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Note on Implementation of Three-Qubit SWAP Gate*

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Abstract *In this paper, the synthesis and implementation of three-qubit SWAP gate is discussed. The three-qubit SWAP gate can be decomposed into product of 2 two-qubit SWAP gates, and it can be realized by 6 CNOT gates. Research illustrated that although the result is very simple, the current methods of matrix decomposition for multi-qubit gate can not get that. Then the implementation of three-qubit SWAP gate in the three spin system with Ising interaction is investigated and the sequence of control pulse and drift process to implement the gate is given. It needs 23 control pulses and 12 drift processes. Since the interaction can not be switched on and off at will, the realization of three-qubit SWAP gate in specific quantum system also can not simply come down to 2 two-qubit SWAP gates.*

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1 Introduction

In quantum computing, the algorithms are commonly described by the quantum circuit model.[1] The building blocks of quantum circuits are quantum gates, i.e., unitary transformations acting on a set of qubits. Optimal implementation of quantum gates is crucial for designing a quantum computer.

Two-qubit SWAP gate is utilized to exchange the state of the first qubit to the second qubit, and vice versa. It is one of the most useful gate in quantum information processing, such as storing the quantum information,[2] establishing the universality of two-qubit gates,[3] programmable gate arrays,[4] quantum teleportation,[5−6] and the basic implementation of other quantum gates.[7] Three-qubit SWAP gate[8−9] is a generalization of two-qubit SWAP gate which acts on three qubits and rotates their states in a cyclic fashion.

In this paper we discuss the synthesis and implementation of three-qubit SWAP gate. The three-qubit SWAP gate can be decomposed into product of 2 two-qubit SWAP gates, and it can be realized by 6 CNOT gates. Although such result is very simple, the investigation reveals that the current methods of matrix decomposition for multi-qubit gate can not get that. It can get by a knack. Then the implementation of three-qubit SWAP gate in the three spin system with Ising interaction is investigated and the sequence of control pulse and drift process to implement the gate is given. It needs 23 control pulses and 12 drift processes. Since the interaction can not be switched on and off at will, the realization of three-qubit SWAP gate in specific quantum system also can not simply come down to 2 two-qubit SWAP gates.

The paper is organized as follows. In Sec. 2, we give a brief review of the matrix decomposition and the synthesis of quantum circuit of general multi-qubit gate. In Sec. 3, the matrix decomposition and the synthesis of the three-qubit SWAP gate is discussed. The implementation of three-qubit SWAP gate in the three spin system with Ising interaction is investigated in Sec. 4. Finally, we give a brief conclusion in Sec. 5.

2 Matrix Decomposition and Synthesis of General Multi-Qubit Gate

The matrix decomposition has been playing an important role to optimize quantum circuit. In 1995, Barenco *et al.* made use of QR decomposition to show that any unitary transformation on n-qubit can be decomposed into a sequence of one-qubit gates and CNOT gates.[10] The complexity can be measured in terms of the number of elementary gates required. However the number of CNOT gates required for n qubits was order $n^3 4^n$ in their work. Though some improvement was made, there was no distinct change in a period. Till 2004, the situation was changed by introducing new decomposition techniques. These techniques are Cartan decomposition[11] based on group theory and Cosine-Sine decomposition (CSD)[12] based on numerical linear algebra and the quantum Shannon decomposition (QSD)[13] proposed by Shende, Bullock and Markov. The most widely used Cartan decomposition in quantum information science is a decomposition of $SU(2^n)$ group for the n-qubit system introduce

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by Khaneja and Glaser (KGD).[14] Furthermore, there are concurrence canonical decomposition (CCD),[15−16] odd-even decomposition (OED),[17] and a kind of Cartan decomposition for bipartite quantum system in high dimension.[9,18−19]

The implementation of a general two-qubit gate is found to require 3 CNOT gates and 15 elementary one-qubit gates.[7,20] The current theoretical lower bound for the number of CNOT gates needed for realizing an arbitrary n-qubit gate, $(4^n - 3n - 1)/4$, is given in Ref. [20]. However, no circuit construction yielding these numbers of CNOT gate has been presented in literature. Using the CSD, the realization of general n-qubit gate was discussed in Ref. [21], and its result is that it needs at most $4^n - 2^{n+1}$ CNOT gates and 4^n one-qubit gates. The best results is that in Ref. [13], it needs at most $(23/48)4^n - (3/2)2^n + (4/3)$ CNOT gates asymptotically which is obtained by the QSD.

