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# Generalized Quantum Two-Qutrit-State Splitting*

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***Abstract*** *A three-party scheme for splitting an arbitrary unknown two-qutrit state is proposed, where two non-maximally-entangled three-qutrit states are taken as the quantum channel among three parties. With the sender's help, if and only if both receivers collaborate together, they can securely share the quantum state in a probabilistic way by introducing an ancilla qutrit and performing appropriate unitary operations. The relation between the success probability and coefficients characterizing the quantum channel is revealed. The security of the present scheme is analyzed and confirmed. Moreover, the generalization of the three-party scheme to more-party case is also sketched.*

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**Key words:** generalized quantum state splitting, partially entangled state, generalized Bell-state measurement, single-qutrit projective measurement

## 1 Introduction

Entanglement as a kind of important quantum resource is widely exploited and utilized in the field of quantum information science in the last two decades. As one of the most striking applications, quantum entanglement is employed to generalize classical secret sharing[1] to quantum secret sharing (QSS).[2] The basic requirements of QSS in the simplest case can be described as that, the sender's secret can be reconstructed only by either of the two receivers if and only if they both collaborate together. QSS is likely to play a key role in both transmitting of classical message and protecting secret quantum information, e.g., in secure operations of distributed quantum computation, sharing difficult-to-construct ancilla states and joint sharing of quantum money, and so on. Since 1999, QSS has progressed quickly and attracted a widespread attention[3−32] over the past several years. These works can be divided into two types, classical message (i.e., bits) sharing[2−16] and quantum information (a quantum state) sharing[2,17−32]. The latter was first termed as quantum state sharing (QSTS) by Lance *et al.* in 2004.[21]

As far as QSTS is concerned, in 1999, Hillery *et al.*[2] presented the first scheme for securely sharing an arbitrary unknown single-qubit state by using a three-qubit or a four-qubit Greenberger–Horne–Zeilinger (GHZ) state as the quantum channel linking different parties. Soon later, Cleve *et al.*[17] investigated a more general quantum $(k, n)$ threshold QSTS scheme. Bandyopadhyay[18] proposed a QSTS scheme using optimal methods in 2000, and Hsu[19] proposed other QSTS scheme based on Grover's algorithm in 2003. Recently, Li *et al.*[20] proposed a QSTS scheme for sharing an unknown single-qubit state with a multipartite joint measurement. Some QSTS schemes were implemented in cavity QED[23]. Zhang *et al.*[24] proposed a multiparty QSTS of an arbitrary unknown single-qubit state via photon pairs. Lance *et al.*[25] proposed other continuous-variable QSTS scheme via quantum disentanglement. Deng *et al.*[26−27] proposed two QSTS schemes for sharing an arbitrary two-qubit state based on entanglement swapping. Li *et al.*[28] proposed an efficient symmetric multiparty QSTS scheme of an arbitrary $m$-qubit state with $m$ GHZ states. Incidentally, for these schemes relational to quantum discrete state sharing, although the quantum channels may be different seemingly, they are all maximally entangled states in common. Inspired by probabilistic teleportation first considered by Agrawal and Pati,[33] in 2006 Gordon and Rigolin[29] first proposed two new QSTS protocols (termed as generalized QSTS scheme by themselves), where the quantum channels are not maximally entangled states. Soon later, Man *et al.*[30] presented a three-party QSTS scheme for probabilistically sharing $n$-qubit state with $n$ non-maximally entangled three-qubit states and generalized Bell-state measurement. Wang *et al.*[31] presented a scheme for probabilistically implementing QSTS of an arbitrary unknown two-qubit state by using two non-maximally entangled three-qubit states as the quantum channel. However, it is worth while noticing that in all these schemes the quantum information to be shared is either a discrete state or a continuous one. For all these discrete states, they are all qubit states. Only very recently, Wang *et al.*[32] proposed a three-party single-qutrit state sharing scheme by using a generalized GHZ state as the quantum channel. This is

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a generalization in dimensionality. Nevertheless, in their scheme the quantum channel is still a maximally entangled state. In this paper, we will propose a generalized QSTS scheme for splitting an arbitrarily unknown two-qutrit state by using two non-maximally-entangled three-qutrit states as the quantum channel.

This paper is organized as follows. In Sec. 2, a three-party QSTS scheme is proposed in full detail. Moreover, the scheme security is analyzed and confirmed. In Sec. 3, the three-party QSTS scheme is concisely generalized to a more-party case. Finally, some summaries are given in Sec. 4.

