

Subluminal and Superluminal Phenomena in a Four-Level Atom*

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Abstract In a four-level atomic system, we investigate the light pulse propagation properties interacting with only one laser field. It is shown that in the steady state, the group velocity of the light pulse can be changed from subluminal to superluminal by varying the field detuning. Meanwhile, the effects of the field intensity on the group velocity are also shown. At last, with special parameters, the analytical solution for the group index is also obtained.

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1 Introduction

The advance of laser technology has stimulated further development and understanding of the propagation of optical pulses under various conditions. Especially, the velocity of light has been thoroughly studied. Many theories and experiments have shown that it is possible to slow down the group velocity,^[1–4] or even stop the light propagation.^[5–7] With electromagnetically induced transparency (EIT), Kasapi *et al.* observed a group velocity of $c/165$ in Pb vapor cell.^[8] Hau *et al.* observed a group velocity of 17 m/s in a Bose–Einstein condensate.^[9] Also, Turukhin *et al.* demonstrated the propagation of slow light through a solid-state material—Pr doped Y_2SiO_5 .^[10]

In contrast to the subluminal light propagation, superluminal phenomena have also been investigated widely. In this case, the group velocity can be faster than vacuum speed (c) without contradicting the causality principle.^[11] For example, using gain-assisted linear anomalous dispersion, the superluminal pulse propagation can be achieved in a resonant medium when the gain is present.^[12] In Ref. [13], a scheme was proposed to show that the propagation of light can be switched from subluminal to superluminal. Bigelow *et al.* reported an observation of both slow and superluminal light propagation at room temperature in an alexandrite crystal.^[14,15] G.S. Agarwal and T.N. Dey modeled sub- and super-luminal propagation of intense pulses in media with saturated and reverse absorption,^[16] which explained the experiments of Bigelow *et al.* on solid state materials. Our group also observed the subluminal and the superluminal group velocity phenomena.^[17–19] These techniques hold promise for uncovering new physical phenomena and for unpractical applications such as controllable optical delay lines, optical data storage, optical memories, and devices for quantum information.^[14,15]

In the present paper, we derive an analytical solution and numerical simulations on describing a propagation of a pumping field through a medium consisting of atoms with a four-level atomic system. It is shown that the change of the pumping field can affect the slope of the dispersion curves and so the group velocity. Also, the dispersion is very sensitive to the field detuning, and it can be changed from normal to anomalous simply. This leads to the group velocity of the pumping field changing from subluminal to superluminal.

2 Theoretical Model

We investigate a four-level atom that was used in Refs. [15] and [16] (see Fig. 1). An intense pumping pulse Ω coupling transition $|1\rangle \leftrightarrow |g\rangle$, can also drive $|2\rangle \leftrightarrow |3\rangle$. The decay of level $|1\rangle$ (Γ_1) is much larger than those of levels $|2\rangle$ (Γ_2) and $|3\rangle$ (Γ_3). This scheme can be at work in a material such as BeAl_2O_4 doped with Cr^{3+} ions and some Cr^{3+} ions replaced by Al^{3+} .^[15] In previous papers,^[15,16] the reverse saturation can produce an antihole in the susceptibility for the probe in the presence of a pumping field. It is this antihole that results in the superluminal propagation. In what follows, we show how the measurement can follow by using the steady state method.

In the interaction picture and rotating wave approximation, the equations of motion for the density matrix can be obtained as follows:

$$\begin{aligned}\dot{\rho}_{gg} &= 2\Gamma_2\rho_{22} + i\Omega(\rho_{1g} - \rho_{g1}), \\ \dot{\rho}_{22} &= 2\Gamma_1\rho_{11} - 2\Gamma_2\rho_{22} + 2\Gamma_3\rho_{33} + i\Omega(\rho_{32} - \rho_{23}), \\ \dot{\rho}_{33} &= -2\Gamma_3\rho_{33} + i\Omega(\rho_{23} - \rho_{32}), \\ \dot{\rho}_{32} &= (i\Delta - \Gamma_3)\rho_{32} + i\Omega(\rho_{22} - \rho_{33}), \\ \dot{\rho}_{1g} &= (i\Delta - \Gamma_1)\rho_{1g} + i\Omega(\rho_{gg} - \rho_{11}),\end{aligned}\quad (1)$$

which are constrained by $\rho_{gg} + \rho_{11} + \rho_{22} + \rho_{33} = 1$ and $\rho_{ij} = \rho_{ji}^*$ ($i, j = g, 1, 2, 3$). Here, $2\Gamma_i$ ($i = 1, 2, 3$) are the

