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Properties of Unsymmetrical Parabolic Confinement Potential Quantum Dot Qubit

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Abstract *On the condition of electric-LO phonon strong coupling in unsymmetrical parabolic confinement potential quantum dot (QD), we obtain the eigenenergies of the ground state and the first-excited state, the eigenfunctions of the ground state, and the first-excited state by using variational method of Pekar type. This system in QD may be employed as a two-level quantum system-qubit. When the electron is in the superposition state of the ground state and the first-excited state, we obtain the time evolution of the electron density. The relations both the probability density of electron and the period of oscillation with the electron-LO-phonon coupling strength, the confinement strengths in the xy-plane and the z-direction are discussed.*

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1 Introduction

Quantum computing combines computer science with quantum mechanics and is a fast growing research field.[1] And the quantum computation intended as information theory performed using qubits, that is, two-level systems evolving by the unitary evolution as obtained by the laws of quantum mechanics, demands an increasing control on such systems. Several schemes have been proposed for realizing quantum computer in recent years.[2–4] In order to show the advantage of quantum computer over the most classical computer, quantum computer needs to be composed of at least thousands of qubits to be feasible. Consequently, it is clear that quantum computer with a large significant number of qubits would be more realizable in solid-state systems. However, selfassembled quantum dots (QDs) have been attracted substantial attention due to their perfect crystal structures. Therefore, it is one of the most popular solid-state quantum information research fields that qubits can be realized by solid-state devices. Many schemes have been proposed for researching QD and have many kinds of contents, but they are in the initial research stage at present.

In this article, the eigenenergies and their relevant eigenwavefunctions of the ground state and the first-excited state of an electron have been obtained in an unsymmetrical parabolic confinement potential QD using the Pekar variational method in the electron-LO-phonon strong-coupling region. A single qubit can be envisaged as this kind of two-level quantum system in a QD. For this single-electron QD qubit, Li *et al.* presented a kind of parameter-phase diagram schemes and defined the parameters region for the use of the an InAs/GaAs as a two-level quantum system.[5,6] We have obtained the probability density of electron oscillating with a period when electron is in a superposition state of the ground state and first-excited state. We discuss the relations both the probability density of electron and the period of oscillation with the electron-LO-phonon coupling strength, the confinement strengths in the xy-plane and the z-direction in this paper. Our results should be meaningful for designing the solid-state implementation of quantum computing both theoretically and experimentally.

2 Theoretical Model

We consider an electron, which is interacting with bulk LO phonons and subject to a 3D unsymmetrical parabolic confinement potential. The Hamiltonian of an electron-phonon interaction system is given by

$$H = -\frac{\hbar^2}{2m_b}\nabla^2 + \frac{1}{2}m_b\omega_\rho^2\rho^2 + \frac{1}{2}m_b\omega_z^2 z^2 + \sum_q \hbar\omega_{\rm LO} b_q^\dagger b_q + \sum_q (V_q\,{\rm e}^{{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}} b_q + {\rm h.c.})\,, \tag{1}$$

where $r = (\rho, z)$ denotes the position of the electron and $b_q^\dagger(b_q)$ is the creation (annihilation) operator of an optical phonon with a wave vector $\boldsymbol{q}(\boldsymbol{q} = \boldsymbol{q}_\parallel, \boldsymbol{q}_z)$. ω_ρ and ω_z are the measures of confinement strengths of the 3D unsymmetrical parabolic confinement potential in the xy-plane and the z-direction, respectively,

$$V_q = {\rm i}\Big(\frac{\hbar\omega_{\rm LO}}{q}\Big)\Big(\frac{\hbar}{2m_b\omega_{\rm LO}}\Big)^{1/4}\Big(\frac{4\pi\alpha}{V}\Big)^{1/2}\,. \tag{2}$$

