Commun. Theor. Phys. (Beijing, China) **54** (2010) pp. 469–472
© Chinese Physical Society and IOP Publishing Ltd Vol. 54, No. 3, September 15, 2010

Classification of Kantowski–Sachs and Bianchi Type III Space-Times According to Their Killing Vector Fields in Teleparallel Theory of Gravitation

Ghulam Shabbir* and Suhail Khan

Faculty of Engineering Sciences, GIK Institute of Engineering Sciences and Technology, Topi, Swabi, NWFP, Pakistan

(Received December 15, 2009)

Abstract *In this paper we classify Kantowski–Sachs and Bianchi type III space-times according to their teleparallel Killing vector fields using direct integration technique. It turns out that the dimension of the teleparallel Killing vector fields are 4 or 6, which are the same in numbers as in general relativity. In case of 4 the teleparallel Killing vector fields are multiple of the corresponding Killing vector fields in general relativity by some function of t. In the case of 6 Killing vector fields the metric functions become constants and the Killing vector fields in this case are exactly the same as in general relativity. Here we also discuss the Lie algebra in each case.*

PACS numbers: 04.20.-q, 04.20.Jb
Key words: teleparallel killing vector fields, Weitzenböck connections, torsion

1 Introduction

General relativity describes gravity as a property of the geometry of the space-time where curvature of the space-time is directly related to matter content present there in, through Einstein's field equations. Teleparallel theory of gravity is an alternative theory of gravitation, which is based on Weitzenböck geometry.[1] In this theory the torsion plays an important role and the curvature of the space-time is considered as zero. Thus like general relativity where gravitation is attributed to the curvature of the space-time having zero torsion, gravitation in teleparallel theory is attributed to the torsion of the space-time having zero curvature, which plays the role of a force.[2–3]

Over the past few years there has been much interest in symmetries of the space-time manifolds in general relativity. Certain symmetry restrictions are imposed on the space-time metrics to solve the Einstein's field equations. These symmetry restrictions are well expressed in terms of Killing vector fields, which give rise to conservation laws.[4] In teleparallel theory, interest has been shown to find the teleparallel versions of the exact solutions of the Einstein's field equations in general relativity but symmetries of the space-time manifolds were ignored. Recently Sharif and Amir[5] introduced the teleparallel version of the Lie derivative for Killing vector fields and used those equations to find the teleparallel Killing vector fields in Einstein universe. They showed that the number of teleparallel Killing vector fields is the same as in general relativity but three out of seven teleparallel Killing vector fields are multiple of the corresponding Killing vector fields in general relativity by a function of r, three are totally different and one is exactly the same. Sharif and Bushra[6] in their work found out the teleparallel Killing vector fields for spherically symmetric static space-times and in Ref. [7] they obtained interesting results for some well-known space-times where they showed that the Lie algebra for teleparallel Killing vector fields is not closed.

The teleparallel Killing equation is defined as[5]

$$\begin{aligned} L_X^T g_{\mu\nu} &= g_{\mu\nu,\rho}X^\rho + g_{\rho\nu}X^\rho_{,\mu} + g_{\mu\rho}X^\rho_{,\nu} \\ &\quad + X^\rho(g_{\theta\nu}T^\theta_{\mu\rho} + g_{\mu\theta}T^\theta_{\nu\rho}) = 0\,, \end{aligned} \tag{1}$$

where L_X^T denotes the teleparallel Lie derivative with respect to the vector field X and $T^\theta_{\mu\,\nu}$ denotes the torsion tensor, which is anti-symmetric with respect to its lower two indices. The procedure for obtaining the torsion tensor can be found in Ref. [8].

In this paper we focus our investigations to classify Kantowski–Sachs and Bianchi type III space-times according to their Killing vector fields in teleparallel theory of gravitation. This classification of the conservation laws in teleparallel theory will help in understanding the physical and geometrical properties of the space-times. Through out this paper $a, b, c, \ldots = 0, 1, 2, 3$ denote the tangent space indices and $\mu, \nu, \rho, \ldots = 0, 1, 2, 3$ denote the space-time indices. Also the Lie algebra of a set of vector fields on a manifold is completely characterized by the structure constants C^a_{bc} given in term of the Lie brackets by $[X_i, X_j] = C^k_{ij}X_k$, $C^k_{ij} = -C^k_{ji}$, where X_i are the generators and $i, j, k = 0, 1, \ldots, n$. The Lie algebra for the teleparallel Killing vector fields is also discussed in each case.

