

Dynamics of Quantum Discord in Asymmetric and Local Non-Markovian Environments*

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Abstract *The non-Markovian decoherence of quantum and classical correlations is analytically obtained when two qubits are asymmetrically subjected to the bit flip channel and phase flip channel. For one class of initial mixed states, quantum correlations quantified by quantum discord decay synchronously with classical correlations. The discovery that the decaying rates of quantum and classical correlations suddenly change at the characteristic time is physically interpreted by the distance from quantum state to the closest classical states. In a large time interval, quantum correlations are greater than classical correlations. The quantum and classical correlations can be preserved over a longer period of time via the kernel characterizing the environment memory effects.*

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1 Introduction

The great obstacle to the practical tasks of quantum information processing is the destruction of quantum correlations arising from the inevitable interactions of the systems of interest with their environments.^[1–2] The fundamental quantum correlations are contained in nonclassical states. It is known that the entanglement measure can only quantify quantum correlations for inseparable states. Recently, a large amount of experimental and theoretical studies have discovered that quantum discord^[3–4] is a more general measure of nonclassical correlations in comparison with the entanglement.^[5–25] Quantum correlation introduced by Henderson and Vedral^[4] is identical to the quantum discord defined by Ollivier and Zurek^[3] in the case of two qubits.^[25] Specially, the separable mixed states with nonzero quantum discord are also considered as one kind of useful resources contributing to the speedup of some quantum computations. Therefore, the dynamics of quantum correlations have received much attention. Until now, the Markovian and non-Markovian decoherences of correlations have been investigated in Refs. [15–22]. It is found that the exponential decay of quantum discord is distinct from the sudden death of the entanglement^[26] at finite time in the Markovian environments. The decline of quantum discord can be delayed in the non-Markovian channels.^[15] But quantum correlations are usually less than classical correlations in a large time interval. In many interesting studies, it is assumed that qubits are coupled to local reservoirs which are similar to each other. However,

the properties of local environments are often different from each other. This means that the practical environments are asymmetric. The asymmetric characterizations of the decoherence channels for every qubit must bring about the positive or negative effects on the behavior of quantum and classical correlations.

In this paper, we study the dynamics of quantum and classical correlations in the influence of asymmetric and non-Markovian local environments. For the spin system consisting of two noninteracting qubits A and B , the non-Markovian decoherence channel of each qubit is given by the post-Markovian phenomenological approach in Ref. [27]. The effects of the bit-flip channel and phase-flip channel are taken into account. In Sec. 2, the time evolution of quantum states is analytically obtained when a general class of Bell-diagonal mixed states are chosen to be initial ones. In Sec. 3, three different types of the behavior of quantum discord and classical correlations are deduced with the change of initial states. A simple discussion concludes the paper.

2 Evolution under Asymmetric Decoherence Channels

The Shabani and Lidar post-Markovian master equation^[27] is used to describe the time evolution of quantum bipartite states $\rho(t)$ in the non-Markovian environments. The general form of the master equation is ex-

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pressed by

$$\dot{\rho}(t) = \hat{L} \int_0^t dt' k(t') \exp(t' \hat{L}) \rho(t-t'), \quad (1)$$

where the operator \hat{L} is the Markovian Liouvillian and $k(t)$ is the kernel characterizing the environment memory effects. Specially, $k(t) = \delta(t)$ in the Markovian case. For a given system and environment, the kernel can be experimentally engineered via quantum state tomography.^[27] For the Pauli channels,

$$\hat{L}\rho(t) = \sum_{j=A,B} \frac{a}{2} (2\sigma_\alpha^j \rho \sigma_\alpha^j - \sigma_\alpha^j \sigma_\alpha^j \rho - \rho \sigma_\alpha^j \sigma_\alpha^j),$$