By making the well-known LLP transformation

$$U = \exp\Big[\sum_q (f_q b_q^\dagger - f_q^* b_q)\Big]\,, \tag{3}$$

where f_q (f_q^*) is the variational function, we obtain

$$H' = U^{-1}HU\,. \tag{4}$$

Supposing that the Gaussian function approximation is valid in the ground state of electron-phonon system, by following variational method of Pekar type[7] we have

$$|\Psi_0\rangle = \Big(\frac{\lambda}{\pi}\Big)^{1/2}\Big(\frac{\mu}{\pi}\Big)^{1/4}\exp\Big(-\frac{\lambda\rho^2}{2}\Big)\exp\Big(-\frac{\mu z^2}{2}\Big)|0_{\rm ph}\rangle\,, \tag{5}$$

where λ and μ are the variational parameters. For convenience, we consider only the unsymmetrical QD, i.e. $\omega_z \geq \omega_\rho$, which ensures that $\mu \geq \lambda$. The above equation satisfies the following normalized relation:

$$\langle\Psi_0|\Psi_0\rangle = 1\,. \tag{6}$$

By minimizing the expectation value of the Hamiltonian, we then obtain the electron ground state energy:

$$E_0 = \langle\Psi_0|H'|\Psi_0\rangle\,. \tag{7}$$

Similarly, the trial wave function of electron-phonon system in the first-excited state may be chosen as

$$|\Psi_1\rangle = \Big(\frac{\lambda}{\sqrt{\pi}}\Big)\Big(\frac{\mu}{\pi}\Big)^{1/4}\rho\exp\Big(-\frac{\lambda\rho^2}{2}\Big) \times \exp(\pm\mathrm{i}\phi)\exp\Big(-\frac{\mu z^2}{2}\Big)|0_{\rm ph}\rangle\,. \tag{8}$$

This satisfies the following relations:

$$\langle\Psi_0|\Psi_1\rangle = 0\,, \tag{9}$$

$$\langle\Psi_1|\Psi_1\rangle = 1\,. \tag{10}$$

We can obtain the energy in the first-excited state:

$$E_1 = \langle\Psi_1|H'|\Psi_1\rangle\,. \tag{11}$$

Then we can get the eigenlevel and the eigenwavefunction. Then, we obtain the two-level system needed by a single qubit. The superposition state of electron can be expressed as

$$|\Psi_{01}\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\,, \tag{12}$$

where $|0\rangle = |\Psi_0\rangle$ and $|1\rangle = |\Psi_1\rangle$. The time evolution of the quantum state of the electron can be written as

$$|\Psi_{01}\rangle(t,r) = \frac{1}{\sqrt{2}}\Psi_0\exp\Big(\frac{-\mathrm{i}E_0 t}{\hbar}\Big) + \frac{1}{\sqrt{2}}\Psi_1\exp\Big(\frac{-\mathrm{i}E_1 t}{\hbar}\Big)\,. \tag{13}$$

The probability density is in the following form,

$$Q(r,t) = |\Psi_{01}(t,r)|^2 = \frac{1}{2}[|\Psi_0(r)|^2 + |\Psi_1(r)|^2 + \Psi_0^*(r)\Psi_1(r) \times \exp(\mathrm{i}\omega_{01}t) + \Psi_1^*(r)\Psi_0(r)\exp(-\mathrm{i}\omega_{01}t)]\,, \tag{14}$$

where

$$\omega_{01} = \frac{E_1 - E_0}{\hbar}\,. \tag{15}$$

3 Results and Discussions

The numerical results of the probability density of electron and the period of oscillation versus the electron-phonon coupling strength, the confinement strengths in the xy-plane and the z-direction in an unsymmetrical parabolic QD are presented in Figs. 1 ∼ 3.