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decay rates from the excited state $|i\rangle$, $\Delta = \omega - \omega_{1g}$, and $\Omega = 2\vec{d}_{1g} \cdot \vec{E}(z, t)/\hbar$ denotes the Rabi frequency, where \vec{d}_{1g} is the dipole moment matrix element. Here, we consider the intense pulse is near resonance with $|1\rangle \leftrightarrow |g\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, and the corresponding detuning Δ are the same.

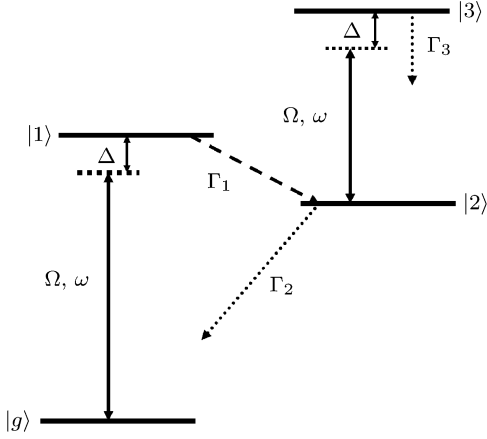


Fig. 1 Scheme of a four-level atomic system.

3 Numerical Simulation and Analysis

The steady state matrix can be used to calculate the susceptibility χ , which is related to^[20]

$$\begin{aligned} \rho_{1g} + \rho_{32} = \frac{1}{C} & [-2\Gamma_1^2\Gamma_3\Delta\Omega^3 - \Gamma_2\Omega(2\Gamma_1\Gamma_3\Omega^2 \\ & + 2\Gamma_1\Gamma_3\Delta(\Gamma_3^2 + \Delta^2) - \Gamma_1^2\Omega(\Gamma_3^2 + \Delta) \\ & + \Delta\Omega^2(\Gamma_1 + \Gamma_3)(\Gamma_1 + 6\Gamma_3))], \end{aligned} \quad (2)$$

where

$$\begin{aligned} C = 2\Gamma_1\Gamma_3\{ & \Gamma_2(\Gamma_1^2 + \Delta^2)(\Gamma_3^2 + \Delta^2) + \Omega^2[\Gamma_1^2\Gamma_2 \\ & + \Gamma_1\Gamma_3^2 + 2\Gamma_2\Gamma_3^2 + \Delta^2(\Gamma_1 + 3\Gamma_2)] \} \end{aligned}$$

$$+ 2\Omega^4(\Gamma_1 + \Gamma_2 + \Gamma_3)\}. \quad (3)$$

Therefore, the susceptibility χ , whose real and imaginary parts determine the dispersion and absorption of the field, is proportional to $(\rho_{1g} + \rho_{32})$.

The group velocity in a dispersive medium can be written as

$$v_g = \text{Re}\left(\frac{d\omega}{dk}\right) \approx \frac{c}{n + \omega \frac{dn}{d\omega}}. \quad (4)$$

In the region of normal dispersion, one has $dn/d\omega > 0$, which results in the reduction of the group velocity. Conversely, in another region of transparent anomalous dispersion, where $dn/d\omega < 0$, we can get superluminal group velocity. In special cases where $dn/d\omega < 0$ and $|\omega dn/d\omega| \gg 1$, the group velocity becomes negative. Here, we will investigate the dispersion characteristics of the pumping field, from which we can simply demonstrate the group velocity.

