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Quantum Non-thermal Effect From Kerr-Newman Black Hole*

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Abstract *We present a short and direct derivation of Hawking radiation by using the Damour-Ruffini method, as taking into account the self-gravitational interaction from the Kerr-Newman black hole, It is found that the radiation is not exactly thermal, and because the derivation obey conservation laws, the non-thermal Hawking radiation can carry information from the black hole. So it can be used to explain the black hole information paradox, and the process satisfies unitary.*

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The Damour and Ruffini suggested[1] that a massive charged particle could be tunneled out by a wave function over the horizon, which gives rise to the creation of a pair: one particle will go out and another antiparticle will fall back toward the singularity, and they obtained the spectrum of Hawking radiation. In 2000, a semi-classical method of modeling Hawking radiation as a tunneling effect has been developed.[2] Its main point is that the Hawking radiation of a black hole does not have a pure thermal spectrum if the self-gravitation taken into account. A key insight is to find a coordinate system well behaving at the event horizon to calculate the emission rate. Tunneling provides not only a useful verification of thermodynamic properties of black holes but also an alternate conceptual means for understanding the underlying physical process of black hole radiation. It has been shown to be very robust, having been successfully applied to a wide variety of exotic space-times.[3−15] However, there is a complicated calculating process of the imaginary part of the action for the outgoing particle. Recently, the authors of Ref. [16] have derived the Hawking radiation from a static spherically symmetric black hole, based on the Damour–Ruffini method and considering the self-gravitation interaction and energy conservation. Their result shows that the radiation is not exactly thermal but the non-thermal Hawking radiation can carry information from the black hole. This can be used to explain the black hole information paradox and in addition the process satisfies unitary. In this paper, we attempt to extend this method to stationary axisymmetric Kerr-Newman black holes. A new method to calculate the corrected non-thermal Hawking radiation of the stationary black hole is given, which is more general.

In a quantum system, the transmission of outgoing-wave can be expressed as

$$\Gamma(i \to f) = |M_{fi}|^2 A \,, \tag{1}$$

where A is a phase factor, $|M_{fi}|^2$ is the square of the probability amplitude in the transition process, and A can be obtained by summing over all final states and averaging over all initial states. The number of final states can be expressed in exponential form through the final states entropy $\mathrm{e}^{\Delta S}$, and similarly, the number of the initial states can be expressed in exponential form through the final states entropy $\mathrm{e}^{\Delta S}$. We obtain

$$\Gamma = \frac{\exp(S_{\text{final}})}{\exp(S_{\text{initial}})} = \mathrm{e}^{\Delta S} \,. \tag{2}$$

Next, we consider the Kerr-Newman space-time system, theirs metric is[17]

$$\begin{aligned} \mathrm{d}s^2 = &-\Big(1 - \frac{2Mr - Q^2}{\rho^2}\Big)\mathrm{d}t^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 \\ &+ \Big[(r^2 + a^2)\sin^2\theta + \frac{(2Mr - Q^2)a^2\sin^4\theta}{\rho^2}\Big]\mathrm{d}\phi^2 \\ &- \frac{2(Mr - Q^2)a\sin^2\theta}{\rho^2}\mathrm{d}t\mathrm{d}\phi \,, \end{aligned} \tag{3}$$

where $\rho = r^2 + a^2\cos^2\theta$, $\Delta = r^2 + a^2 + Q^2 - 2Mr = (r - r_+)(r - r_-)$. So we can obtain

$$r_\pm = M \pm \sqrt{M^2 - a^2 - Q^2} \,. \tag{4}$$

In this expression, r_+ and r_- are the event horizon and the inner horizon, M is the total mass of the black hole, and a is the angular momentum per unit mass of the black hole. The surface gravity of the event horizon is

$$\kappa = \frac{r_+ - r_-}{2(r_+^2 + a^2 - Q^2)} \,. \tag{5}$$

There is a coordinate singularity in the metric (3) at the radius of the event horizon. To extend Damour–Ruffini's work to Kerr-Newman space-time, we should first find a

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coordinate system from which we expect that it well behave at the event horizon, and its coordinate clock synchronization can be transmitted from one place to another. We first investigate the dragging coordinate system. Let there be