Figure 1 shows the time evolution of the electron probability density $|\Psi_{01}(t,r)|^2$ when the electron is in the superposition state of $(1/\sqrt{2})\,(|0\rangle + |1\rangle)$ for electron-LO-phonon coupling constant $\alpha = 6$, the confinement strength in the xy-plane $\omega_\rho = 5\omega_{\rm LO}$, the confinement strength in the z-direction $\omega_z = 20\omega_{\rm LO}$ and phase difference $\phi = 2\pi$. We find that the probability density of electron oscillates with the period of oscillation $T_0 = h/(E_1 - E_0)$ for the above material and shape parameters. The time t in Figs. 1(a), 1(b), 1(c), 1(d), and 1(e) are 0, 0.25, 0.5, 0.75, and 1 T_0, respectively.

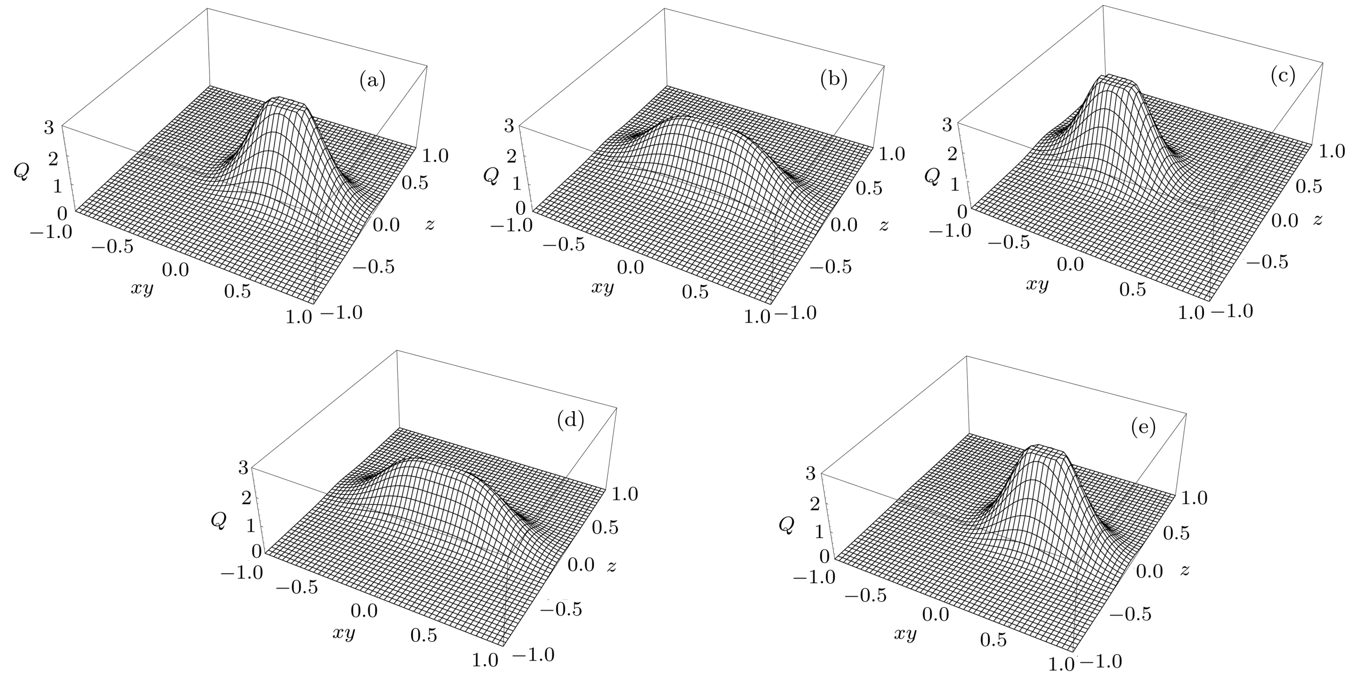

Fig. 1 The time evolution of the electron probability density in superposition of $|0\rangle$ and $|1\rangle$.