## 2 Three-Party Two-Qutrit State Splitting Scheme

Our three-party QSTS scheme contains the following 6 steps [see Fig. 1]:

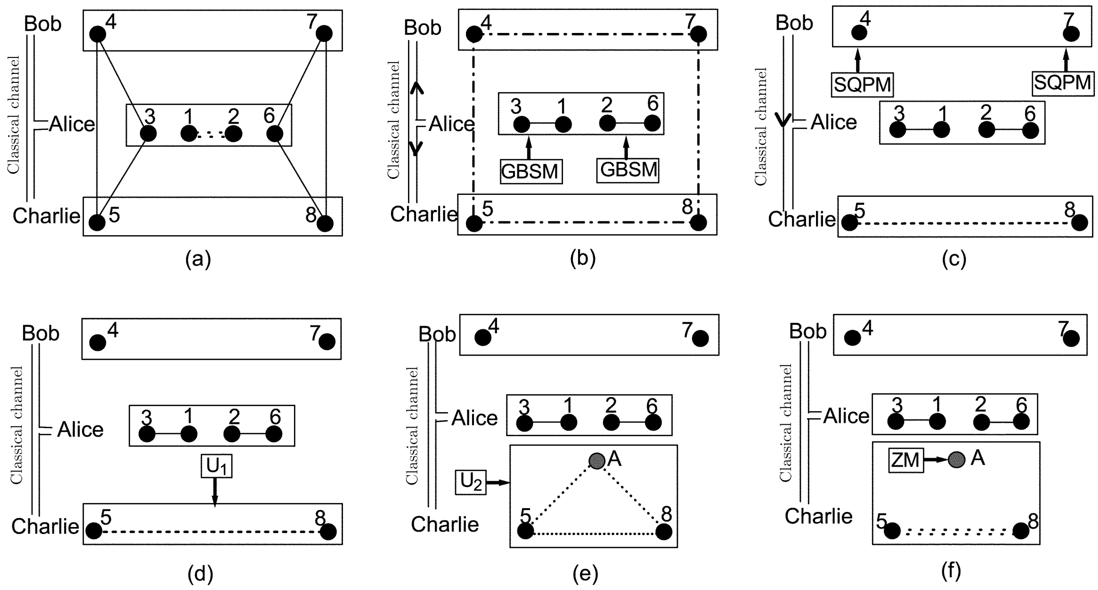


**Fig. 1** Generalized three-party scheme for splitting an arbitrary unknown two-qutrit state $|\Lambda\rangle_{12}$ by using two non-maximally entangled three-qutrit states $|\Xi\rangle_{345}$ and $|\Upsilon\rangle_{678}$ as the quantum channel. The line linking qutrits represents their entanglement. (a) → (b) → (c) → (d) → (e) → (f). (a) The quantum information and the quantum channel; (b) Alice performs two GBSMs on qutrit pairs $(1,3)$ and $(2,6)$ and informs Bob and Charlie of the measurement results $R_{13}$ and $R_{26}$ via a classical channel; (c) Bob is required to make two SQPMs on qutrits 4 and 7 and tells Charlie the measurement outcomes $R_4$ and $R_7$ over a classical channel; (d) Charlie performs the unitary operation $U_1$ on his two qutrits; (e) Charlie introduces the auxiliary qutrit $A$ and executes the unitary operation $U_2$ on his three qutrits. (f) Charlie performs the $Z$-basis measurement (ZM) on the ancilla qutrits $A$ to realize the sharing.

(i) Suppose there are three legitimate parties, Alice, Bob and Charlie. Alice is the sender of quantum information (i.e., a two-qutrit state), Bob and Charlie are her two agents. The quantum information initially inhabits Alice's qutrit pair (1,2), that is, the qutrit pair is in the state

$$|\Lambda\rangle_{12} = \alpha|00\rangle_{12} + \beta|11\rangle_{12} + \gamma|22\rangle_{12}\,, \tag{1}$$

where $\alpha$, $\beta$, and $\gamma$ are complex and satisfy $|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1$. Alice wants to let her agents share the quantum information in such a way that she can assign either of them to reconstruct the original state only with the other's help. Without loss of generality, hereafter we assume that Alice assigns Charlie to reconstruct the original state with the help of Bob. Moreover, Alice has another qutrit pair (3,6). Her agents Bob and Charlie have qutrit pairs (4,7) and (5,8), respectively. These three qutrit pairs are in the state