2 Main Results

Consider Kantowski–Sachs and Bianchi type III space-times in usual coordinates (t, r, θ, ϕ) (labeled by (x^0, x^1, x^2, x^3), respectively) with the line element

$$ds^2 = -dt^2 + A^2(t)dr^2 + B^2(t)\,(d\theta^2 + f^2(\theta)d\phi^2)\,, \tag{2}$$

where A and B are no where zero functions of t only. For $f(\theta) = \sinh\theta$ or $f(\theta) = \sin\theta$ the space-time (2) becomes

---

*E-mail: shabbir@giki.edu.pk

Bianchi type III or Kantowski–Sachs space-time, respectively. The above space-times (2) admits four linearly independent Killing vector fields,[9] which are

$$\frac{\partial}{\partial r},\quad \frac{\partial}{\partial \phi},\quad \cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$

$$\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,, \tag{3}$$

where prime denotes the derivative with respect to θ. The tetrad components S_μ^a and its inverse S_a^μ can be obtained by adopting a procedure given in Ref. [8] as

$$S_\mu^a=\mathrm{diag}\,(1,A(t),B(t),B(t)f(\theta))\,,$$

$$S_a^\mu=\mathrm{diag}\left(1,\frac{1}{A(t)},\frac{1}{B(t)},\frac{1}{B(t)f(\theta)}\right). \tag{4}$$

The corresponding non-vanishing Weitzenböck connections for Eq. (4) are

$$\Gamma_{10}^1=\frac{A^{\cdot}}{A}\,,\quad \Gamma_{20}^2=\frac{B^{\cdot}}{B}\,,\quad \Gamma_{30}^3=\frac{B^{\cdot}}{B}\,, \tag{5}$$

where "dot" denotes the derivative with respect to t. Thus the non-vanishing torsion components are obtained as

$$T_{01}^1=\frac{A^{\cdot}}{A}\,,\quad T_{02}^2=\frac{B^{\cdot}}{B}\,,\quad T_{03}^3=\frac{B^{\cdot}}{B}\,. \tag{6}$$

A vector field X is said to be teleparallel Killing vector field if it satisfies Eq. (1). One can write Eq. (1) explicitly using Eqs. (2) and (6) as

$$X_{,0}^0=0\,,\quad X_{,1}^1=0\,,\quad X_{,2}^2=0\,, \tag{7}$$

$$B^2X_{,1}^2+A^2X_{,2}^1=0\,, \tag{8}$$

$$A^2X_{,3}^1+B^2f^2(\theta)X_{,1}^3=0\,, \tag{9}$$

$$X_{,3}^2+f^2(\theta)X_{,2}^3=0\,, \tag{10}$$

$$f'(\theta)X^2+f(\theta)X_{,3}^3=0\,, \tag{11}$$

$$A^2X_{,0}^1-X_{,1}^0+AA^{\cdot}X^1=0\,, \tag{12}$$

$$B^2X_{,0}^2-X_{,2}^0+BB^{\cdot}X^2=0\,, \tag{13}$$

$$B^2f^2(\theta)X_{,0}^3-X_{,3}^0+BB^{\cdot}f^2(\theta)X^3=0\,. \tag{14}$$

Equation (7) gives

$$X^0=P^1(r,\theta,\phi)\,,\quad X^1=P^2(t,\theta,\phi)\,,$$

$$X^2=P^3(t,r,\phi)\,, \tag{15}$$

and Eq. (11) gives

$$X^3=-\frac{f'(\theta)}{f(\theta)}\int P^3(t,r,\phi)d\phi+P^4(t,r,\theta)\,, \tag{16}$$

where $P^1(r,\theta,\phi)$, $P^2(t,\theta,\phi)$, $P^3(t,r,\phi)$, and $P^4(t,r,\theta)$ are functions of integration, which are to be determined using the remaining six equations. To avoid lengthy and tedious calculations here we shall present only results which are:

