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Friedmann Cosmology with Bulk Viscosity: A Concrete Model for Dark Energy*

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Abstract *The universe content is considered as a non-perfect fluid with bulk viscosity and can be described by a general equation of state (endowed some deviation from the conventionally assumed cosmic perfect fluid model). An explicitly bulk viscosity dark energy model is proposed to confront consistently with the current observational data sets by statistical analysis and is shown consistent with (not deviated away much from) the concordant Λ Cold Dark Matter (CDM) model by comparing the decelerating parameter. Also we compare our relatively simple viscosity dark energy model with a more complicated one by contrast with the concordant ΛCDM model and find our model improves for the viscosity dark energy model building. Finally we discuss the perspectives of dark energy probes for the coming years with observations.*

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1 Introduction

Lots of astrophysics observations have indicated that the cosmic expansion is accelerating.[1] In the past decade, modified gravity model,[2] decaying Λ Cold Dark Matter (CDM) model,[3] modified equation of state (hereafter EOS) and so on are explored to explain the cosmic acceleration mechanism. To consider causality and the hydrodynamical instability, an interesting barotropic dark energy model is proposed[4] that includes a linear EOS,[5] which incorporated into cosmological model can describe the hydrodynamically stable dark energy behaviors.

The astrophysical observations also indicate some evidences that the cosmic media is not a perfect fluid,[6] and the viscosity effect could be concerned in the evolution of the universe.[7–9] On the other hand, in the standard cosmological model, if the EOS parameter ω is less than -1, so-called phantom, the universe shows the future finite time singularity called the Big Rip[10,11] or Cosmic Doomsday. Several mechanisms are proposed to prevent the future big rip, like by considering quantum effects terms in the action,[12] or by including viscosity effects for the Universe evolution.[13] So some interesting questions naturally arise: Are there some new kinds of singularities emerge when concerns the cosmic viscosity? And what kind of role the cosmic viscosity element can play for helping the above two facets: dark energy and cosmic singularity? Considering some deviation from the ideal fluid model is also very helpful to nowadays cosmology probes advancement.

In past papers,[14–20] the bulk viscosity effects in cosmology have been studied in various aspects. Dissipative processes are thought to be present in any realistic theory of the evolution of the universe. The early universe thermodynamics system is far from equilibrium, the viscosity deviation should be concerned in the studies of the early stage for cosmological evolution. But even in the later cosmic evolution stage, for example, the temperature for the intergalactic medium (IGM), baryonic gas, generally is about 10^4 K to 10^6 K and the complicated IGM is rather non-trivial. The sound speed c_s in the baryonic gas is only a few km$\cdot$s^{-1} to a few tens km$\cdot$s^{-1} and the Jeans length λ yields a term as an effective viscosity $c_s\lambda$. On the other hand, the bulk velocity of the baryonic gas is of the order of hundreds km$\cdot$s^{-1}.[21] So it is helpful to consider the viscosity element in the later cosmic evolution. It is well known that in the framework of homogeneous and isotropy Friedmann–Robertson–Walker (FRW) metric, the shear viscosity has no contribution in the energy-momentum tensor, the bulk viscosity term has the same dimension as pressure, and it behaves like an effective pressure. In the late universe, since we do not know clearly the nature of the universe content (dark matter and dark energy components), concerning the bulk viscosity contribution in the energy-momentum is reasonable and practical. Moreover, the cosmic viscosity here can also be regarded as an effective quantity as caused by some complicated astrophysics mechanisms and may play a role as a dark energy candidate[22–24] or a possible unification scheme for the two mysterious dark components (dark matter and dark energy)[24,33,34] as they may be facets of the same problem, and then evidence for dark matter is also evidence for dark energy.