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{g_{03}}{g_{33}} = \Omega\,. \tag{6}$$

The space-time metric corresponding to the Kerr-Newman black hole can be rewritten as

$$\begin{aligned}\mathrm{d}s^2 &= -\frac{\rho^2\Delta}{(r^2+a^2)^2-\Delta a^2\sin^2\theta}\mathrm{d}t^2 + \frac{\rho^2}{\Delta}\mathrm{d}r^2 + \rho^2\mathrm{d}\theta^2 \\ &= \hat{g}_{00}\,\mathrm{d}t^2 + g_{11}\,\mathrm{d}r^2 + g_{22}\,\mathrm{d}\theta^2\,. \end{aligned}\tag{7}$$

In fact, the line element (7) represents a three-dimensional hyper-surface in four-dimensional Kerr-Newman space-time. We can get the pure thermal spectrum of the Hawking radiation by using the Damour–Ruffini method. It means that we can also discuss the non-thermal Hawking radiation in the dragging coordinate system. From Eq. (7), we have

$$\begin{aligned}\hat{g}^{00} &= -\frac{(r^2+a^2)^2-\Delta a^2\sin^2\theta}{\rho^2\Delta}\,,\\ g^{11} &= \frac{\Delta}{\rho^2},\quad g^{22} = \frac{1}{\rho^2}\,,\\ \sqrt{-g} &= \frac{\rho^2\sqrt{\rho^2}}{\sqrt{(r^2+a^2)^2-\Delta a^2\sin^2\theta}}\,.\end{aligned}\tag{8}$$

In curved space-time the Klein–Gordon equation is

$$\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) - m_0^2\varphi = 0\,. \tag{9}$$

With Eq. (7), the Klein–Gordon equation can be reduced to

$$\begin{aligned}&g^{11}\frac{\mathrm{d}^2R(r)}{\mathrm{d}r^2} + \frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}(\sqrt{-g}g^{11})\frac{\mathrm{d}R(r)}{\mathrm{d}r} + \frac{R(r)}{\psi(\theta)}F(r,\theta)\\ &\quad= \Big[m_0^2 + \Big(\omega + \frac{g_{03}}{g_{33}}\Big)^2\Big]R(r)\,,\end{aligned}\tag{10}$$

where

$$\begin{aligned}F(r,\theta) &= g^{22}\frac{\mathrm{d}^2\psi(\theta)}{\mathrm{d}\theta^2} + \frac{1}{\sqrt{-g}}\frac{\partial\varphi}{\partial\theta}\frac{\partial}{\partial\theta}(\sqrt{-g}g^{22})\\ &\quad\times\frac{\mathrm{d}\psi(\theta)}{\mathrm{d}\theta}\,.\end{aligned}\tag{11}$$

And the wave function φ has been separated as

$$\varphi = \mathrm{e}^{-\mathrm{i}\omega t}R(r)\psi(\theta)\,\mathrm{e}^{\mathrm{i}m\phi}\,. \tag{12}$$

Introduce the Tortoise coordinate

$$r_* = \frac{1}{2\kappa}\ln\Big(\frac{r-r_+}{r_+}\Big)\,. \tag{13}$$

Thus

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}r} &= \frac{1}{2\kappa(r-r_+)}\frac{\mathrm{d}}{\mathrm{d}r_*}\,,\\ \frac{\mathrm{d}^2}{\mathrm{d}r^2} &= \frac{1}{4\kappa^2(r-r_+)^2}\frac{\mathrm{d}^2}{\mathrm{d}r_*^2} - \frac{1}{2\kappa(r-r_+)^2}\frac{\mathrm{d}}{\mathrm{d}r_*}\,.\end{aligned}\tag{14}$$

Substituting Eq. (14) into Eq. (10), we obtain

$$\begin{aligned}&\frac{\mathrm{d}^2R(r)}{\mathrm{d}r_*^2} - 2\kappa\frac{\mathrm{d}R(r)}{\mathrm{d}r_*} + 2\kappa(r-r_+)\Big(\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\sqrt{-g} + \frac{1}{g^{11}}\frac{\partial g^{11}}{\partial r}\Big)\frac{\mathrm{d}R(r)}{\mathrm{d}r_*}\\ &\quad+\frac{4\kappa^2(r-r_+)^2}{g^{11}}\frac{R(r)}{\psi(\theta)}F(r,\theta) = \frac{4\kappa^2(r-r_+)^2}{g^{11}}\Big[m_0^2 + \Big(\omega + m\frac{g_{03}}{g_{33}}\Big)^2 g^{00}\Big]R(r)\,.\end{aligned}\tag{15}$$

