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Improved $Z^{1/3}$ Law of Nuclear Charge Radius*

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Abstract *An improved $Z^{1/3}$ law of nuclear charge radius is presented. The comparison between the calculated and experimental nuclear charge radii now available shows that this new formula is better than the other conventional formulae.*

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Key words: nuclear charge radius, improved $Z^{1/3}$ law, isospin dependence

The radius of nuclear charge distribution is one of the nuclear fundamental properties and usually described by the $A^{1/3}$ law,[1,2]

$$R_c = r_0 A^{1/3}\,, \tag{1}$$

where A is the nuclear mass number and $R_c = \sqrt{(5/3)\langle r^2\rangle}$, $\sqrt{\langle r^2\rangle}$ is the root-mean-square (rms) radius. Many years ago, from the analysis of very limited data of charge radii of β-stable nuclei then available it was noted that the "radius constant" r_0 decreases systematically with increasing A rather than keeps a constant, i.e.

$$r_0 \approx \begin{cases} 1.30\ \text{fm, for light nuclei (e.g. } ^{32}\text{S, } ^{40}\text{Ca, etc.)}\,, \\ 1.20\ \text{fm, for heavy nuclei (e.g. } ^{197}\text{Au, } ^{208}\text{Pb, etc.)}\,. \end{cases} \tag{2}$$

It is well known that various nuclear bulk properties, such as the nuclear binding energy,[1,2] the Wigner's isobaric mass multiplet equation (IMME),[3] the separate neutron shells and proton shells in a nucleus, and the shell model harmonic oscillator potential strength $\hbar\omega_p$ and $\hbar\omega_n$,[4] etc, are all isospin-dependent. Thus, it is not surprised that such a simple isospin-independent $A^{1/3}$-law is unable to satisfactorily describe the global variation in R_c with mass number A.

Moreover, it was noted[5] that the experimental radius change $\delta R_c = R_c(Z, A+\delta A) - R_c(Z, A)$ within an isotopic chain significantly smaller than that predicted by the $A^{1/3}$-law, $\delta R_c = R_c \delta A/3A$. On the contrary, the observed charge radius difference between neighboring isotones, $\delta R_c = R_c(Z+\delta Z, N) - R_c(Z, N)$, is usually much larger than that predicted by the $A^{1/3}$ law. In view of these reasons, it was suggested[5] that the charge radii for the most β-stable nuclei would follow the $Z^{1/3}$-law,

$$R_c = r_p Z^{1/3}\,. \tag{3}$$

It was found that, in contrast to the $A^{1/3}$ law, the radius parameter of $Z^{1/3}$ law r_p keeps almost a constant, $r_p \approx 1.63$ fm. Considering $Z = A/2 - T_z$, we have $(Z/A)^{1/3} \approx (1/2)^{1/3}(1 - 2T_z/3A)$, T_z is the isospin. For the most β-stable light nuclei ($T_z \approx 0$), $(Z/A)^{1/3} \approx (1/2)^{1/3}$, whereas for the most heavy nuclei, (e.g. ^{197}Au, ^{208}Pb, etc.), $(Z/A)^{1/3}/(1/2)^{1/3} \approx 0.93$. Thus, it is understandable why $r_0 \approx 1.30$ fm for the most β-stable light nuclei, and r_0 decreases gradually to $r_0 \approx 1.20$ fm for the most β-stable heavy nuclei. Furthermore, in the semiempirical nuclear mass formula, if the coulomb energy term $\propto Z^2/A^{1/3}$ (based on $R_c \propto A^{1/3}$) is replaced by a term $\propto Z^{5/3}$ (based on $R_c \propto Z^{1/3}$), the agreement between the calculated and experimental binding energies and the location of β-stability line is improved.[6]

In the past two decades a vast amount of new experimental informations on the electromagnetic structure of nuclear ground state became available and the accuracy was much improved. Almost all the β-stable nuclei have been measured by the muon X-ray transition technique and the corresponding charge radii have been accurately obtained (the experimental relative error is about 10^{-3}). Moreover, modern techniques for optical isotope shift measurement have made it possible to reach short-lived (down to 1 s) unstable isotopes, and more and more nuclei far from the β-stability line become available experimentally.[7,8] Particularly, a set of about 800 ground state nuclei charge radii deduced from the experimental data of elastic electron scattering, muonic atom X-ray, K_α isotope shift, and optical isotope shift have been presented in the article by I. Angeli.[8] First, let us reexamine

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the validity of both the $A^{1/3}$ law and $Z^{1/3}$ law for describing the global variation in charge radii for the most β-stable nuclei.