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The bulk viscosity $\zeta = \alpha\rho^s$ has been studied in the full causal theory of bulk viscosity, and the cases $s = 1/2$ and other specific values have possessed exact solutions with interesting properties as shown in Refs. [16] and [17]. We argue that the non-causal approximation is reasonable and useful in the late stage of the universe evolution. In our previous papers,[13] we have shown that the Friedmann equations can be solved with both a more general EOS and bulk viscosity detailed as follows. (Note that the EOS with a constant pressure gets degenerate with ΛCDM model)

$$p = (\gamma - 1)\rho + p_0 \,, \tag{1}$$

where p_0 and γ are two parameters. The bulk viscosity is expressed as

$$\zeta = \zeta_0 + \zeta_1 \frac{\dot{a}}{a} \,, \tag{2}$$

where ζ_0 and ζ_1 are two constants conventionally, and the overhead dot stands for derivative with respect to time. The motivation of considering this bulk viscosity is that by fluid mechanics we know the transport (dissipation)/viscosity phenomenon is involved with the "velocity" $\dot{a}$, which is related to the scalar expansion $\theta = 3\dot{a}/a$. A linear combination of $\zeta = \zeta_0$ (constant) and $\zeta \propto \theta$ is a more general form of viscosity term than considering them separately in the previous papers.[7,25] Inequality in our previous case should be constrained by present observation data as

$$-1.38 < \gamma - 1 + \frac{p_0}{\rho} < -0.82 \,. \tag{3}$$

The parameter p_0 can be positive (attractive force) or negative (repulsive force), and conventionally ζ_0 and ζ_1 are regarded as positive. To choose the parameters properly, it can prevent the Big Rip problem or some other kinds of singularity for the cosmology model, like in the phantom energy phase, as demonstrated before.

This present paper is a continuous work following our previous efforts, which is organized as follows. In Sec. 2 we present a relatively simple cosmology model with extremely non-relativistic dark matter and viscosity dark energy by an explicit form. With these we give out the exact solution and discuss the acceleration phase in this model. In the last section (Sec. 3) we discuss and summarize our conclusions.

2 Model and Calculations

We consider the Friedmann–Roberson–Walker metric in the flat space geometry ($k = 0$) as the case favored by WMAP satellite mission on cosmic background radiation (CMB) data

$$ds^2 = -dt^2 + a(t)^2(dr^2 + r^2 d\Omega^2) \,, \tag{4}$$

and assume that the cosmic fluid possesses a bulk viscosity. The energy-momentum tensor can be written as

$$T_{\mu\nu} = \rho U_\mu U_\nu + (p + \Pi) H_{\mu\nu} \,, \tag{5}$$

where in the co-moving coordinates $U^\mu = (1, 0)$, and $H_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$.[25] By defining the effective pressure as $\tilde{p} = p + \Pi$ and from the Einstein equation $R_{\mu\nu} - g_{\mu\nu}R/2 = 8\pi G T_{\mu\nu}$, we obtain the Friedmann equations

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \,, \tag{6a}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3\tilde{p}) \,. \tag{6b}$$

The covariant conservation equation for energy $T^{0\nu}_{;\nu}$, yields

$$\dot{\rho} + (\rho + \tilde{p})\theta = 0 \,, \tag{7}$$

where the expansion parameter $\theta = U^\mu_{;\mu} = 3\dot{a}/a$.

Then the Friedmann equations give:

$$8\pi G\rho = 3\Big(\frac{\dot{a}}{a}\Big)^2 = 3H^2 \,.$$

If we assume that the cosmic fluid possesses a bulk viscosity as shown explicitly in the pressure as $\Pi = -\xi\theta$ (we take another Greek character to represent the bulk viscosity which is different from our previous papers as briefly summarized in the above introduction section), the energy-momentum tensor could be written fully as:

$$T_{\mu\nu} = \rho U_\mu U_\nu + (p - \xi\theta) H_{\mu\nu} \,,$$

where $\theta = 3H$ in FRW background, and the explicit form of the bulk viscosity ξ will be presented below.

The cold dark matter has been assumed to be extremely non-relativistic so that we can take its pressure $p = 0$, and we now suppose that the effect of dark energy on cosmos evolution is included in the viscous term $-\xi\theta$ which has the dimension of pressure. In this present work we treat it as the effective pressure of dark energy. Then the covariant conservation of $T_{\mu\nu}$ (Eq. (7)) yields:

$$\dot{\rho} + (\rho - \xi\theta)\theta = 0 \,,$$

so,

$$\dot{\rho} + \rho\theta = \xi\theta^2 \,. \tag{8}$$

Here, it will be convenient to define a dimensionless parameter below:

$$h^2 = \frac{H^2}{{H_0}^2} = \frac{\rho}{\rho_{\rm cr}} \,,$$

where $\rho_{\rm cr}$ is the critical density. Using this definition, we could rewrite Eq. (8) as:

$$\frac{dh^2/dt}{H_0} + 3h^3 = 9\lambda h^2 \,, \tag{9}$$

where the bulk viscosity is redefined as $\lambda = \xi H_0/\rho_{\rm cr}$.

