4^2 & \frac{1}{\sqrt{3}}(v_1^2 - v_2^2 - v_4^2) & -\frac{2}{3}t(v_1^2 + 2v_2^2 + 2v_4^2) \\ \frac{1}{\sqrt{3}}(v_1^2 - v_2^2 - v_4^2) & \frac{1}{3}(v_1^2 + v_2^2 + 4v_3^2 + v_4^2) & -\frac{2}{3\sqrt{3}}t(v_1^2 - 2v_2^2 - 2v_3^2 - 2v_4^2) \\ -\frac{2}{3}t(v_1^2 + 2v_2^2 + 2v_4^2) & -\frac{2}{3\sqrt{3}}t(v_1^2 - 2v_2^2 - 2v_3^2 - 2v_4^2) & \frac{4}{9}t^2(v_1^2 + 4v_2^2 + v_3^2 + 4v_4^2) \end{bmatrix},$$

where $t = g_x/g$. By partially diagonalizing this matrix we can identify three physical states A (photon), Z and Z' ,

$$\begin{aligned} A_\mu &= W_\mu^3 \sin \theta_w + \left(-\frac{1}{\sqrt{3}}W_\mu^8 \text{tg } \theta_w + X_\mu \sqrt{1 - \frac{1}{3} \text{tg}^2 \theta_w} \right) \cos \theta_w, \\ Z_\mu &= W_\mu^3 \cos \theta_w - \left(-\frac{1}{\sqrt{3}}W_\mu^8 \text{tg } \theta_w + X_\mu \sqrt{1 - \frac{1}{3} \text{tg}^2 \theta_w} \right) \sin \theta_w, \\ Z'_\mu &= -W_\mu^8 \sqrt{1 - \frac{1}{3} \text{tg}^2 \theta_w} - \frac{1}{\sqrt{3}}X_\mu \text{tg } \theta_w. \end{aligned}$$

In the (A, Z, Z') basis the mass matrix for Z and Z' is

$$\mathcal{M}^2 = \begin{bmatrix} M_Z^2 & M_{ZZ'}^2 \\ M_{ZZ'}^2 & M_{Z'}^2 \end{bmatrix} \quad (1a)$$

with

$$\begin{aligned} M_Z^2 &= \frac{1}{4}(g^2/\cos^2 \theta_w)(v_1^2 + v_2^2 + v_4^2), \\ M_{ZZ'}^2 &= \frac{1}{4}(g^2/\cos^2 \theta_w)[v_1^2(2\sin^2 \theta_w - 1) + v_2^2 + v_4^2]/\sqrt{3 - 4\sin^2 \theta_w}, \\ M_{Z'}^2 &= \frac{1}{4}(g^2/\cos^2 \theta_w)[-4v_1^2 \sin^2 \theta_w \cos^2 \theta_w + (v_1^2 + v_2^2 + v_4^2) + 4v_3^2 \cos^4 \theta_w]/(3 - 4\sin^2 \theta_w). \end{aligned} \quad (1b)$$

The mass eigenstates are

$$Z_{1\mu} = Z_\mu \cos \theta - Z'_\mu \sin \theta, \quad Z_{2\mu} = Z_\mu \sin \theta + Z'_\mu \cos \theta,$$

where Z_1 corresponds to the neutral gauge boson of the SM and Z_2 is a heavy partner.

We now turn to the estimate of mass bounds on the exotic gauge bosons. It is reasonable to set $v_1 = v_4 = 0$ in

Eq. (1), since the symmetry breaking hierarchy requires $v_3 \gg v_{1,2} > v_4$. Thus \mathcal{M}^2 becomes

$$\mathcal{M}^2 = \begin{bmatrix} M_Z^2 & M_Z^2/\sqrt{3 - 4\sin^2 \theta_w} \\ M_Z^2/\sqrt{3 - 4\sin^2 \theta_w} & M_{Z'}^2 \end{bmatrix}, \quad (2)$$

where $M_Z^2 = (1/4)g^2v_2^2/\cos^2 \theta_w = M_W^2/\cos^2 \theta_w$. From

Eq. (2) we can deduce that the mass shift is of the order

$$\Delta M^2 = M_{Z'_1}^2 - M_Z^2 = -M_Z^4/[M_{Z'}^2(3 - 4\sin^2\theta_w)]$$

with $M_Z = M_W/\cos\theta_w$. Requiring ΔM^2 to be less than experimental value^[4] we get the lower bound on $M_{Z'}$,