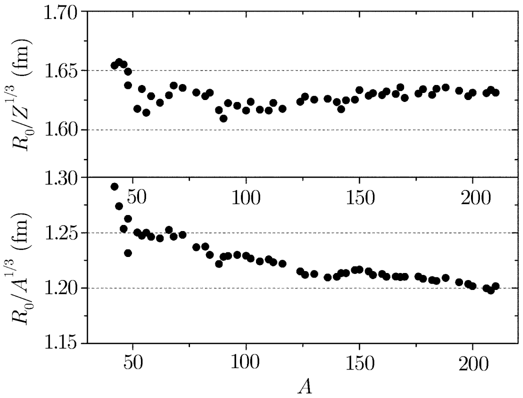

Fig. 1 The global variation in the experimental nuclear charge radius parameters for 75 most β-stable nuclei, $r_0 = R_0/A^{1/3}$, and $r_p = R_0/Z^{1/3}$.

In Fig. 1 the nuclear quadrupole deformation effect on the charge radii R_c has been taken into account,

$$R_0 = \frac{R_c}{\sqrt{1 + (5\beta^2/4\pi)}}\,, \tag{4}$$

where R_0 is the counterpart of R_c for a spherical nucleus, and β is the quadrupole deformation derived from the experimental intrinsic quadrupole moment Q_0,[9]

$$Q_0 = \frac{5Z\langle r^2\rangle}{\sqrt{5\pi}}\beta(1 + 0.158\beta)\,. \tag{5}$$

Note that the deformation parameter β presented in Ref. [9] is derived by the relation $Q_0 = (5Z\langle r^2\rangle/\sqrt{5\pi})\beta$ (neglecting the smaller β^2-term), so is a little larger than that derived by Eq. (5) for the well deformed nuclei with $\beta \geq 0.20$. In Fig. 1 the experimental $R_0/A^{1/3}$ and $R_0/Z^{1/3}$ for 75 most β-stable nuclei ($20 \leq Z \leq 94$) are presented. It is found that the charge radius parameter and the relative root-mean-square deviation χ derived by the $A^{1/3}$ law and the $Z^{1/3}$ law are respectively,

$$R_0 = r_0 A^{1/3}, \quad r_0 = 1.220 \text{ fm}, \quad \chi = 16.2\times10^{-3}\,, \tag{6}$$

$$R_0 = r_p Z^{1/3}, \quad r_p = 1.627 \text{ fm}, \quad \chi = 4.70\times10^{-3}\,. \tag{7}$$

As expected, for the most β-stable nuclei the isospin-dependent $Z^{1/3}$ law works much better than the isospin-independent $A^{1/3}$ law; i.e. while r_0 gradually decreases with increasing A, r_p approximately keeps constant (except the other shell effects).

Though the $Z^{1/3}$ law of nuclear charge radii works much better than the $A^{1/3}$ law for the most β-stable nuclei, it is extravagant to hope that such a simple one-parameter $Z^{1/3}$-law could work well also for the exotic nuclei far away from the β-stable line with high $\mid T_z - T_z^* \mid$ values, (T_z^* is the isospin of most β-stable nucleus). However, the $Z^{1/3}$ law may be served as a good starting point for further improvements.

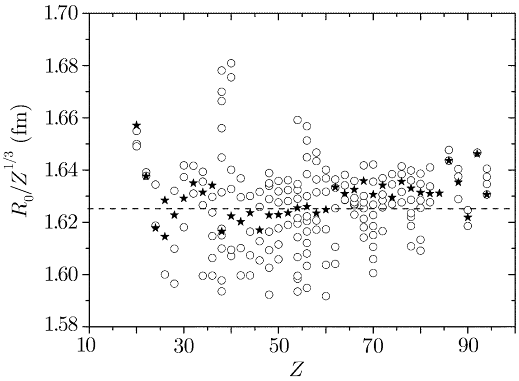

Fig. 2 The experimental $R_0/Z^{1/3}$ for the even-even nuclei ($20 \leq Z \leq 94$). The most β-stable nucleus in each isotopic chain is marked by a star $\star$.