For a general three-qubit gate, based on the KGD, Vatan and Williams get the result that using at most 98 one-qubit gates and 40 CNOT gates to implement the gate.[22] Based on the modified KGD, we find that the gate can be implemented by using at most 73 one-qubit gates to rotate about the y and z axis and 26 CNOT gates.[23] The best result is also given in Ref. [13] and obtained by the QSD, it needs at most 20 CNOT gates.

3 Note on Decomposition and Synthesis of Three-Qubit SWAP Gate

Although much progress on the matrix decomposition to implement and optimize of quantum circuit is obtained, the synthesis and optimal of two-qubit circuit is well solved, there remains many open questions for that. Even if for a simple three-qubit gate, the problem may still occur. Now we use three-qubit SWAP gate to illustrate that.

For three-qubit SWAP gate, its action is defined in the tensor product basis as

$$X_{\rm swap} = |i\rangle_1 \otimes |j\rangle_2 \otimes |k\rangle_3 \to |k\rangle_1 \otimes |i\rangle_2 \otimes |j\rangle_3\,, \tag{1}$$

where $i, j, k = 0, 1$ and $\{|0\rangle, |1\rangle\}_{1,2,3}$ are orthonormal basis for the Hilbert spaces of the three subsystems. The matrix representation of this operator is given by

$$X_{\rm swap} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}. \tag{2}$$

The CCD of three-qubit SWAP gate has been given in Ref. [8]. Another decomposition of the gate has been given in Ref. [9], in which the first step of the decomposition is the Cartan decomposition of 2×4 bipartite quantum system. Moreover, the decomposition can be carried out by use of the KGD, CSD, and QSD. Among the various decompositions described above, the best result of the decomposition for three-qubit SWAP gate is that based on the QSD. Based on this result, the three-qubit SWAP gate can be implemented by using 7 nearest-neighbor CNOT gates and 11 one-qubit gates to rotate about the y and z axis. Another good result is based on the CCD, it needs 10 CONT gates and 4 one-qubit gates to rotate about the y and z axis.

In fact, the decomposition of three-qubit SWAP gate has simpler result than that obtained above. By the definition of the gate and the knowledge of permutation group, the three-qubit SWAP gate can be decomposed into product of 2 two-qubit SWAP gates. Since each two-qubit SWAP gate can be realized by 3 CNOT gates,[7] the three-qubit SWAP gate can be realized by 6 CNOT gates. So we have

$$X_{\rm swap} = W_1^2 W_2^3 = C_1^2 C_2^1 C_1^2 C_2^3 C_3^2 C_2^3\,. \tag{3}$$

Here we denote by W_i^j a 2-qubit SWAP gate between i-th qubit and j-th qubit, and C_i^j a CNOT gate with control on the i-th qubit and target on the j-th qubit. This result is better than all of the current methods of matrix decomposition for multi-qubit gate.

Nevertheless, the decomposition in Eq. (3) can be obtained by a knack. Taking the cosine-sine decomposition as its first step, we have

$$X_{\rm swap} = A\Sigma B\,. \tag{4}$$

In order to get decomposition as simple as possible, we take $A = I_4 \oplus A_2$. By calculation, we get

$$A_2 = \begin{pmatrix} 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix},$$

$$\Sigma = \begin{pmatrix} I_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 \\ 0 & 0 & I_2 & 0 \\ 0 & -I_2 & 0 & 0 \end{pmatrix}, \tag{5}$$

$$B = {\rm diag}\left\{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}\right\}. \tag{6}$$

Take A, Σ together and re-factor the product, we have

$$A\Sigma = \begin{pmatrix} I_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 \\ 0 & 0 & I_2 & 0 \\ 0 & I_2 & 0 & 0 \end{pmatrix} \mathrm{diag}\left\{ I_4, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\} = C_2^1 C_1^2 \,. \tag{7}$$

Then we factor B as

$$B = \mathrm{diag}\left\{ I_4, \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \right\} \mathrm{diag}\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \right\} = C_1^2 W_2^3 \,. \tag{8}$$

Since $W_1^2 = C_1^2 C_2^1 C_1^2$ and $W_2^3 = C_2^3 C_3^2 C_2^3$, we get the synthesis of three-qubit SWAP gate in Eq. (3).