As we know, $(\rho_{1g} + \rho_{32})$ corresponds to the dispersion and absorption coefficient. In Fig. 2, the real and the imaginary parts of $(\rho_{01} + \rho_{02})$ on Δ with different Ω is plotted. According to Fig. 2(a), it is shown that around $\Delta = 0$, the dispersion curves present anomalous dispersion under different pumping rates. Meanwhile, with the increase of the pumping field Ω , its value increases, too. This results in the increasing of the superluminal phenomenon. However, around $\Delta = 5\gamma$, normal dispersion occurs, i.e., subluminal phenomenon is also presented. Meanwhile, as shown in Fig. 2(b), the imaginary part of $(\rho_{1g} + \rho_{32})$ is always negative, i.e., absorption always occurs. That is, superluminal behavior occurs within an absorption line. From these figures, it can be obtained that the slope of the dispersion changes from large negative values to positive values. Then, in principle, we show that the group velocity can be varied from subluminal to superluminal by varying the detuning Δ .

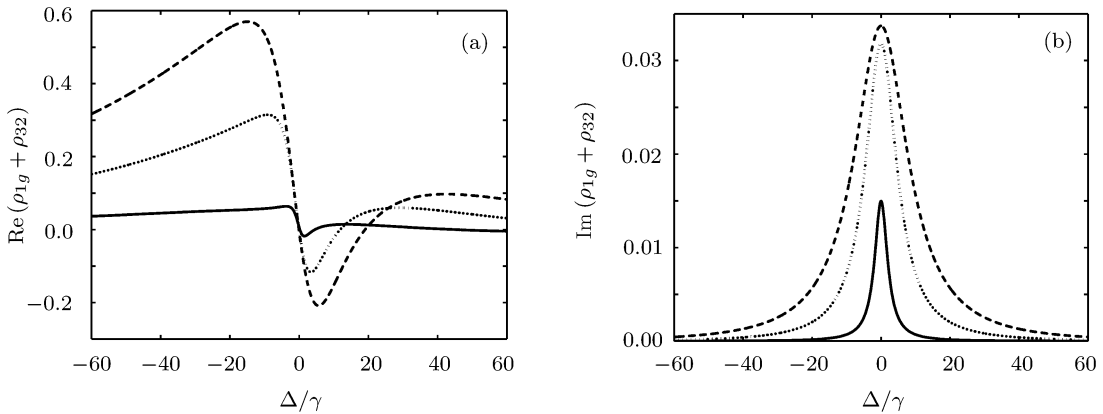


Fig. 2 Variation of $\gamma\text{Re}(\rho_{1g} + \rho_{32})$ on Δ/γ with different Ω . The solid, dotted and dashed lines are $\Omega = 2\gamma$, 5γ and 8γ , respectively. Here $\Gamma_1 = 50\gamma$, $\Gamma_2 = 2\gamma$ and $\Gamma_3 = \gamma$.

The complex refractive index n of the medium follows the relation

$$n = n' + in'' = 1 + \frac{1}{2}\chi, \quad (5)$$

with real part n' and imaginary part n'' . The definition of the group index is

$$n_g = n' + \omega \frac{\partial n'(\omega)}{\partial \omega}, \quad (6)$$

where n' is the real part of the refractive index. The relation of the susceptibility and the density matrix element can be easily derived

$$\chi(\omega) = \frac{2N|\vec{d}_{1g}|^2}{\hbar\epsilon_0\Omega}(\rho_{1g} + \rho_{32}), \quad (7)$$

where N is the atomic density. Substitute Eqs. (2) and (6) into Eq. (5), and then the group index can be obtained.

The numerical results of the group index n_g can be easily obtained. Choosing $N = 2 \times 10^{19} \text{ cm}^{-3}$, the dependence of n_g on Δ is shown in Fig. 3. It is shown that n_g increases with the enhancement of Δ , and then decreases. Its value is large and positive at first, and then, it becomes smaller, and even becomes larger negative. Around $\Delta = 0$, n_g becomes the largest negative value ($n_g \approx -8.95 \times 10^8$ with the parameters we choose). After this, it increases with Δ , and even becomes large positive value. From Eqs. (4) and (6), it is obtained that the group velocity of the pumping field is changed between subluminal and superluminal. This means, subluminal and superluminal phenomena can be effectively unified in this system.