Obviously, for $r\to r_+$, we get

$$\begin{aligned}&2\kappa(r-r_+)\Big(\frac{1}{\sqrt{-g}}\frac{\partial}{\partial r}\sqrt{-g}\Big) = 0\,,\\ &2\kappa(r-r_+)\Big(\frac{1}{g^{11}}\frac{\partial g^{11}}{\partial r}\Big) = 2\kappa\,,\\ &\frac{4\kappa^2(r-r_+)^2}{g^{11}}\Big[m_0^2 + \Big(\omega + m\frac{g_{03}}{g_{33}}\Big)^2 g^{00}\Big]R(r)\\ &\quad = -(\omega - m\Omega_h)^2R(r)\,.\end{aligned}\tag{16}$$

Thus, we get the standard wave equation near the event horizon

$$\frac{\mathrm{d}^2R(r)}{\mathrm{d}r_*^2} + (\omega-\omega_0)^2R(r) = 0\,, \tag{17}$$

where $\omega_0 = m\Omega_h$. By a simple calculation, we can get the radial wave solutions of Eq. (17) as

$$R_1(r) = \mathrm{e}^{-\mathrm{i}(\omega-\omega_0)r_*},\quad R_2(r) = \mathrm{e}^{\mathrm{i}(\omega-\omega_0)r_*}\,. \tag{18}$$

From $R(t,r) = \mathrm{e}^{-\mathrm{i}\omega t}R(r)$, we obtain the ingoing-wave and outgoing-wave solutions

$$R^{\mathrm{in}} = R(t,r)R_1(r) = \mathrm{e}^{-\mathrm{i}\omega v}\,, \tag{19}$$

$$R^{\mathrm{out}} = R(t,r)R_2(r) = \mathrm{e}^{2\mathrm{i}(\omega-\omega_0)r_*}\,\mathrm{e}^{-\mathrm{i}\omega v}\,, \tag{20}$$

where $v = t + [(\omega-\omega_0)/\omega]r_*$ is the advanced Eddington–Finkelstein coordinate.

In the following discussion, we first rewrite R^{out} as follows

$$R^{\mathrm{out}} = \mathrm{e}^{-\mathrm{i}\omega v}\Big(\frac{r-r_+}{r_+}\Big)^{\mathrm{i}(\omega-\omega_0)/2\kappa_+}\,, \tag{21}$$

which shows that the outgoing wave is not analytic on the event horizon r_+. Therefore we have to extend it by analytical continuation to inside the horizon through the lower-half complex r-plane

$$(r\to r_+)\to|r-r_+|\,\mathrm{e}^{-\mathrm{i}\pi} = (r_+-r)\,\mathrm{e}^{-\mathrm{i}\pi}\,. \tag{22}$$

Then near the event horizon r_+, R^{out} can be rewritten as

$$R^{\mathrm{out}} = \mathrm{e}^{-\mathrm{i}\omega v}\,\mathrm{e}^{\pi(\omega-\omega_0)/\kappa_+}\,\mathrm{e}^{2\mathrm{i}(\omega-\omega_0)r_*}\,. \tag{23}$$

The scattering probability of the outgoing wave at the event horizon is

$$\Gamma = \Big|\frac{R^{\mathrm{out}}(r>r_+)}{R^{\mathrm{out}}(r<r_-)}\Big|^2 = \mathrm{e}^{-2\pi(\omega-\omega_0)/\kappa_+}\,. \tag{24}$$

If we consider the influence of the radiant particle the self-gravitational interaction to time-space background and considers radial process of energy as an integral process, the equation (24) can be rewritten as

$$\Gamma = \prod_i \Gamma_i = \exp\Big(-2\pi \int_0^{\omega} \frac{1}{\kappa_+} \mathrm{d}\omega'\Big)\,. \tag{25}$$