$$|\Pi\rangle_{345678} = |\Xi\rangle_{345} \otimes |\Upsilon\rangle_{678} \equiv (x|000\rangle_{345} + y|111\rangle_{345} + z|222\rangle_{345}) \otimes (r|000\rangle_{678} + s|111\rangle_{678} + t|222\rangle_{678})\,, \tag{2}$$

where $x$, $y$ $z$, $r$, $s$, and $t$ are real and satisfy $x^2 + y^2 + z^2 = r^2 + s^2 + t^2 = 1$, $|x| > |y| > |z|$ and $|r| > |s| > |t|$. This state forms in essence the quantum channel linking the three parties for QSTS. See Fig. 1(a) for illustration.

(ii) Alice performs generalized Bell-state measurements (GBSMs) on her qutrit pairs (1,3) and (2,6), respectively [See Fig. 1(b)]. Before the measurements, the combined state of the eight qutrits can be expressed as

$$\begin{aligned}|\Lambda\rangle_{12} \otimes |\Pi\rangle_{345678} = \frac{1}{3}[&(|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{00} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{20} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{10})(\alpha xr|0000\rangle + \beta ys|1111\rangle + \gamma zt|2222\rangle)_{4578}\\ &+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{10} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{00} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{20})(\alpha xr|0000\rangle + e^{4\pi i/3}\beta ys|1111\rangle + e^{2\pi i/3}\gamma zt|2222\rangle)_{4578}\end{aligned}$$

$$
\begin{aligned}
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{20} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{10} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{00})(\alpha xr|0000\rangle + \mathrm{e}^{2\pi i/3}\beta ys|1111\rangle + \mathrm{e}^{4\pi i/3}\gamma zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{02} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{22} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{12})(\beta xr|0000\rangle + \gamma ys|1111\rangle + \alpha zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{12} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{02} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{22})(\mathrm{e}^{4\pi \mathrm{i}/3}\beta xr|0000\rangle + e^{2\pi i/3}\gamma ys|1111\rangle + \alpha zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{22} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{12} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{02})(\mathrm{e}^{2\pi \mathrm{i}/3}\beta xr|0000\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\gamma ys|1111\rangle + \alpha zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{01} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{21} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{11})(\gamma xr|0000\rangle + \alpha ys|1111\rangle + \beta zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{11} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{01} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{21})(\mathrm{e}^{2\pi \mathrm{i}/3}\gamma xr|0000\rangle + \alpha ys|1111\rangle + e^{4\pi i/3}\beta zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{21} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{11} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{01})(e^{4\pi i/3}\gamma xr|0000\rangle + \alpha ys|1111\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}\beta zt|2222\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{01} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{21} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{11})(\alpha xs|0011\rangle + \beta yt|1122\rangle + \gamma zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{11} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{01} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{21})(\alpha xs|0011\rangle + \mathrm{e}^{4\pi i/3}\beta yt|1122\rangle + e^{2\pi i/3}\gamma zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{21} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{11} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{01})(\alpha xs|0011\rangle + e^{2\pi i/3}\beta yt|1122\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\gamma zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{00} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{20} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{10})(\beta xs|0011\rangle + \gamma yt|1122\rangle + \alpha zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{10} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{00} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{20})(e^{4\pi i/3}\beta xs|0011\rangle + \mathrm{e}^{2\pi i/3}\gamma yt|1122\rangle + \alpha zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{20} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{10} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{00})(\mathrm{e}^{2\pi \mathrm{i}/3}\beta xs|0011\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\gamma yt|1122\rangle + \alpha zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{02} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{22} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{12})(\gamma xs|0011\rangle + \alpha yt|1122\rangle + \beta zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{12} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{02} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{22})(\mathrm{e}^{2\pi \mathrm{i}/3}\gamma xs|0011\rangle + \alpha yt|1122\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\beta zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{22} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{12} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{02})(\mathrm{e}^{4\pi \mathrm{i}/3}\gamma xs|0011\rangle + \alpha yt|1122\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}\beta zr|2200\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{02} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{22} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{12})(\alpha xt|0022\rangle + \beta yr|1100\rangle + \gamma zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{12} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{02} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{22})(\alpha xt|0022\rangle + e^{4\pi i/3}\beta yr|1100\rangle + e^{2\pi i/3}\gamma zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{00}|\Psi\rangle_{26}^{22} + |\Psi\rangle_{13}^{10}|\Psi\rangle_{26}^{12} + |\Psi\rangle_{13}^{20}|\Psi\rangle_{26}^{02})(\alpha xt|0022\rangle + e^{2\pi i/3}\beta yr|1100\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\gamma zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{01} + |\Psi\rangle_{21}^{12}|\Psi\rangle_{26}^{21} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{11})(\beta xt|0022\rangle + \gamma yr|1100\rangle + \alpha zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{11} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{01} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{21})(\mathrm{e}^{4\pi \mathrm{i}/3}\beta xt|0022\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}\gamma yr|1100\rangle + \alpha zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{02}|\Psi\rangle_{26}^{21} + |\Psi\rangle_{13}^{12}|\Psi\rangle_{26}^{11} + |\Psi\rangle_{13}^{22}|\Psi\rangle_{26}^{01})(\mathrm{e}^{2\pi \mathrm{i}/3}\beta xt|0022\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\gamma ys|1100\rangle + \alpha zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{00} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{20} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{10})(\gamma xt|0022\rangle + \alpha yr|1100\rangle + \beta zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{10} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{00} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{20})(\mathrm{e}^{2\pi \mathrm{i}/3}\gamma xt|0022\rangle + \alpha yr|1100\rangle + \mathrm{e}^{4\pi \mathrm{i}/3}\beta zs|2211\rangle)_{4578} \\
&+ (|\Psi\rangle_{13}^{01}|\Psi\rangle_{26}^{20} + |\Psi\rangle_{13}^{11}|\Psi\rangle_{26}^{10} + |\Psi\rangle_{13}^{21}|\Psi\rangle_{26}^{00}) \\
&\times (\mathrm{e}^{4\pi \mathrm{i}/3}\gamma xt|0022\rangle + \alpha yr|1100\rangle + \mathrm{e}^{2\pi \mathrm{i}/3}\beta zs|2211\rangle)_{4578}]\,,
\end{aligned}
\tag{3}
$$