Figure 2 presents the period of oscillation $T_0 = h/(E_1 - E_0)$ as a function of the confinement strength in the xy-plane with the different coupling strength ($\alpha = 0, 6, 10$) for $\omega_z = 20\omega_{\rm LO}$. It is shown that the period of oscillation increases with decreasing the confinement strength in the xy-plane. The reason is that, the energy spacing between the ground state and the first-excited state increases with increasing the confinement strength in the xy-plane. Figure 2 also shows that the period of oscillation decreases with increasing the electron-phonon coupling strength. Because the coupling strength of the electron-phonon interaction is weaker in the first-excited state than that in the ground state, the energy spacing increases with increasing the coupling strength. The increase in energy spacing causes a decrease in the period of oscillation.

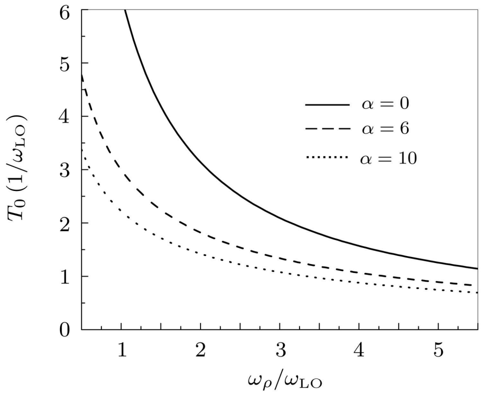

Fig. 2 The period of oscillation as a function of the confinement strength in the xy-plane with the fixed coupling strength for $\omega_z = 20\omega_{\rm LO}$.

Figure 3 presents the period of oscillation as a function of the confinement strength in the z-direction with the different coupling strength ($\alpha = 6, 8, 10$) for $\omega_\rho = 5\omega_{\rm LO}$. It can be seen that the period of oscillation is slowly decreasing function of the confinement strength in the z-direction. The reason is that, the energy spacing between the ground state and the first-excited state increases slowly with increasing the confinement strength in the z-direction. Figure 3 also shows that the period of oscillation decreases with increasing the electron-phonon coupling strength. A qubit cannot be independent of environment and must be interacted with the heat bath. As a result, the interaction destroys the superposition state of a qubit, which is decoherence.[8] The period of oscillation T_0 decreases, that is the life time of a qubit reduces, so the process of decoherence is quicken. It is very harmful to store information which makes the QD as its elementary unit. But, in principle, this effect can be minimized by a more precise fabrication technology, by cooling the crystal and by choosing the state and the physical parameters properly.[9]

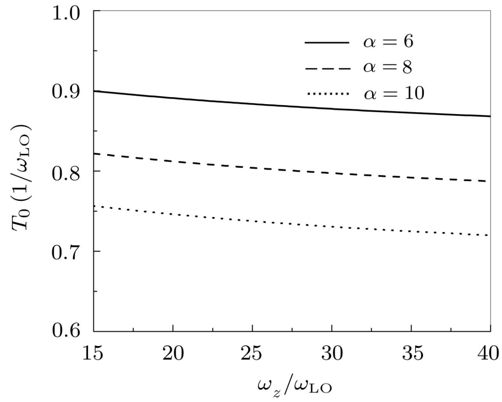

Fig. 3 The period of oscillation as a function of the confinement strength in the z-direction with the fixed coupling strength for $\omega_\rho = 5\omega_{\rm LO}$.

4 Summary

The eigenenergies and the relevant eigenwavefunctions of the ground state and the first-excited state of electron have been obtained in an unsymmetrical parabolic confinement potential QD using the Pekar type variational method in the electron-LO-phonon strong-coupling region. The single qubit can be envisaged as this kind of two-level quantum system in a QD. The probability density of electron oscillates with a period when the electron is in the superposition state of the ground state and the first-excited state. It is found that the period of oscillation decreases with increasing the electron-phonon coupling strength, the confinement strengths in the xy-plane and the z-direction in an unsymmetrical parabolic QD.

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