There is another kind of three-qubit SWAP gate which is defined as that

$$X'_{\mathrm{swap}} = |i\rangle_1 \otimes |j\rangle_2 \otimes |k\rangle_3 \to |j\rangle_1 \otimes |k\rangle_2 \otimes |i\rangle_3 \,. \tag{9}$$

The synthesis of this gate is given by

$$X'_{\mathrm{swap}} = W_2^3 W_1^2 = C_2^3 C_3^2 C_2^3 C_1^2 C_2^1 C_1^2 \,. \tag{10}$$

4 Note on Implementation of Three-Qubit SWAP Gate in Spin System

We now discuss the implementation of the gate in a system of coupled spins. The Hamiltonian of the system can be written as

$$H = H_d + \sum_i u_i(t) H_i \,, \tag{11}$$

where H_d is the part of Hamiltonian that is internal to the system and we call it the free evolution Hamiltonian and $\sum_i u_i(t) H_i$ is the part of Hamiltonian that can be externally changed called control Hamiltonian. For convenience, the single parameter one-qubit operators $\exp(-i\theta\sigma_{lz})$ acting on l-th qubit are denoted by $R_{lz}(\theta)$, and that analogous are denoted by R_{lx}, R_{ly}. Single parameter two-qubit operators $\exp(-i\theta\sigma_{l\alpha}\sigma_{k\beta})$ $(\alpha, \beta \in (x, y, z))$ acting on l-th and k-th qubits are denoted by $R_{l\alpha,k\beta}(\theta)$.

The implementation of the CONT gate is given in Ref. [24],

$$\begin{aligned} C_i^j = {} & e^{i\pi/4} R_{iz}\Big(\frac{\pi}{4}\Big) R_{jx}\Big(\frac{\pi}{4}\Big) R_{jy}\Big(\frac{\pi}{4}\Big) R_{iz,jz}\Big(-\frac{\pi}{4}\Big) \\ & \times R_{jy}\Big(-\frac{\pi}{4}\Big) \,. \end{aligned} \tag{12}$$

For two-qubit SWAP gate, we have

$$\begin{aligned} W_i^j & = \exp\Big\{ i\frac{\pi}{4}(\sigma_{ix}\sigma_{jx} + \sigma_{iy}\sigma_{jy} + \sigma_{iz}\sigma_{jz} - I) \Big\} \\ & = e^{-i\pi/4} R_{ix,jx}\Big(-\frac{\pi}{4}\Big) R_{iy,jy}\Big(-\frac{\pi}{4}\Big) \\ & \quad \times R_{iz,jz}\Big(-\frac{\pi}{4}\Big) \,. \end{aligned} \tag{13}$$

Using the transformation relation of two-qubit operators

$$\begin{aligned} R_{ix,jx}(\theta) = {} & R_{iy}\Big(\frac{\pi}{4}\Big) R_{jy}\Big(\frac{\pi}{4}\Big) R_{iz,jz}(\theta) R_{jy}\Big(-\frac{\pi}{4}\Big) \\ & \times R_{iy}\Big(-\frac{\pi}{4}\Big) \,, \end{aligned} \tag{14}$$

$$\begin{aligned} R_{iy,jy}(\theta) = {} & R_{ix}\Big(-\frac{\pi}{4}\Big) R_{jx}\Big(-\frac{\pi}{4}\Big) R_{iz,jz}(\theta) R_{jx}\Big(\frac{\pi}{4}\Big) \\ & \times R_{ix}\Big(\frac{\pi}{4}\Big) \,, \end{aligned} \tag{15}$$