where

$$
|\Psi\rangle^{nm} = \frac{1}{\sqrt{3}}\sum_{j=0}^{2}\mathrm{e}^{2\pi \mathrm{i} jn/3}|j\rangle \otimes |(j+m) \bmod 3\rangle, \quad n \in \{0,1,2\},\ m \in \{0,1,2\}\,. \tag{4}
$$

Hence, Alice will get one of 81 possible results. Without loss of generality, we suppose Alice's measurement results on qutrit pairs (1,3) and (2,6) are $|\Psi\rangle_{13}^{10}$ and $|\Psi\rangle_{26}^{20}$, respectively, and she publicly announces her measurement results via a classical channel [See Fig. 1(b) also]. In this case, the joint state of qutrits 4, 5, 7, and 8 collapses to

$$
|\Gamma\rangle_{4578} = \frac{1}{3}(xr\alpha|0000\rangle_{4578} + ys\beta|1111\rangle_{4578} + zt\gamma|2222\rangle_{4578})\,. \tag{5}
$$

This state can be rewritten as

$$
\begin{aligned}
|\Gamma\rangle_{4578} &= \frac{1}{3}(xr\alpha|0000\rangle_{4578} + ys\beta|1111\rangle_{4578} + zt\gamma|2222\rangle_{4578}) \\
&= \frac{1}{9}[|\xi\rangle_4|\xi\rangle_7(xr\alpha|00\rangle_{58} + ys\beta|11\rangle_{58} + zt\gamma|22\rangle_{58}) \\
&\quad + |\xi\rangle_4|\zeta\rangle_7(xr\alpha|00\rangle_{58} + e^{4\pi i/3}ys\beta|11\rangle_{58} + e^{2\pi i/3}zt\gamma|22\rangle_{58}) \\
&\quad + |\xi\rangle_4|\varsigma\rangle_7(xr\alpha|00\rangle_{58} + \mathrm{e}^{2\pi \mathrm{i}/3}ys\beta|11\rangle_{58} + \mathrm{e}^{4\pi \mathrm{i}/3}zt\gamma|22\rangle_{58}) \\
&\quad + |\zeta\rangle_4|\xi\rangle_7(xr\alpha|00\rangle_{58} + \mathrm{e}^{4\pi \mathrm{i}/3}ys\beta|11\rangle_{58} + \mathrm{e}^{2\pi \mathrm{i}/3}zt\gamma|22\rangle_{58})
\end{aligned}
$$