$$M_{Z'} > 600 \text{ GeV}. \quad (3)$$

In the approximation $v_3 \gg v_{1,2} > v_4$ we have

$$M_U \simeq M_V \simeq M_{Z'}\sqrt{3 - 4\sin^2\theta_w}/2\cos\theta_w, \quad (4)$$

which implies, together with Eq. (3), the following lower bounds $M_U (M_V) > 490 \text{ GeV}$. We will see that these lower bounds can be improved by considering flavor-changing neutral currents (FCNC).

4 FCNC Processes

We first write down the gauge interactions for Z and Z' bosons explicitly.

(i) Gauge interaction of Z boson. The interaction Lagrangian among fermions and Z is

$$\mathcal{L}(Z) = -(g/2\cos\theta_w)Z_\mu\bar{\psi}\gamma^\mu[g_L(\psi)(1 - \gamma_5) + g_R(\psi)(1 + \gamma_5)]\psi.$$

The neutral current coefficients are given as

$$g_{L,R} = T^3(\psi_{L,R}) - Q(\psi)\sin^2\theta_w,$$

where T^3 is the weak isospin of fermion ψ and Q is the charge of ψ in units e . It is easy to verify that for usual fermions the respective coefficients g_L and g_R coincide with those of the SM, hence there is no FCNC coupled to Z boson.

(ii) Gauge interaction of Z' boson. The interaction Lagrangian among fermions and Z' is

$$\mathcal{L}(Z') = -(g/2\cos\theta_w)Z'_\mu\bar{\psi}\gamma^\mu[g'_L(\psi)(1 - \gamma_5) + g'_R(\psi)(1 + \gamma_5)]\psi,$$

where the new neutral current coupling coefficients are given as

$$g'_{L,R} = \frac{1}{2}\sqrt{3 - 4\sin^2\theta_w}Y(\psi_{L,R}) - \frac{3}{2}\frac{\cos^2\theta_w}{\sqrt{3 - 4\sin^2\theta_w}}X(\psi_{L,R}).$$

In the left-handed sector the third family has different coupling to Z' from the other two families. This leads to the FCNCs. In particular, the FCNC in the down sector is

$$\mathcal{L}_{\text{FCNC}} = -\frac{g}{2\cos\theta_w}Z'_\mu\delta_L U_{3d_i}^* U_{3d_j} \bar{d}_i \gamma^\mu (1 - \gamma_5) d_j \quad (5)$$

and

$$\delta_L = g'_L(d_3) - g'_L(d_1) = -\cos^2\theta_w/\sqrt{3 - 4\sin^2\theta_w},$$

where $d_i (= d, s, b)$ are mass eigenstates. U is the unitary matrix that relates mass eigenstates to weak eigenstates. There is no FCNC for right-handed currents, since usual right-handed fermions transform identically.

As an example of FCNC process we consider the $K_L - K_S$ mass difference. The Z' exchange contribution

to the mass difference appears in Fig. 1. From Eq. (5) we can deduce the corresponding effective Lagrangian

$$\mathcal{L}_{\text{eff}} = -\frac{g^2}{4M_{Z'}^2\cos^2\theta_w}\delta_L^2|U_{3d}^*U_{3s}|^2[\bar{d}\gamma_\mu(1 - \gamma_5)S]^2,$$

which gives the physical $K^0 \leftrightarrow \bar{K}^0$ transition amplitude

$$A(K^0 \leftrightarrow \bar{K}^0) = \langle \bar{K}^0 | \mathcal{H}_{\text{eff}}(\bar{d}S \leftrightarrow d\bar{S}) | K^0 \rangle \quad (6)$$

with $\mathcal{H}_{\text{eff}} = -\mathcal{L}_{\text{eff}}$. The matrix element in Eq. (6) can be estimated by the ‘‘vacuum insertion’’ approximation.^[5] The $K_L - K_S$ mass difference is then computed by means of the equation