According to the first law of thermodynamics, we have

$$\mathrm{d}S' = -\frac{\mathrm{d}\omega'}{T}\,. \tag{26}$$

Therefore, Eq. (25) can be expressed as

$$\Gamma = \exp\Big[-\int_0^{\omega} \Big(\frac{1}{T}\Big)\mathrm{d}\omega'\Big] = \mathrm{e}^{\Delta S}\,, \tag{27}$$

this result is the same as Eq. (2), where

$$\begin{aligned}\Delta S &= \frac{1}{4} \times 4\pi \times \Big[(M-\omega) + \sqrt{(M-\omega)^2 - a^2 - Q^2}\,\Big]^2 - \frac{1}{4} \times 4\pi \times \Big(M + \sqrt{M^2 - a^2 - Q^2}\,\Big)^2 \\ &\approx -2\pi\Big[\frac{\omega}{\kappa_+} - \Big(1 + \frac{3M}{2\sqrt{M^2-a^2-Q^2}} + \frac{M^3}{4(M^2-a^2-Q^2)^{3/2}}\Big)\omega^2 + \cdots\Big]\,. \end{aligned} \tag{28}$$

ΔS is the entropy change of the black hole between before and after the emission. This result is obviously consists with an underlying unitary theory.

In this Letter, we obtained the result that the permeation ratio of the outgoing-wave revises the thermal radiation spectrum from the Kerr-Newman black hole, while taking into account the self-gravitational interaction of the radiant particle energy to space-time background. This derived result is in contrast with previous analysis of the same subject by using the tunneling method, and satisfies unitary. However, we use quite a different way, which is more simple, more direct, and clearer in its physical meaning. Moreover, the calculation is easy, and we do not have to care whether a radiant particle has restmass. It is obvious that if we consider the self-gravitational interaction, the transmission ratio of outgoing-wave at the event area appears to deviate from the thermal radiation spectrum of the black hole, which might contain related information about the material that makes up the black hole. This result may lead to resolving the problem of information loss. In fact, if we calculate the Hawking radiation of the Kerr-Newman black hole by using the Damour–Ruffini method without considering the self-gravitational interaction of the radiation energy to time-space background, we get a precise radiation spectrum of the tunneling process through the event area. The outgoing-wave has a potential barrier in Eq. (15), which lies between the event area and infinite distance. Therefore, in the opinion of an observer looking at the event area from an infinite distance the black hole radiation spectrum, dispersed by the shape, a gray spectrum.

References

[1] T. Damuor and R. Ruffini, Phys. Rev. D **14** (1976) 332.

[2] M.K. Parikh and F. Wiclzek, Phys. Rev. Lett. **85** (2000) 5042.

[3] E.C. Vagenas, Phys. Lett. B **503** (2001) 399.

[4] A.J.M. Medved, Phys. Rev. D **66** (2002) 124009.

[5] M.K. Parikh, Int. J. Mod. Phys. D **13** (2004) 2355.

[6] J.Y. Zhang and Z. Zhao, Phys. Lett. B **618** (2005) 14.

[7] J.Y. Zhang and Z. Zhao, Mod. Phys. Lett. A **21** (2006) 1865.

[8] S.Z. Yang, Q.Q. Jiang, and H.L. Li, Chin. Phys. **14** (2005) 2411.

[9] S.Z. Yang, Chin. Phys. Lett. **22** (2005) 2492.

[10] J.Y. Zhang and Z. Zhao, Phys. Lett. B **638** (2005) 110.

[11] H.Z. Fang, Y.P. Hu, and Z. Zhao, Chin. Phys. Lett. **22** (2005) 1611.

[12] W.B. Liu, Phys. Lett. B **634** (2006) 541.

[13] Q.Q. Jiang, S.Q. Wu, and X. Cai, Phys. Rev. D **73** (2006) 064003.

[14] J. Ren, J.Y. Zhang, and Z. Zhao, Chin. Phys. Lett. **23** (2006) 2019.

[15] Y.W. Han, Chin. Phys. Lett. **24** (2007) 333.

[16] W.B. Liu, Acta Phys. Sin. **56** (2007) 6164.

[17] B. Carter, Phys. Rev. **174** (1968) 1559.