$$
\begin{aligned}
&+|\zeta\rangle_4|\zeta\rangle_7(xr\alpha|00\rangle_{58}+\mathrm{e}^{2\pi\mathrm{i}/3}ys\beta|11\rangle_{58}+\mathrm{e}^{4\pi\mathrm{i}/3}zt\gamma|22\rangle_{58})\\
&+|\zeta\rangle_4|\varsigma\rangle_7(xr\alpha|00\rangle_{58}+ys\beta|11\rangle_{58}+zt\gamma|22\rangle_{58})\\
&+|\varsigma\rangle_4|\xi\rangle_7(xr\alpha|00\rangle_{58}+\mathrm{e}^{2\pi\mathrm{i}/3}ys\beta|11\rangle_{58}+\mathrm{e}^{4\pi\mathrm{i}/3}zt\gamma|22\rangle_{58})\\
&+|\varsigma\rangle_4|\zeta\rangle_7(xr\alpha|00\rangle_{58}+ys\beta|11\rangle_{58}+zt\gamma|22\rangle_{58})\\
&+|\varsigma\rangle_4|\varsigma\rangle_7(xr\alpha|00\rangle_{58}+\mathrm{e}^{4\pi\mathrm{i}/3}ys\beta|11\rangle_{58}+\mathrm{e}^{2\pi\mathrm{i}/3}zt\gamma|22\rangle_{58})]\,,
\end{aligned}
\tag{6}
$$

where

$$
\begin{aligned}
|\xi\rangle&=\frac{1}{\sqrt{3}}(|0\rangle+|1\rangle+|2\rangle)\,,\\
|\zeta\rangle&=\frac{1}{\sqrt{3}}(|0\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|1\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|2\rangle)\,,\\
|\varsigma\rangle&=\frac{1}{\sqrt{3}}(|0\rangle+\mathrm{e}^{4\pi\mathrm{i}/3}|1\rangle+\mathrm{e}^{2\pi\mathrm{i}/3}|2\rangle)\,.
\end{aligned}
\tag{7}
$$

The three states $\{|\xi\rangle,|\zeta\rangle,|\varsigma\rangle\}$ are related to the computation basis vectors $\{|0\rangle,|1\rangle,|2\rangle\}$, and form a complete orthogonal basis set of a single-qutrit Hilbert space.

(iii) As mentioned just, Charlie is assigned to reconstruct the state with Bob's help. Hence, after Alice's publication on her measurement results, Bob performs single-qutrit projective measurements (SQPMs) on his qutrits 4 and 7, respectively [See Fig. 1(c)]. Without loss of generality, suppose Bob's measurement results on qutrits 4 and 7 are $|\xi\rangle_4$ and $|\varsigma\rangle_7$, respectively. The joint state of Charlie's qutrits 5 and 8 collapses to

$$
\begin{aligned}
|\Theta\rangle_{58}=\frac{1}{9}(&xr\alpha|00\rangle_{58}+\mathrm{e}^{2\pi\mathrm{i}/3}ys\beta|11\rangle_{58}\\
&+\mathrm{e}^{4\pi\mathrm{i}/3}zt\gamma|22\rangle_{58})\,.
\end{aligned}
\tag{8}
$$

Then he communicates his outcomes to Charlie over a classical channel.

(iv) After having received Bob's classical message about his measurement outcomes, Charlie first performs a unitary operation $U_1=\sum_{j=0}^{2}\mathrm{e}^{4\pi\mathrm{i}j/3}|jj\rangle\langle jj|$ on her qutrits 5 and 8 [See Fig. 1(d)], which transforms $|\Theta\rangle_{58}$ into

$$
|\Theta'\rangle_{58}=\frac{1}{9}(xr\alpha|00\rangle_{58}+ys\beta|11\rangle_{58}+zt\gamma|22\rangle_{58})\,. \tag{9}
$$

(v) Charlie introduces an auxiliary qutrit $A$ in the state $|0\rangle_A$. The joint state of the three qutrits 5, 8, and $A$ is

$$
\begin{aligned}
|\Theta''\rangle_{58A}=\frac{1}{9}(&xr\alpha|00\rangle_{58}+ys\beta|11\rangle_{58}\\
&+zt\gamma|22\rangle_{58})|0\rangle_A\,.
\end{aligned}
\tag{10}
$$