$$\Delta m_K = m_L - m_S = 2\text{Re} A(K^0 \leftrightarrow \bar{K}^0),$$

which gives

$$\Delta m_K = \frac{16}{3}\frac{G_F}{\sqrt{2}}\frac{M_W^2\delta_L^2}{M_{Z'}^2\cos^2\theta_w}|U_{3d}^*U_{3s}|^2f_K^2m_K, \quad (7)$$

where $f_K = 1.23 m_\pi$. Assuming $\Delta m_K \leq 1 \times 10^{-15} \text{ GeV}$ we get from Eq. (7),

$$M_{Z'}^2 > 1.55 \times 10^{12}|U_{3d}^*U_{3s}|^2 (\text{GeV})^2.$$

In Table 1 we give our results for the lower bound for a number of values of $|U_{3d}^*U_{3s}|$. We see that stronger lower bounds on $M_{Z'}$, hence on M_U and M_V , can be obtained, if $|U_{3d}^*U_{3s}| \geq 1 \times 10^{-3}$.

At the end of this section we will derive upper mass bounds on the exotic gauge bosons. To do this we assume a normalization prompted by potential unification in a more complete theory^[6]

$$\frac{1}{4}g_x^2 \text{Tr}(X)^2 = g^2 \text{Tr}(T_3)^2,$$

which gives $g_x^2 = \frac{1}{2}g^2$ for the fermion representations in our model. Combining with the matching conditions we find that $\sin^2\theta_w$ should be less than 0.3. With this constraint the upper bound on $M_{Z'}$ can be computed by considering the one-loop running of $\sin^2\theta_w$. Including the Higgs multiplets and taking $\sin^2\theta_w(M_Z) = 0.231$ and $\alpha^{-1} = 127.9$ we find $M_{Z'} < 5.8 \times 10^4 \text{ TeV}$. Equation (4) then gives $M_U (M_V) < 4.6 \times 10^4 \text{ TeV}$.

Table 1 The lower bound on $M_{Z'}$ for different values of $|Y_{3d}^*U_{3s}|$.

$ U_{3d}^*U_{3s} $	$M_{Z'} (\text{TeV})$
0.004×0.035	0.2
4.8×10^{-4}	0.6
1×10^{-3}	1.3
1×10^{-2}	12.5

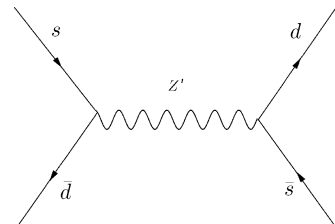


Fig. 1 Diagram responsible for $K^0 - \bar{K}^0$ transition due to Z' exchange.

5 Cosmological Bound on N -Particle Mass

We now proceed to derive the upper bound on the N 's mass by invoking cosmological theories. We assume that the N is still relativistic when it decouples. The present mass density of the N is then^[7]

$$\begin{aligned} \rho_{N_0} &= n_{N_0} M_N = 825 [g_{\text{eff}}/g_{*s}(T_f)] (M_N/\text{eV}) \text{eV} \cdot \text{cm}^{-3}, \\ \Omega_{N_0} h^2 &= 7.83 \times 10^{-2} [g_{\text{eff}}/g_{*s}(T_f)] (M_N/\text{eV}), \end{aligned} \quad (8)$$

where $\Omega_{N_0} = \rho_{N_0}/\rho_c$. Since $\Omega_{N_0} h^2 < \Omega_0 h^2 \lesssim 1$, we get from Eq. (8),

$$M_N \lesssim 12.8 [g_{*s}/g_{\text{eff}}(T_f)] \text{eV}.$$

We see that the upper bound of N 's mass depends on freeze-out temperature T_f , which is related to ν - N mixing angle ϕ . The upper bound of the ϕ can be estimated by considering the μ -decay experiment. We note that the μ -decay rate ($1/\tau_2$) in our model differs from $1/\tau_1$ in the SM by a factor $\cos^4 \phi$. Hence the difference in lifetimes can be approximated by

$$\Delta\tau = \tau_2 - \tau_1 = 2\tau\phi^2,$$

since $\phi \ll 1$. The experimental value for τ is $(2.19703 \pm 0.00004) \times 10^{-6}$ s. Therefore $\phi \lesssim 3 \times 10^{-3}$.