Case 1 In this case we have $A=A(t)$, $B=B(t)$, and $A(t)\neq B(t)$. The space-time is given in Eq. (2). Solution of Eqs. (7) to Eq. (14) is given by

$$X^0=0,\quad X^1=\frac{1}{A(t)}c_1,\quad X^2=\frac{1}{B(t)}(c_2\cos\phi+c_3\sin\phi),$$

$$X^3=\frac{1}{B(t)}\frac{f'(\theta)}{f(\theta)}(c_3\cos\phi-c_2\sin\phi)+\frac{1}{B(t)}c_4\,, \tag{17}$$

where c_1, c_2, c_3, $c_4\in R$. Here the above space-time (2) admits four linearly independent teleparallel Killing vector fields which are

$$\frac{1}{A(t)}\frac{\partial}{\partial r},\quad \frac{1}{B(t)}\frac{\partial}{\partial\phi},\quad \frac{1}{B(t)}\Big(\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\Big)\,,$$

$$\frac{1}{B(t)}\Big(\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\Big)\,.$$

Killing vector fields in general relativity are

$$\frac{\partial}{\partial r},\quad \frac{\partial}{\partial \phi},\quad \cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$

$$\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,.$$

One can easily see that Killing vector fields in both theories are same in number but the teleparallel Killing vector fields are the inverse multiple of the metric functions $A(t)$ and $B(t)$. The generators for Lie algebra in this case are

$$X_1=\frac{1}{A(t)}\frac{\partial}{\partial r},\quad X_2=\frac{1}{B(t)}\Big(\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\Big)\,,$$

$$X_3=\frac{1}{B(t)}\Big(\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\Big),\quad X_4=\frac{1}{B(t)}\frac{\partial}{\partial\phi}\,.$$

Here the non-zero components of the Lie brackets are

$$[X_2,X_3]=\frac{1}{B(t)}X_4\,,\quad [X_2,X_4]=\frac{1}{B(t)}X_3\,,$$

$$[X_3,X_4]=-\frac{1}{B(t)}X_2\,.$$

In this case the Lie algebra is not closed. The reasons for this are the presence of torsion and rotation in the space-time (2). It is important to remind the reader that B is not constant.

Case 2 In this case we have $A=A(t)$ and $B=\eta$, where $\eta\in R\setminus\{0\}$. The space-time (2) takes the form

$$ds^2=-dt^2+A^2(t)dr^2+\eta^2(d\theta^2+f^2(\theta)d\phi^2)\,, \tag{18}$$

Solution of Eqs. (7) to Eq. (14) is given by

$$X^0=0\,,\quad X^1=\frac{1}{A(t)}c_1\,,\quad X^2=c_2\cos\phi+c_3\sin\phi\,,$$

$$X^3=\frac{f'(\theta)}{f(\theta)}(c_3\cos\phi-c_2\sin\phi)+c_4\,, \tag{19}$$

where c_1, c_2, c_3, and $c_4\in R$. Here the above space-time (18) admits four linearly independent teleparallel Killing vector fields which are

$$\frac{1}{A(t)}\frac{\partial}{\partial r},\quad \frac{\partial}{\partial\phi},\quad \cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$

$$\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,.$$

Killing vector fields in general relativity are

$$\frac{\partial}{\partial r},\quad \frac{\partial}{\partial \phi},\quad \cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$

$$\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,.$$

Comparison shows that only one teleparallel Killing vector field is different from Killing vector fields in general relativity. The one teleparallel Killing vector field, which is different from general relativity is a multiple of the corresponding element of the inverse tetrad field. The generators for Lie algebra in this case are

$$X_1=\frac{1}{A(t)}\frac{\partial}{\partial r}\,,\quad X_2=\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$
$$X_3=\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,,\quad X_4=\frac{\partial}{\partial\phi}\,.$$

Here the non-zero components of the Lie brackets are $[X_2,X_3]=X_4$, $[X_2,X_4]=X_3$, and $[X_3,X_4]=-X_2$. In this case the Lie algebra is closed.

Case 3 In this case we have $B=B(t)$ and $A=\eta$, where $\eta\in R\setminus\{0\}$. The space-time (2) can after a suitable rescaling of r, be written in the form

$$ds^2=-dt^2+dr^2+B^2(t)\,(d\theta^2+f^2(\theta)d\phi^2)\,. \tag{20}$$

The teleparallel Killing vector fields in this case are

$$X^0=0\,,\quad X^1=c_1\,,\quad X^2=\frac{1}{B(t)}(c_2\cos\phi+c_3\sin\phi)\,,$$
$$X^3=\frac{1}{B(t)}\frac{f'(\theta)}{f(\theta)}(c_3\cos\phi-c_2\sin\phi)+\frac{1}{B(t)}c_4\,, \tag{21}$$

where c_1, c_2, c_3, and $c_4\in R$. Here the above space-time (20) admits four linearly independent teleparallel Killing vector fields, which are

$$\frac{\partial}{\partial r},\quad \Big(\frac{1}{B(t)}\Big)\frac{\partial}{\partial\phi},\quad \frac{1}{B(t)}\Big(\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\Big)\,,$$
$$\frac{1}{B(t)}\Big(\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\Big)\,.$$