In order for Charlie to reconstruct the original state, he then performs another unitary operation $U_2$ on particles 5, 8, and $A$ [See Fig. 1(e)]. The unitary operation $U_2$ takes the form of the following $9\times 9$ matrices under the basis $\{|000\rangle_{58A},\ |110\rangle_{58A},\ |220\rangle_{58A},\ |001\rangle_{58A},\ |111\rangle_{58A},\ |221\rangle_{58A},\ |002\rangle_{58A},\ |112\rangle_{58A},\ |222\rangle_{58A}\}$,

$$
U_2=\begin{pmatrix} A_1 & A_2 & 0\\ A_2 & -A_1 & 0\\ 0 & 0 & I\end{pmatrix}\,, \tag{11}
$$

where $I$ is the $3\times 3$ identity matrix, $A_1$ and $A_2$ are the $3\times 3$ diagonal matrices with off-diagonal elements being zeros, i.e.,

$$
\begin{aligned}
A_1&=\mathrm{diag}\Big(\frac{zt}{xr},\frac{zt}{ys},1\Big)\,,\\
A_2&=\mathrm{diag}\left(\sqrt{1-\Big(\frac{zt}{xr}\Big)^2},\sqrt{1-\Big(\frac{zt}{ys}\Big)^2},0\right)\,.
\end{aligned}
\tag{12}
$$

After Charlie's collective unitary operation $U_2$ on the qutrits 5, 8 and $A$, the state $|\Theta''\rangle_{58A}$ is transformed into

$$
\begin{aligned}
U_2|\Theta''\rangle_{58A}=\frac{1}{9}\big[&zt(\alpha|00\rangle_{58}+\beta|11\rangle_{58}+\gamma|22\rangle_{58})|0\rangle_A\\
&+\big(\sqrt{x^2r^2-z^2t^2}\alpha|00\rangle_{58}\\
&+\sqrt{y^2s^2-z^2t^2}\beta|11\rangle_{58}\big)|1\rangle_A\big]\,.
\end{aligned}
\tag{13}
$$

By the way, it is obvious that, the unitary operation $U_1$ and $U_2$ may be incorporated into a single transformation $U=U_1\otimes U_2$.

(vi) Charlie measures the auxiliary qutrit $A$ in the $Z$-basis $\{|0\rangle,|1\rangle\}$ [See Fig. 1(f)]. If the measurement result is $|0\rangle_A$, Charlie knows he has already successfully reconstructed the original state $|\Lambda\rangle$ on his qutrits 5 and 8. Otherwise, the scheme fails. Through a complicated calculation we have worked out the success probability, it is $|zt|^2/81$ in this case. One can easily see that the success probability is determined by two smallest coefficients $|z|$ and $|t|$.

Note that, in Step (ii), when Alice performs GBSMs on her qutrit pairs (1,3) and (2,6), there are 81 possible results [shown in Eq. (3)]. In Step (iii), when Bob executes SQPMs on qutrits 4 and 7 in the basis $\{|\xi\rangle,|\zeta\rangle,|\varsigma\rangle\}$, there are also 9 possible results [See Eq. (6)]. Hence, there are 729 cases in total. For each case, it can be dealt with the same method we have just described. After tedious deduction repeats, fortunately we find that for each case the quantum information sharing can realized with the equal probability of $|zt|^2/81$ by executing similar performances. Consequently, for our generalized QSTS scheme the total success probability is $|zt|^2/81\times 729=9|zt|^2$. Furthermore, if $|x|=|y|=|z|=|r|=|s|=|t|=1/\sqrt{3}$, i.e., the quantum channel is composed of two maximally entangled three-qutrit states, the total probability equals 1, i.e, the generalized probabilistic QSTS is reduced to a deterministic one. Moreover, it is clear and natural that in this situation the unitary operation $U$ executed by the receiver Charlie is reduced to the unitary operation $U_1$ only, that is, the unitary operation $U_2$ is equal to an identity operation and the auxiliary particle $A$ does not need to be introduced. Obviously, this reduction coincides with the usual knowledge about QSTS.