In Fig. 2 we present the experimental $R_0/Z^{1/3}$ for the even-even nuclei ($20 \leq Z \leq 94$). The $R_0/Z^{1/3}$ for the most β-stable nucleus in each isotopic chain is marked by a star $\star$. It is noted that in each isotopic chain, the $R_0/Z^{1/3}$ values diverse uniformly with increasing $(T_z - T_z^*)$ except the other shell effects on charge radii. To show this more clearly, as a typical example, the $R_0/Z^{1/3}$ vs. T_z for the $_{56}$Ba isotopic chain is shown in Fig. 3. Obviously, an approximate linear dependence on T_z of $R_0/Z^{1/3}$ for a given isotopic chain is seen (except for an obvious neutron shell effect near $N = 82$). It is noted that a linear T_z dependence was also approximately reproduced by the RMF (relativistic mean field) calculations for nuclear radii within a chain of isotopes.[10,11]

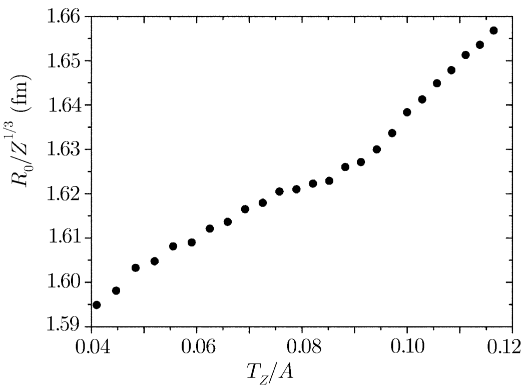

Fig. 3 $R_0/Z^{1/3}$ vs. T_z/A for the $_{56}$Ba isotopic chain.

Therefore, it is expected that the following improved $Z^{1/3}$-law may be a promising candidate for describing the global variation in charge radii including the exotic nuclei far away from the β-stable line,

$$R_0 = r_p Z^{1/3}\Big[1 + b(T_z - T_z^*)\frac{1}{A}\Big]\,, \tag{8}$$

where $T_z^* \approx 3.0 \times 10^{-3}\ A^{5/3}$ is the value for the most β-stable nuclei.[1] The experimental $\sqrt{\langle r^2 \rangle}$ of 441 nuclei $(20 \le Z \le 94)$[8] whose deformation β have been derived from the experimental intrinsic quadrupole moment Q_0[9] by Eq. (5) are analyzed by using this improved $Z^{1/3}$-law (8). The results are as follows,

$$r_p = 1.623 \text{ fm}, \quad b = 0.452, \quad \chi = 5.93 \times 10^{-3} . \quad (9)$$

For comparison, an improved $A^{1/3}$-law including a similar linear $(T_z - T_z^*)/A$ term is also used to analyze these 441 nuclei,

$$R_0 = r_0 A^{1/3} \left[1 + b(T_z - T_z^*) \frac{1}{A} \right] , \quad (10)$$

and the results are

$$r_0 = 1.221 \text{ fm}, \quad b = -0.283, \quad \chi = 13.72 \times 10^{-3} . \quad (11)$$

It is seen that the rms deviation for the improved $A^{1/3}$ law (10) is much larger than that for the improved $Z^{1/3}$-law (8). An expression similar to Eq. (10) was proposed by B. Nerlo–Pomorska and K. Pomorski,[12]

$$R_0 = r_0 A^{1/3} [1 + \alpha(I - I_\beta)] , \quad (12)$$

where $\alpha = 0.223$, $I = (N-Z)/A$ is the reduced isospin and I_β is that for the β-stable isotope, $I_\beta = 0.4A/(200 + A)$ (Green estimation).

If the deformation effect is not taken into account, the 716 charge radius data $(20 \le Z \le 94)$ presented in Ref. [8] are also analyzed using the improved $Z^{1/3}$ law, $R_c = r'_p Z^{1/3}[1 + b'(T_z - T_z^*)]$, and the result is

$$r'_p = 1.639 \text{ fm}, \quad b' = 0.459, \quad \chi = 8.729 \times 10^{-3} . \quad (13)$$

Similarly, for the improved $A^{1/3}$-law , $R_c = r'_0 A^{1/3}[1 + b'(T_z - T_z^*)]$, one gets

$$r'_0 = 1.226 \text{ fm}, \quad b' = -0.140, \quad \chi = 14.96 \times 10^{-3} . \quad (14)$$

In summary, from the analysis of experimental nuclear charge radii now available, it is found that the improved $Z^{1/3}$-law (8) is superior to the other conventional formulae based on the $A^{1/3}$-law. This fact seems to be consistent with the experimental evidence that $E_x A^{1/3}$ increases gradually with increasing A,[13] where E_x ($\propto 1/R_0$) is the nuclear giant monopole resonance energy.

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