For comparison with the observational data, we should derive h from Eq. (9) by writing it as the equation of redshift z as:

$$-2(1 + z)\frac{dh}{dz} + 3h = 9\lambda \,. \tag{10}$$

In the last part of Ref. [33] condition with constant viscosity has been considered. It proves that it has a good fitting with SNe Ia data sets. Recently, a complicated viscosity form dependent on h has been proposed in Ref. [32]. To overcome old cosmological constant problem we believe that a variable viscosity will give a better result. Here we propose an explicit red-shift dependent bulk viscosity, which gets a constant limit (an effective cosmological constant) today as $z = 0$. And we will show that this relatively simple form is better than the complex one in Ref. [32] when compared with ΛCDM model. The explicit red-shift dependent viscosity is

$$9\lambda = \lambda_0 + \lambda_1(1+z)^n\,, \tag{11}$$

where n is an arbitrary number, λ_0, and λ_1 are two arbitrary constants, which could all be best fitted from the observational data sets. This is the main ansatz we make for a relatively simple bulk viscosity model building, which is not equivalent to the previous case as demonstrated in Eq. (2) that can be converted by using of $1/a(z) = 1+z$ to $\zeta = \zeta_0 - \zeta_1 \dot{z} a(z)$, nor a special case for it though they look like in form, as they have got different behavior function forms.

With this new bulk viscosity, we could write Eq. (10) as:

$$-2(1+z)\frac{dh}{dz} + 3h = \lambda_0 + \lambda_1(1+z)^n\,, \tag{12}$$

which is a first-order differential equation of h and the exact solution of h from this equation can be obtained exactly as:

$$h = \lambda_2(1+z)^{1.5} - \frac{\lambda_1}{2n-3}(1+z)^n + \frac{\lambda_0}{3}\,, \tag{13}$$

where λ_2 is an integration constant. The obtained relation above could be regarded that the cosmic expansion rate is from a combined result of the viscosity term: the matter component effect plus the effective dark energy (including the cosmological constant like term $\lambda_0/3$) contributions. Because of the consistent requirement $h = 1$ for the spatial flat universe at present, from the above we have

$$\frac{\lambda_0}{3} = 1 - \lambda_2 + \frac{\lambda_1}{2n-3}\,.$$

With the analysis of dimension of term λ_2 and the fact that matter density of our cosmos $\rho_m \propto a(t)^{-3}$, where $a(t)$ is the scalar scale factor we will show that the value of λ_2 from data fitting below does consist with the result of matter component given by WMAP5 data sets.[28] To make the fitting result compared with ΛCDM model, we make the assumption that $\lambda_2 = \sqrt{\Omega_{m0}}$, where $\Omega_{m0} \equiv 8\pi G\rho_{m0}/(3{H_0}^2)$. So in our viscosity dark energy model, the first term as labelled by λ_2 could be looked as matter contribution and the two other terms of $h(z)$ are due to the viscosity dark energy, which with limit case ($z = 0$ today) as the concordant ΛCDM model. The consistent requirement relation also gets clear meaning, that is, the relationship between the matter and dark energy (including the cosmological constant contribution) components.

The observations of the SNe Ia have provided the first "direct" evidence of the accelerating expansion for our current universe. So any model attempting to explain the acceleration mechanism should be consistent with the SNe Ia data implying results, as a basic requirement. As we know the observations of supernovas measure essentially the apparent magnitude m, which is related to the luminosity distance d_L by

$$m = M + 5\log_{10} D_L(z)\,, \tag{14}$$

where the distance $D_L(z) \equiv (H_0)d_L(z)$ is the dimensionless luminosity and

$$d_L = (1+z)d_M(z)\,, \tag{15}$$

where d_M is the co-moving distance given by

$$d_M = \int_0^z \frac{1}{H(z')}dz'\,. \tag{16}$$

Also,

$$\mathcal{M} = M + 5\log_{10}\Big(\frac{1/H_0}{1\mathrm{Mpc}}\Big) + 25\,, \tag{17}$$