where $\alpha = x, y, z$ corresponds to the bit flip, the bit-phase and phase flip channel respectively. The parameter a represents the decaying rate. According to Ref. [28], the post-Markovian channels are adopted to describe the actual dynamics approximately. For simplicity, qubits A and B are subjected to the two different channels of bit flip ε_{BF} and phase flip one ε_{PF} respectively. The bit flip channel for the j -th qubit can be given by $\varepsilon_{\text{BF}}^j(|e\rangle_j \langle e|) = (1/2)[I^j + p(t)\sigma_z^j]$, $\varepsilon_{\text{BF}}^j(|e\rangle_j \langle g|) = (1/2)[\sigma_x^j + ip(t)\sigma_y^j]$, $\varepsilon_{\text{BF}}^j(|g\rangle_j \langle e|) = (1/2)[\sigma_x^j - ip(t)\sigma_y^j]$, and $\varepsilon_{\text{BF}}^j(|g\rangle_j \langle g|) = (1/2)[I^j - p(t)\sigma_z^j]$. Here $|e(g)\rangle_j$ is the eigenstate of σ_z^j with the corresponding eigenvalue ± 1 and I^j is the identity operator of qubit j . If the kernel is assumed to be $k(t) = A \exp(-\gamma t)$, the parameter $p(t)$ is decided by

$$p(t) = \exp\left(-\frac{2a+\gamma}{2}t\right) \left(\cos \omega_0 t + \frac{2a+\gamma}{2\omega_0} \sin \omega_0 t\right),$$

where $\omega_0 = \sqrt{2aA - [(2a+\gamma)/2]^2}$. The phase flip channel can also be defined by $\varepsilon_{\text{PF}}^j(|e\rangle_j \langle e|) = (1/2)(I^j + \sigma_z^j)$, $\varepsilon_{\text{PF}}^j(|e\rangle_j \langle g|) = (p/2)(\sigma_x^j + i\sigma_y^j)$, $\varepsilon_{\text{PF}}^j(|g\rangle_j \langle e|) = (p/2)(\sigma_x^j - i\sigma_y^j)$, and $\varepsilon_{\text{PF}}^j(|g\rangle_j \langle g|) = (1/2)(I^j - \sigma_z^j)$.

The initial states of two qubits are one important Bell-diagonal mixed states in the general form of

$$\rho^{AB}(0) = \frac{1}{4} \left(I^A \otimes I^B + \sum_{\alpha=x,y,z} c_\alpha \sigma_\alpha^A \sigma_\alpha^B \right), \quad (2)$$

where the real number satisfies $0 \leq |c_\alpha| \leq 1$. These symmetric states are ones with maximally mixed marginals, i.e., $\text{Tr}_{A(B)} \rho \propto I^{B(A)}$. Under the two asymmetric decoherence channels, quantum bipartite states at any time can be analytically calculated by

$$\rho^{AB}(t) = \varepsilon_{\text{BF}}^A \otimes \varepsilon_{\text{PF}}^B [\rho^{AB}(0)]. \quad (3)$$

The expression of $\rho^{AB}(t)$ is given by

$$\rho^{AB}(t) = \lambda_1(t) |\Psi^+\rangle \langle \Psi^+| + \lambda_2(t) |\Phi^+\rangle \langle \Phi^+| + \lambda_3(t) |\Phi^-\rangle \langle \Phi^-| + \lambda_4(t) |\Psi^-\rangle \langle \Psi^-|, \quad (4)$$

where the four Bell states are defined by $|\Psi^\pm\rangle = (1/\sqrt{2})(|eg\rangle \pm |ge\rangle)$ and $|\Phi^\pm\rangle = (1/\sqrt{2})(|ee\rangle \pm |gg\rangle)$. The coefficients $\lambda_i(t)$ are written by $\lambda_{1,2}(t) = (1/4)[1 + c_x(t) \pm c_y(t) \mp c_z(t)]$ and $\lambda_{3,4}(t) = (1/4)[1 - c_x(t) \mp c_y(t) \pm c_z(t)]$, where $c_x(t) = p(t)c_x$, $c_y(t) = p^2(t)c_y$, and $c_z(t) = p(t)c_z$. It is shown that quantum states also keep the symmetric Bell-diagonal expression under the asymmetric Pauli

channels. The mixing proportions of Bell states are varied in time.