Now let us briefly analyze the scheme security against eavesdropping and cheating. Generally, there may be existing an outside eavesdropper Eve or/and an inside dishonest party. Their aim is to obtain Alice's quantum information solely without being detected. The scheme security completely depends on the process for setting up the quantum channel. In our scheme, we have assumed that the three parties have already securely shared the quantum channel described in Eq. (2). In the practical condition, we can adopt several decoy qubits as suggested in Refs. [15], [30], [34], and [35] as the checking qubits to ensure the security. In this case, the outside eavesdropper Eve can be detected as the fact that her attacks will inevitably introduce detectable errors. Concerning the insider's cheating, either of Bob and Charlie may be dishonest, but Alice does not exactly know who is the dishonest one beforehand. Fortunately, in our scheme, either of them has the equal chance to reconstruct Alice's quantum information. If Bob is dishonest and try to cheat the other insider such that he can get the quantum information safely and solely. Only when Bob is assigned by Alice to reconstruct the original state, can he successfully obtain the original state without being detected. Otherwise, his cheating can be revealed when Alice publishes a subset of her quantum information for comparison check. The success probability for the dishonest Bob is only 50%. Furthermore, Bob's success probability decreases with the number of agents increasing.

## 3 More-Party Two-Qutrit State Splitting Scheme

Now let us briefly introduce the generalization of the three-party two-qutrit state splitting scheme to more-party case. Similar to the three-party case, our this more-party generalized scheme also contains 6 steps:

(i) Suppose there are $N$ legitimate parties. Alice is the quantum information sender. By the way, the quantum information is still given by the Eq. (1). The other $N-1$ parties are Alice's agents, say Bob$_i$ $(i=1,2,\ldots,N-1)$. The quantum channel linking $N$ legitimate parties are two non-maximally entangled $N$-qutrit states

$$|\Xi'\rangle_{1'2'3'\cdots N'} = x|00\cdots0\rangle_{1'2'3'\cdots N'} + y|11\cdots1\rangle_{1'2'3'\cdots N'} + z|22\cdots2\rangle_{1'2'3'\cdots N'}\,, \tag{14}$$

$$|\Upsilon'\rangle_{1''2''3''\cdots N''} = r|00\cdots0\rangle_{1''2''3''\cdots N''} + s|11\cdots1\rangle_{1''2''3''\cdots N''} + t|22\cdots2\rangle_{1''2''3''\cdots N''}\,, \tag{15}$$

where $x$, $y$, $z$, $r$, $s$, and $t$ are defined as above. Qutrit pair $(1',1'')$ belongs to Alice, and qutrit pairs $(2',2'')$, $(3',3'')$, $\ldots$, $(N',N'')$ belong to Bob$_i$ $(i=1,2,\ldots,N-1)$, respectively.

(ii) Alice performs GBSMs on her two qutrit pairs $(1,1')$ and $(2,1'')$, and declares the measurement results via a classical channel. Without loss of generality, we suppose Alice's measurement results are $|\Psi\rangle_{11'}^{10}$ and $|\Psi\rangle_{21''}^{20}$, respectively. In this case, the joint state of qutrits $2',3',\ldots,N'$, $2'',3'',\ldots,N''$ collapses to

$$|\Gamma'\rangle_{2'3'\cdots N'2''3''\cdots N''} = \frac{1}{3}(xr\alpha|00\cdots0\rangle_{2'3'\cdots N'2''3''\cdots N''} + ys\beta|11\cdots1\rangle_{2'3'\cdots N'2''3''\cdots N''} + zt\gamma|22\cdots2\rangle_{2'3'\cdots N'2''3''\cdots N''})\,. \tag{16}$$