In this case Killing vector fields in general relativity are

$$\frac{\partial}{\partial r},\quad \frac{\partial}{\partial\phi},\quad \cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$
$$\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,.$$

Teleparallel Killing vector field are same in number and one is exactly the same as in general relativity. Other three teleparallel Killing vector fields are different from Killing vector fields in general relativity. They are the inverse multiple of the metric function $B(t)$. The generators for Lie algebra in this case are

$$X_1=\frac{\partial}{\partial r}\,,\quad X_2=\frac{1}{B(t)}\Big(\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\Big)\,,$$
$$X_3=\frac{1}{B(t)}\Big(\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\Big),\quad X_4=\frac{1}{B(t)}\frac{\partial}{\partial\phi}\,.$$

Here the non-zero components of the Lie brackets are

$$[X_2,X_3]=\frac{1}{B(t)}X_4\,,\quad [X_2,X_4]=\frac{1}{B(t)}X_3\,,$$
$$[X_3,X_4]=-\frac{1}{B(t)}X_2\,.$$

In this case the Lie algebra is not closed. The reasons for this are the presence of torsion and rotation in the space-time (22). It is important to remind the reader that B is not constant.

Case 4 In this case we have $A=A(t)$, $B=B(t)$, and $A(t)=B(t)$. The space-time (2) takes the form

$$ds^2=-dt^2+A^2(t)\,(dr^2+d\theta^2+f^2(\theta)d\phi^2)\,. \tag{22}$$

Solution of Eqs. (7) to Eq. (14) is given by

$$X^0=0,\quad X^1=\frac{1}{A(t)}c_1,\quad X^2=\frac{1}{A(t)}(c_2\cos\phi+c_3\sin\phi)\,,$$
$$X^3=\frac{1}{A(t)}\frac{f'(\theta)}{f(\theta)}(c_3\cos\phi-c_2\sin\phi)+\frac{1}{A(t)}c_4\,, \tag{23}$$

where c_1, c_2, c_3, and $c_4\in R$. Here the above space-time (22) admits four linearly independent teleparallel Killing vector fields which are

$$\frac{1}{A(t)}\frac{\partial}{\partial r},\quad \frac{1}{A(t)}\frac{\partial}{\partial\phi},\quad \frac{1}{A(t)}\Big(\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\Big)\,,$$
$$\frac{1}{A(t)}\Big(\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\Big)\,.$$

Killing vector fields in general relativity are

$$\frac{\partial}{\partial r},\quad \frac{\partial}{\partial\phi},\quad \cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\,,$$
$$\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\,.$$

One can easily see that Killing vector fields in both theories are same in number but the teleparallel Killing vector fields are the inverse multiple of the metric functions $A(t)$. The generators for Lie algebra in this case are

$$X_1=\frac{1}{A(t)}\frac{\partial}{\partial r}\,,\quad X_2=\frac{1}{A(t)}\Big(\cos\phi\frac{\partial}{\partial\theta}-\frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial\phi}\Big)\,,$$
$$X_3=\frac{1}{A(t)}\Big(\sin\phi\frac{\partial}{\partial\theta}+\frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial\phi}\Big),\quad X_4=\frac{1}{A(t)}\frac{\partial}{\partial\phi}\,.$$

Here the non-zero components of the Lie brackets are

$$[X_2,\,X_3]=\frac{1}{A(t)}X_4\,,\quad [X_2,X_4]=\frac{1}{A(t)}X_3\,,$$
$$[X_3,\,X_4]=-\frac{1}{A(t)}X_2\,.$$

In this case the Lie algebra is not closed. The reasons for this are the presence of torsion and rotation in the space-time (22). It is important to remind the reader that A is not a constant.