where M is the absolute magnitude which is believed to be constant for all supernovaes of type Ia. In this paper, we use the 307 Union SNe Ia data sets compiled in Ref. [26]. The data points in these samples are given in terms of the distance modulus

$$\mu_{\mathrm{obs}} \equiv m(z) - M_{\mathrm{obs}}(z)\,. \tag{18}$$

We employ it for doing the standard statistic analysis. So the χ^2 is calculated from

$$\chi^2 = \sum_{i=1}^{n}\Big[\frac{\mu_{\mathrm{obs}}(z_i) - M' - 5\log_{10} D_{\mathrm{Lth}}(z_i;c_\alpha)}{\sigma_{\mathrm{obs}}(z_i)}\Big]^2\,, \tag{19}$$

where $M' = \mathcal{M} - M_{\mathrm{obs}}$ is a free parameter and $D_{\mathrm{Lth}}(z_i;c_\alpha)$ is the theoretical prediction for the dimensionless luminosity distance of a supernovae at a particular distance, for a given model with parameter c_α.

On the other hand, the shift parameter $\mathcal{R}$ and the distance parameter $\mathcal{A}$ are considered to give contributions to data fitting. The shift parameter $\mathcal{R}$ is defined in Refs. [29] and [30] as

$$\mathcal{R} \equiv \sqrt{\Omega_m}\int_0^{z_*}\frac{dz'}{h(z')}\,, \tag{20}$$

and WMAP5 results[28] have updated the redshift of recombination to be $z_* = 1090$. Its detail meaning can be found in Ref. [31]. The distance parameter $\mathcal{A}$ is defined as

$$\mathcal{A} \equiv \sqrt{\Omega_m}\,h(z_b)^{-1/3}\Big(\frac{1}{z_b}\int_0^{z_b}\frac{dz'}{h(z')}\Big)^{2/3}\,, \tag{21}$$

where $z_b = 0.35$.

Considering $\mathcal{R}$ and $\mathcal{A}$, we use the total χ^2 to make the standard statistic analysis, and data fitting:

$$\chi^2_{\mathrm{total}} = \chi^2 + \Big(\frac{\mathcal{R} - \mathcal{R}_{\mathrm{obs}}}{\sigma_{\mathcal{R}}}\Big)^2 + \Big(\frac{\mathcal{A} - \mathcal{A}_{\mathrm{obs}}}{\sigma_{\mathcal{A}}}\Big)^2 \tag{22}$$

The result of data fitting is listed in Table 1.

Table 1 Fitting results for model parameters.

n	λ_0	λ_1	λ_2	χ^2
−1	0.180 27	2.0923	0.521 45	312.8
−0.8	−0.046 72	2.2784	0.520 27	313.0747
−2	0.638 24	1.8331	0.525 38	312.2193
1	1.0464	−0.151 36	0.500 16	323.3296

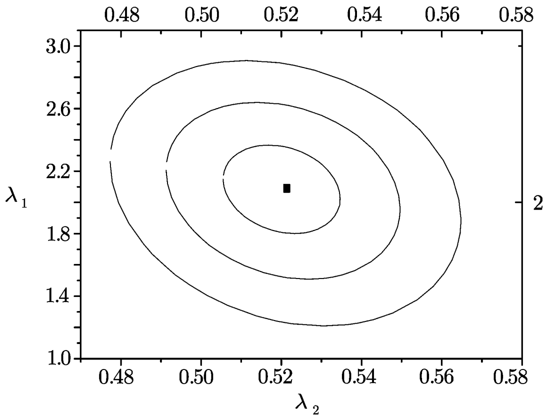

Fig. 1 The 1σ, 2σ, 3σ C.L. contours of λ_1 and λ_2 in the best fitting condition $n = -1$. The black dot corresponds to the best fitting values.