This state can be rewritten as

$$\begin{aligned}|\Gamma'\rangle_{2'3'\cdots N'2''3''\cdots N''} &= \frac{1}{3}(xr\alpha|00\cdots0\rangle_{2'3'\cdots N'2''3''\cdots N''} + ys\beta|11\cdots1\rangle_{2'3'\cdots N'2''3''\cdots N''} + zt\gamma|22\cdots2\rangle_{2'3'\cdots N'2''3''\cdots N''})\\ &= \Big(\frac{1}{3}\Big)^{N-1}\sum_{l_1=0}^{2}\sum_{l_2=0}^{2}\cdots\sum_{l_{m-1}=0}^{2}\sum_{l_{m+1}=0}^{2}\cdots\sum_{l_{N-1}=0}^{2}\sum_{l_N=0}^{2}\cdots\sum_{l_{N+m-2}=0}^{2}\sum_{l_{N+m}=0}^{2}\cdots\sum_{l_{2N-2}=0}^{2}\\ &\quad\times[|\chi_{l_1}\rangle_{2'}|\chi_{l_2}\rangle_{3'}\cdots|\chi_{l_{m-1}}\rangle_{m'}|\chi_{l_{m+1}}\rangle_{m'+2}\cdots|\chi_{l_{N-1}}\rangle_{N'}|\chi_{l_N}\rangle_{2''}\cdots|\chi_{l_{N+m-2}}\rangle_{m''}\\ &\quad\times|\chi_{l_{N+m}}\rangle_{m''+2}\cdots|\chi_{l_{2N-2}}\rangle_{N''}\otimes(xr\alpha|00\rangle_{(m'+1)(m''+1)} + ys\beta\,\mathrm{e}^{4\pi\mathrm{i}L/3}|11\rangle_{(m'+1)(m''+1)}\\ &\quad + zt\gamma\,\mathrm{e}^{2\pi\mathrm{i}L/3}|22\rangle_{(m'+1)(m''+1)})]\,,\end{aligned} \tag{17}$$

where $L=\sum_{k=1(k\neq m,N+m-1)}^{2N-2} l_k$ and $|\chi_{l_k}\rangle\in\{|\xi\rangle,|\zeta\rangle,|\varsigma\rangle\}$.

(iii) As the symmetry, anyone of Bob$_i$ $(i=1,2,\ldots,N-1)$ can reconstruct Alice's original state if and only if he/she gets the other agents' help. Without loss of generality, we assume Alice chooses the $m$-th agent Bob$_m$ to reconstruct her original state. Then the other $N-2$ agents Bob$_i$ $(i=1,2,\ldots,N-2)$ are asked to perform SQPMs on their respective two qutrits and announce their measurement results to Bob$_m$ via a classical channel. According to Eq. (17), after the other $N-2$ agents' measurement the two qutrits in the $m$-th agent's possession is left into

$$\Big(\frac{1}{3}\Big)^{N-1}(xr\alpha|00\rangle_{(m'+1)(m''+1)} + ys\beta\,\mathrm{e}^{4\pi\mathrm{i}L/3}|11\rangle_{(m'+1)(m''+1)} + zt\gamma\,\mathrm{e}^{2\pi\mathrm{i}L/3}|22\rangle_{(m'+1)(m''+1)})\,.$$

(iv) After having received the classical message, $\text{Bob}_m$ first performs a unitary operation $U_1' = \sum_{j=0}^{2} \mathrm{e}^{2\pi \mathrm{i} jL/3}|jj\rangle\langle jj|$ on qutrit pair $(m'+1, m''+1)$.

(v) $\text{Bob}_m$ introduces an auxiliary qutrit $A$ with its initial state $|0\rangle_A$ and performs another unitary operation $U_2'$ with the same form of Eqs. (11) and (12). Then $\text{Bob}_m$ can get the same result as Eq. (13) except the coefficient.

(vi) $\text{Bob}_m$ measures the qutrit $A$ in the $Z$-basis $\{|0\rangle, |1\rangle\}$. If the measurement result is $|0\rangle_A$, $\text{Bob}_m$ knows she/he has already successfully reconstructed the original state $|\Lambda\rangle$ on her/his qutrits $m'+1$ and $m''+1$. Otherwise, the scheme fails. Obviously, since the probability is only determined by the small coefficients of the states taken as the quantum channel, the success probability in the more-party case is also $9|zt|^2$.

The security of this more-party generalized QSTS scheme is as same as that of the three-party case.

## 4 Conclusion

To summarize, in this paper we have proposed a generalized scheme for splitting an arbitrary unknown two-qutrit state by using two non-maximally-entangled three-qutrit states as the quantum channel among three parties. In this scheme, the state sender Alice needs to perform two GBSMs on her qutrit pairs and then publishes her measurement results via a classical channel. As the symmetry, anyone (the receiver) of the agents has the chance to be assigned by Alice to regenerate the original state. After Alice's assignment, the other agents are required to perform SQPMs on their respective two qutrits. If they collaborate with the assignee, then the assignee can probabilistically recover the quantum information by introducing an ancilla qutrit and performing appropriate unitary operations. After tedious works, we have worked out that the success probability is $9|zt|^2$, which is completely determined by the smallest two coefficients $|z|$ and $|t|$. Further, the scheme security is briefly analyzed and confirmed. Moreover, the generalization of the three-party scheme to more-party case is concisely introduced.

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