Case 5 In this case we have $A=\beta$ and $B=\eta$, where $\beta,\eta\in R\setminus\{0\}$. The space-time (2) in this case after a suitable rescaling of r, becomes

$$ds^2=-dt^2+dr^2+\eta^2\,(d\theta^2+f^2(\theta)d\phi^2)\,. \tag{24}$$

Solution of Eqs. (7) to Eq. (14) is given by

$$X^0=rc_1+c_2\,,\quad X^1=tc_1+c_3\,,$$
$$X^2=c_4\cos\phi+c_5\sin\phi\,,$$
$$X^3=\frac{f'(\theta)}{f(\theta)}(c_5\cos\phi-c_4\sin\phi)+c_6\,, \tag{25}$$

where c_1, c_2, c_3, c_4, c_5, and $c_6 \in R$. Here the above space-time (24) admits six linearly independent teleparallel Killing vector fields which are

$$\frac{\partial}{\partial t}, \quad \frac{\partial}{\partial r}, \quad \frac{\partial}{\partial \phi}, \quad t\frac{\partial}{\partial r} + r\frac{\partial}{\partial t}\,,$$
$$\sin\phi\frac{\partial}{\partial \theta} + \frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial \phi}\,,$$
$$\cos\phi\frac{\partial}{\partial \theta} - \frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial \phi}\,,$$

which are exactly the same as in general relativity.[10] The generators for Lie algebra in this case are

$$X_1 = r\frac{\partial}{\partial t} + t\frac{\partial}{\partial r}\,, \quad X_2 = \frac{\partial}{\partial t}\,, \quad X_3 = \frac{\partial}{\partial r}\,,$$
$$X_4 = \cos\phi\frac{\partial}{\partial \theta} - \frac{f'(\theta)}{f(\theta)}\sin\phi\frac{\partial}{\partial \phi}\,,$$
$$X_5 = \sin\phi\frac{\partial}{\partial \theta} + \frac{f'(\theta)}{f(\theta)}\cos\phi\frac{\partial}{\partial \phi}\,, \quad X_6 = \frac{\partial}{\partial \phi}\,.$$

Here the non-zero components of the Lie brackets are $[X_1, X_3] = -X_2$, $[X_4, X_5] = X_6$, $[X_4, X_6] = X_5$, and $[X_5, X_6] = -X_4$. In this case the Lie algebra is closed.

3 Conclusion

In this paper we classified Kantowski–Sachs and Bianchi type III space-times according to their teleparallel Killing vector fields. From the above study it is shown that the above space-times admit 4 or 6 teleparallel Killing vector fields, which are same in number as in general relativity. It also turns out that the teleparallel Killing vector fields are multiple of some specific functions of t, (theses functions are basically the inverse multiple of metric functions A and B) in the case when the above space-times admit 4 Killing vector fields. These functions appear in teleparallel Killing vector fields because of the non-zero torsion components. In the case of 6 teleparallel Killing vector fields the space-time metrics becomes constants and these 6 Killing vector fields are same as in general relativity. From the above discussion it is clear that the presence of torsion in the Kantowski–Sachs and Bianchi type III space-times does not increase or decrease the number of conservation laws. Lie algebra of teleparallel Killing vector fields in each case is also studied. There exist four different cases when the above space-times admit 4 Killing vector fields. In these four cases only one case (which is case 2) closed the Lie algebra. For the 6 Killing vector fields these exists only one case (which is case 5) closed the Lie algebra.

References

[1] R. Weitzenböck, *Invarianten Theorie*, Noordhoft, Gronningen (1923).

[2] R. Aldrovandi and J.G. Pereira, *An Introduction to Geometrical Physics*, World Scientific, Singapore (1995).

[3] J.G. Pereira, T. Vargas, and C.M. Zhang, Class. Quantum Gravity **18** (2001) 833.

[4] A.Z. Petrov, *Phys. Einstein Spaces*, Oxford University Press, Pergamon (1969).

[5] M. Sharif and M.J. Amir, Mod. Phys. Lett. A **23** (2008) 963.

[6] M. Sharif and B. Majeed, Commun. Theor. Phys. **52** (2009) 435.

[7] B. Majeed and M. Phil, Thesis Punjab University (2008).

[8] R. Aldrovendi and J.G. Pereira, *An Introduction to Gravitation Theory*, preprint.

[9] G. Shabbir and A.B. Mehmood, Mod. Phys. Lett. A **22** (2007) 807.

[10] G. Shabbir and A. Ali, Mod. Phys. Lett. A **22** (2007) 2209.