we can get

$$\begin{aligned} W_i^j = {} & e^{-i\pi/4} R_{iy}\Big(\frac{\pi}{4}\Big) R_{jy}\Big(\frac{\pi}{4}\Big) R_{iz,jz}\Big(-\frac{\pi}{4}\Big) R_{iy}\Big(-\frac{\pi}{4}\Big) \\ & \times R_{jy}\Big(-\frac{\pi}{4}\Big) R_{ix}\Big(-\frac{\pi}{4}\Big) R_{jx}\Big(-\frac{\pi}{4}\Big) R_{iz,jz}\Big(-\frac{\pi}{4}\Big) \\ & \times R_{ix}\Big(\frac{\pi}{4}\Big) R_{jx}\Big(\frac{\pi}{4}\Big) R_{iz,jz}\Big(-\frac{\pi}{4}\Big) \,. \end{aligned} \tag{16}$$

In the two spin system with Ising interaction, we have

$$\begin{aligned} & H_d = J\sigma_{1z}\sigma_{2z}/4 \,, \quad H_i = \sigma_{l\alpha} \,, \quad l \in (1, 2) \,, \\ & \alpha \in (x, y, z) \,. \end{aligned} \tag{17}$$

The $R_{1z,2z}$ is a drift process of free evolution, and single parameter one-qubit operators usually correspond to control pulses. So in the two spin system, implementing a CNOT gate needs 1 drift process and 4 control pulses, and a two-qubit SWAP gate needs 3 drifts and 8 control pulses. The number of the control pulses to implement a two-qubit SWAP gate is less than that to implement 3 CNOT gates.

Although the three-qubit SWAP gate can be decomposed into product of 2 two-qubit SWAP gates, we can not simply use the result in Eq. (16) for its implementation. The reason is that the interaction can not be switched on and off at will. In the three spin system, we have

$$\begin{aligned} & H_d = J(\sigma_{1z}\sigma_{2z} + \sigma_{2z}\sigma_{3z})/4, \ H_i = \sigma_{l\alpha} \,, \\ & l \in (1, 2, 3), \ \alpha \in (x, y, z) \,. \end{aligned} \tag{18}$$

Now the drift process of the system is given by

$$M(\theta) = \exp(-i\theta\sigma_{1z}\sigma_{2z}) \exp(-i\theta\sigma_{2z}\sigma_{3z}) \,. \tag{19}$$

But we find that

$$\begin{aligned} R_{1z,2z}(\theta) & = M\Big(\frac{\theta}{2}\Big) R_{3\alpha}\Big(\pm\frac{\pi}{2}\Big) M\Big(\frac{\theta}{2}\Big) R_{3\alpha}\Big(\mp\frac{\pi}{2}\Big) \\ & = R_{3\alpha}\Big(\pm\frac{\pi}{2}\Big) M\Big(\frac{\theta}{2}\Big) R_{3\alpha}\Big(\mp\frac{\pi}{2}\Big) M\Big(\frac{\theta}{2}\Big) \,, \end{aligned} \tag{20}$$

$$R_{2z,3z}(\theta) = M\Big(\frac{\theta}{2}\Big) R_{1\alpha}\Big(\pm\frac{\pi}{2}\Big) M\Big(\frac{\theta}{2}\Big) R_{1\alpha}\Big(\mp\frac{\pi}{2}\Big)$$

$$= R_{1\alpha}\Big(\pm\frac{\pi}{2}\Big)M\Big(\frac{\theta}{2}\Big)R_{1\alpha}\Big(\mp\frac{\pi}{2}\Big)M\Big(\frac{\theta}{2}\Big)\,, \quad (21)$$

where $\alpha \in (x, y)$. By substituting Eq. (20) to Eq. (12), we get the sequence of pulse and drift process to implement the CNOT gate in the three spin system. It needs 2 drifts and 6 control pulses. Substituting Eq. (20) and Eq. (21) into Eq. (16) and utilizing commute technique, we get

$$\begin{aligned} W_1^2 = {} & e^{-i\pi/4}R_{1y}\Big(\frac{\pi}{4}\Big)R_{2y}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{3y}\Big(\frac{\pi}{2}\Big)M\Big(-\frac{\pi}{8}\Big) \\ & \times R_{1y}\Big(-\frac{\pi}{4}\Big)R_{2y}\Big(-\frac{\pi}{4}\Big)R_{1x}\Big(-\frac{\pi}{4}\Big)R_{2x}\Big(-\frac{\pi}{4}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{3y}\Big(-\frac{\pi}{2}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1x}\Big(\frac{\pi}{4}\Big)R_{2x}\Big(\frac{\pi}{4}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{3x}\Big(-\frac{\pi}{2}\Big)M\Big(-\frac{\pi}{8}\Big)R_{3x}\Big(\frac{\pi}{2}\Big)\,, \quad (22) \end{aligned}$$