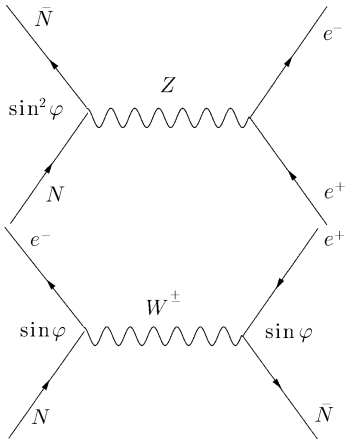


Fig. 2 Diagrams contributing to $N\bar{N}$ annihilation.

We now compute the upper mass bound of the N for two cases.

(i) $T_f \ll M_W$. We assume that Z - Z' mixing angle is so small that the dominant reactions which contribute to $N\bar{N}$ annihilation are those described by the diagrams in Fig. 2. The cross section for these reactions is given by

$\sigma \simeq G_F^2 T^2 \sin^4 \phi$, where G_F is the Fermi constant. The N 's number density is $n \simeq T^3$. So the annihilation rate (per N -particle) is

$$\Gamma = n\sigma|v| \simeq G_F^2 T^5 \sin^4 \phi.$$

The ratio of Γ to the expansion rate is

$$\frac{\Gamma}{H} = \frac{G_F^2 T^5 \sin^4 \phi}{T^2/M_{\text{pl}}} \simeq (T \sin^{4/3} \phi / 1 \text{ MeV})^3,$$

where M_{pl} is the Planck mass. Since the N decouples when $\Gamma/H \simeq 1$, we deduce that

$$T_f = 1/\sin^{4/3} \phi \text{ MeV}.$$

As an illustrative value we choose $\phi \simeq 3 \times 10^{-3}$. Then $T_f = 2.3$ (GeV) $\ll M_W$. At this temperature $g_{*s} = 91.5$. For a single four-component N -particle $g_{\text{eff}} = 3$. This implies that

$$M_N < 390 \text{ eV}.$$

(ii) $T_f \gg M_W$. On dimensional grounds the cross section of $N\bar{N}$ annihilation is

$$\sigma \simeq G_F^2 M_W^2 \sin^4 \phi / T^2.$$

Following the same steps as in the case (i) we find $T_f = 300$ GeV (Choose $\sin \phi = 2.26 \times 10^{-4}$) and $g_{*s} = 174$ (Assume that all particles have masses less than 300 GeV except $U^0, \bar{U}^0, V^\pm, Z'$ and ϕ_3). This implies that

$$M_N < 740 \text{ eV}.$$

Finally, we note that our results are insensitive to the freeze-out temperature.

6 Conclusions

So far we have considered an $SU(3)_L \times U(1)_X$ model in detail. We first derive the mass expressions for gauge bosons. By requiring ΔM^2 to be less than the experimental value we get a lower bound on $M_{Z'}$. The relation between $M_{Z'}$ and M_U (M_V) then gives a lower bound on M_U (M_V). Furthermore the gauge interactions for Z and Z' bosons are recalled and then used to estimate the $K_L - K_S$ mass difference due to the exchange of Z' boson. Next we assume a new normalization, which together with the matching conditions gives $\sin^2 \theta_w(M_{Z'}) < 0.3$. From this constraint an upper bound on $M_{Z'}$ is obtained. Equation (4) then gives an upper bound on M_U and M_V . Finally, we get an upper bound on N 's mass on cosmological grounds.

References

- [1] J. Schechter and Y. Ueda, Phys. Rev. **D8** (1973) 484.
- [2] CHEN Feng-Zhi, NIE Chuan-Hui and WANG Ping, High Energy Phys. Nucl. Phys. **15** (1991) 259; F. Pisano and V. Pleitez, Phys. Rev. **D46** (1992) 410; P.H. Frampton, Phys. Rev. Lett. **69** (1992) 2889.
- [3] CHEN Feng-Zhi, Phys. Lett. **B442** (1998) 223.
- [4] *Particle Physics Booklet*, (1998) p. 8.
- [5] R.E. Shrock and S.B. Treiman, Phys. Rev. **D19** (1979) 2148.
- [6] See Frampton (Ref. [2]).
- [7] E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley, Redwood City, California (1990).