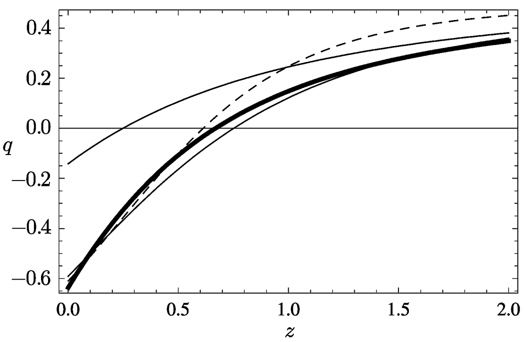

Fig. 2 q-z relation. The lowest thin line represents ΛCDM, thick line and the highest solid line are our models with $n = -1$ and $n = 1$ respectively. By including the more complex viscosity form in [32], we get another result (dashed line) for illustration.

In Fig. 2, we plot the deceleration parameter q relation with the redshift as model comparison:

$$q = \frac{1+z}{H}\frac{dH}{dz} - 1\,. \tag{23}$$

At the same time, we plot the deceleration parameter relations of $q(z)$ in ΛCDM model and the more complex bulk viscosity in Ref. [32] for contrast in Fig. 2, too.

To demonstrate clearly how and when the viscosity dark energy components catch up of the matter contribution, and overpass it, we also plot the two components evolution in Fig. 3. With our best fitting result of the $n = -1$ case, we could calculate the EOS of viscosity dark energy. The pressure of dark energy is $-\xi\theta$, so the equation of state parameter from the starting point can be expressed as:

$$\omega_X = -\frac{\xi\theta}{\rho_X}\,, \tag{24}$$

where $\rho_X = \rho(1-\Omega_m)$ is the density of dark energy. Then we have,

$$\omega_X = -\frac{3H\xi}{\rho(1-\Omega_m)}\,. \tag{25}$$

With the relation of ξ and λ defined above, we can analytically get the ω_X today:

$$\omega_0 = -\frac{\lambda_0 + \lambda_1}{3(1-\Omega_{m0})}\,. \tag{26}$$

By the best fitting data we can obtain an approximated value of the EOS at present ($h = 1$) $\omega \simeq -1.04$. There is a little deviation from ΛCDM model by a value of -0.04.

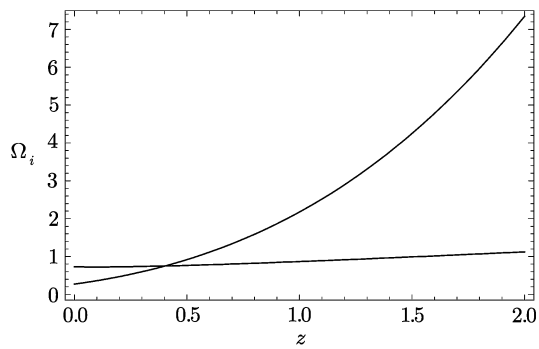

Fig. 3 The bulk viscosity dark energy part (the slow changing almost horizontal line) evolutions vs matter component.

Recently, work in Ref. [27] has proposed Om diagnose method to differentiate a new model from the ΛCDM model with the constant equation of state parameter exactly as -1. The diagnostic parameter Om is defined as:

$$Om = \frac{h^2 - 1}{x^3 - 1}\,, \tag{27}$$

where $x = 1 + z$. The dimensionless expansion parameter $h(z)^2$ for a two-component: matter (mostly cold dark matter) and dark energy model with constant equation of state can be written clearly as:

$$h(x)^2 = \Omega_{m0}x^3 + (1-\Omega_{m0})x^{3(1+\omega)}\,. \tag{28}$$

So, for the ΛCDM model with its EOS parameter $\omega = -1$, $Om = \Omega_{m0}$. This result also gives us a *null-test* of cosmology constant. For the model we proposed (here we use the best fitting results with $n = -1$, which gives a better $q(z)$ evolution curve compared with the ΛCDM model than a complicated viscosity form model[32] as shown in Figs. 2 and 5), its h^2 could be written as:

$$h^2 = \Big(\lambda_2 x^{1.5} + \frac{\lambda_1}{5}x^{-1} + \frac{\lambda_0}{3}\Big)^2\,, \tag{29}$$

so the Om evolution for this viscosity dark energy model is shown in Fig. 4 to compare with the ΛCDM model which is a horizontal line in the plot.