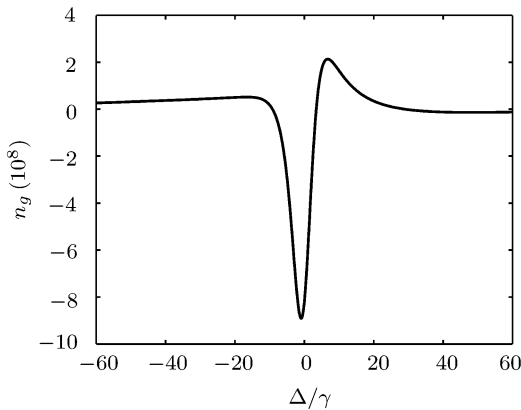


Fig. 3 Variation of n_g on Δ/γ . $N = 2 \times 10^{19} \text{ cm}^{-3}$, $\Omega = 5\gamma$ and $\gamma = 1 \times 10^6 \text{ Hz}$. The other parameters are the same as those in Fig. 2.

Dependence of the group refractive index n_g on Ω with different Δ is shown in Fig. 4. In the case of $\Delta = 5\gamma$, n_g firstly increases, then decreases with the enhancement of the pumping rate Ω , i.e., the value of it is negative at first, and increases to large positive with the increase of Ω , and then decreases to negative values. There exists an Ω value which makes the group index reach the maximum ($n_g \approx 2.385 \times 10^8$ here). Therefore, the group velocity

can be subluminal and superluminal with proper parameters. Especially, the pumping field can be a knob to change the group velocity. For the case of $\Delta = 0$, n_g is always negative. It firstly decreases with enhancement of the pumping field, and then increases. An Ω value which makes the group velocity index reach the minimum (here, $n_g \approx -9.96 \times 10^8$), corresponds to the maximum value of the group velocity. The result may be helpful for the experimental studies of measuring v_g .

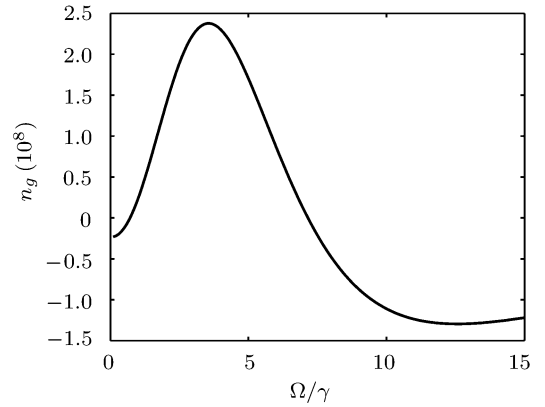


Fig. 4 Variation of the group refractive index n_g with Ω . $\Delta = 5\gamma$, other parameters are the same as those in Fig. 2.

Under the case of $\Delta = 0$, the group index can be analytically obtained as follows:

$$n_g = 1 + \frac{N|\vec{d}_{1g}|^2\Gamma_1\Gamma_2\Gamma_3\Omega}{D\hbar\epsilon_0}(\Gamma_1\Gamma_3 - 2\Omega^2) - \frac{\omega N|\vec{d}_{1g}|^2}{D\hbar\epsilon_0}\{2\Omega^2\Gamma_1^2\Gamma_3 + \Gamma_2[2\Gamma_3^2\Gamma_1 + \Omega^2(\Gamma_1 + \Gamma_3)(\Gamma_1 + 6\Gamma_3)]\}, \quad (8)$$

where

$$D = 2\Gamma_1\Gamma_3[\Gamma_1^2\Gamma_3^2\Gamma_2 + \Omega^2(\Gamma_1^2\Gamma_2 + \Gamma_3^2\Gamma_1 + 2\Gamma_3^2\Gamma_2) + 2\Omega^4(\Gamma_1 + \Gamma_2 + \Gamma_3)].$$

4 Conclusions

In conclusion, we have studied the dispersion property of a four-level atom where the laser beam acts with only one laser pulse. It is shown that in the steady state, the dispersion of the pumping field is separated into two parts: the normal and the anomalous dispersion regions, which correspond to the subluminal and superluminal propagations respectively. Therefore, controlling the group velocity from subluminal to superluminal propagations is possible “in principle”. These methods and results are different from Refs. [14] ~ [16] where the antihole producing by the reverse saturation results in the superluminal phenomenon. Thus, it is easy to observe the group velocity changing phenomenon in experiment. Also, our results can be extended to the case that the pumping field is large.

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