$$\begin{aligned} W_2^3 = {} & e^{-i\pi/4}R_{2x}\Big(-\frac{\pi}{4}\Big)R_{3x}\Big(-\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1x}\Big(\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{2x}\Big(\frac{\pi}{4}\Big)R_{3x}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1x}\Big(-\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{2y}\Big(\frac{\pi}{4}\Big)R_{3y}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1y}\Big(\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{1y}\Big(-\frac{\pi}{2}\Big)R_{3y}\Big(-\frac{\pi}{4}\Big)R_{2y}\Big(-\frac{\pi}{4}\Big)\,. \quad (23) \end{aligned}$$

Finally we get

$$\begin{aligned} X_{\rm swap} = {} & W_1^2 W_2^3 \\ = {} & e^{-i\pi/2}R_{1y}\Big(\frac{\pi}{4}\Big)R_{2y}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{3y}\Big(\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{1y}\Big(-\frac{\pi}{4}\Big)R_{1x}\Big(-\frac{\pi}{4}\Big)R_{2y}\Big(-\frac{\pi}{4}\Big) \\ & \times R_{2x}\Big(-\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{3y}\Big(-\frac{\pi}{2}\Big)M\Big(-\frac{\pi}{8}\Big) \\ & \times R_{1x}\Big(\frac{\pi}{4}\Big)R_{2x}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{3x}\Big(-\frac{\pi}{2}\Big)M\Big(-\frac{\pi}{8}\Big) \\ & \times R_{2x}\Big(-\frac{\pi}{4}\Big)R_{3x}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1x}\Big(\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{2x}\Big(\frac{\pi}{4}\Big)R_{3x}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1x}\Big(-\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{2y}\Big(\frac{\pi}{4}\Big)R_{3y}\Big(\frac{\pi}{4}\Big)M\Big(-\frac{\pi}{8}\Big)R_{1y}\Big(\frac{\pi}{2}\Big) \\ & \times M\Big(-\frac{\pi}{8}\Big)R_{1y}\Big(-\frac{\pi}{2}\Big)R_{2y}\Big(-\frac{\pi}{4}\Big)R_{3y}\Big(-\frac{\pi}{4}\Big)\,. \quad (24) \end{aligned}$$

To implement a two-qubit SWAP gate in three spin system needs 6 drifts and 12 control pulses, and a 3-qubit SWAP gate needs 12 drifts and 23 control pulses. The number of pulse is reduced by one in the combination of the 2 two-qubit SWAP gates. The result for another kind of three-qubit SWAP gate in Eq. (10) can be gotten in a similar way.

5 Conclusions

In this work, we discuss the synthesis and implementation of three-qubit SWAP gate. Although the three-qubit SWAP gate can be decomposed to product of 2 two-qubit SWAP gates, and it can be realized by 6 CNOT gates, although the methods of matrix decomposition for multi-qubit gate have made much progress, none of them can get this simple result. There is much work to do for the optimal implementation of quantum gates, even if for the rather simple three-qubit gates. Otherwise to investigate the "small circuit" structure[25] of three-qubit gates is an important topic for us at present.

Moreover, to implement a CNOT gate in a two spin system needs 1 drift process and 4 control pulses, a two-qubit SWAP gate in a two spin system needs 3 drifts and 8 control pulses, and that to implement a three-qubit SWAP gate in a three spin system needs 12 drifts and 23 control pulses. So the realization of three-qubit SWAP gate in specific quantum system also can not simply come down to 2 two-qubit SWAP gates. The reason for that is the interaction can not be switched on and off at will. Equations (20) and (21) provide the methods to "switch off" part of interaction, but its complexity is also increased. It is also very significative to investigate the direct implementation of specific multi-qubit gates without resorting to the synthesis of single- and two-qubit gates.

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