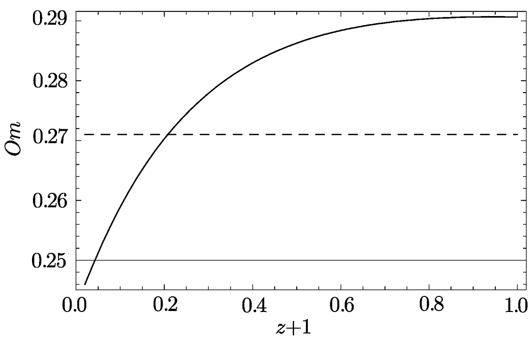

Fig. 4 Om-z relation clearly shows the viscosity dark energy model deviations from ΛCDM model. The dashed line corresponds ΛCDM model while the thick curve corresponds to our viscosity dark energy model contribution.

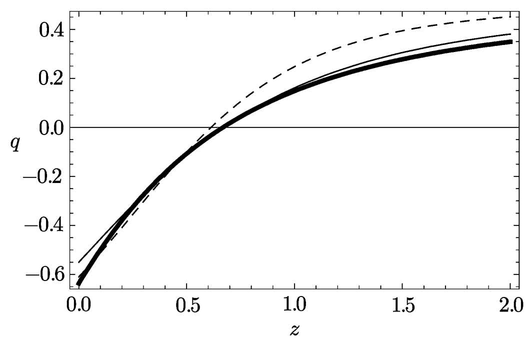

Fig. 5 $q(z)$-z relations for three different models. The nearby two lines are ours with the explicit viscosity form and the concordant ΛCDM model ($\Omega_{m0} = 0.3$). The dashed line corresponds to the complicated one, which can degenerate the previous two models for certain redshift values.

With this diagnostic method, we could also see that this model has different properties from ΛCDM model with the exact constant EOS parameter $\omega = -1$.

Further elaboration to the viscosity dark energy model follows. Compared with the usual Fredmann equation by expansion the right hand side of Eq. (29) it is clear to see that besides the non-relativistic matter-like and cosmological constant-like scaling parts the viscosity dark energy term also brings up various coupling terms with more scaling behaviors that are different from the well known typical ones, such as the radiation, (dark) matter, curvature, and cosmological constant components. In this sense we address it the effective dark energy or unification of dark matter with dark energy (dark fluid). So the EoS parameter in Eqs. (24)–(26) can be regarded as the effective dark energy (the bulk viscosity dark energy) contribution with the approximation that the pressure from matter component is negligible, which is consistent with observational fact that dark energy is dominating today. This picture is also consistent with previous works starting with a two-component model.

With the above discussions we can see our relatively simple viscosity model fits the concordant ΛCDM model better. It is due to the simpler viscosity form characterized by two parameters that can be best fitted. As a matter of fact so far we still have not obtained a systematic way to express the cosmic viscosity effects. In view of the astrophysical reality any progresses in this field is very helpful. To show the similarities and differences among the concordant ΛCDM model, our model and the complicated one[32] we present a much clear figure with the hope that there will be much related works to appear soon. We expect the study for viscosity dark energy can be stimulated further.

3 Summary and Discussions

In this letter we continue previous work on bulk viscous models as potential dark energy candidates by presenting an explicit viscosity form to mimic dark energy behaviors and confront it with current observational data sets. The specific feature here is a variable coefficient of bulk viscosity, characterized by two parameters that can be best fitted by astrophysics observational data sets. Though the viscosity dark energy form is simple, the results are better than a more complicated form as shown in Figs. 2 and 5. The possible universe evolution fate, like possible future singularity types from this viscosity model can be also discussed and we will work it out elsewhere. We emphasize that perfect fluid is just a limit case of a general viscosity media that is more practical in the astrophysics sense.

Discovery of dark energy is about ten years old, but its nature and origin have been still puzzling. Fundamental as it has promised to physics foundations there are several possibilities to develop with great expectations. One of these is the unification scenario of dark matter and dark energy, that is, they are two facets of one secret. If we finally discover the dark matter either by accelerators such as LHC or future ILC or by satellite missions, like PAMELA and Fermi (former GLAST) now on the running, we may infer the existence of dark energy therefore. In this aspect viscosity dark energy has already shown its unification efforts, to write uniformly all the possible cosmic components in a formula with distinct scaling behaviors with respect to scalar factor.[24,33,34] It is worthy further studying.

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