3 Behavior of Quantum and Classical Correlations

To clearly describe the dynamics of correlations, we use quantum discord as one general measure for quantum correlation. For an arbitrary bipartite state ρ^{AB} , the total correlations are expressed by quantum mutual information $I(\rho^{AB}) = \sum_{j=A,B} S(\rho^j) - S(\rho^{AB})$, where ρ^j represents the reduced density matrix of subsystem j and $S(\rho) = -\text{Tr}(\rho \log_2 \rho)$ is the von Neumann entropy. Henderson and Vedral^[4] proposed one measure of bipartite classical correlation $C(\rho^{AB})$ based on a complete set of local projectors $\{\Pi_i^B\}$ on one subsystem B . After the local measurements, the reduced state of subsystem A can be written by $\rho_i^A = (1/p_i) \text{Tr}_B[(I^A \otimes \Pi_i^B) \rho^{AB} (I^A \otimes \Pi_i^B)]$. Here p_i is the measurement probability for the i -th local projector. Then the classical correlation in the bipartite quantum state ρ^{AB} can be given by

$$C(\rho^{AB}) = S(\rho^A) - \sup_{\{\Pi_i^B\}} S(\rho^{A|B}), \quad (5)$$

where $S(\rho^{A|B}) = \sum_{i=0,1} p_i S(\rho_i^A)$ is the conditional entropy of subsystem A and $\sup(\cdot)$ signifies the minimal value of the entropy with respect to a complete set of local measurements. Quantum discord is simply defined by $D(\rho^{AB}) = I(\rho^{AB}) - C(\rho^{AB})$.

According to Ref. [8], the quantum mutual information for Bell-diagonal mixed states $\rho^{AB}(t)$ in Eq. (4) is given by

$$I(\rho^{AB}) = 2 + \sum_{i=1}^4 \lambda_i(t) \log_2 \lambda_i(t), \quad (6)$$

and the classical correlation

$$C(\rho^{AB}) = \sum_{j=1,2} \frac{1 + (-1)^j \Lambda_{\max}(t)}{2} \times \log_2 [1 + (-1)^j \Lambda_{\max}(t)]. \quad (7)$$

Here $\Lambda_{\max}(t) = \max\{|c_x(t)|, |c_y(t)|, |c_z(t)|\}$. To obtain the interesting behavior, we focus on three kinds of initial states.

If the initial states $\rho^{AB}(0)$ satisfy the relation of $c_x = \mp c_z$ and $c_y = \pm c_x^2$, the mutual information is expressed by

$$I(\rho^{AB}) = \sum_{j=1,2} [1 + (-1)^j p(t) c_x] \log_2 [1 + (-1)^j p(t) c_x], \quad (8)$$

and the classical and quantum correlations are given by

$$D = C = \frac{I(\rho^{AB})}{2}. \quad (9)$$

This point means that the decoherence of quantum correlations is identical to that of classical correlations. Because the classical correlations are easily measured in the practical situations, the behavior of quantum correlations can also be observed indirectly. Figure 1 clearly shows

the case of $D = C$. When the kernel of the environments satisfies $A = a = \gamma$, the exponential decay of classical and quantum correlations is plotted in Fig. 1(a). Compared to the Markovian case, the decline of quantum and classical correlations is slow and delayed to a large time interval. It is also proven that the environment memory effects can yield some positive results for the realization of quantum information technology owing to the control of the system-reservoir interactions. When the kernel is changed to be $A = 10a \gg \gamma$, the values of $D = C$ are decaying sinusoidally and the continuous changes between nonzero values and zero ones are illustrated in Fig. 1(b).

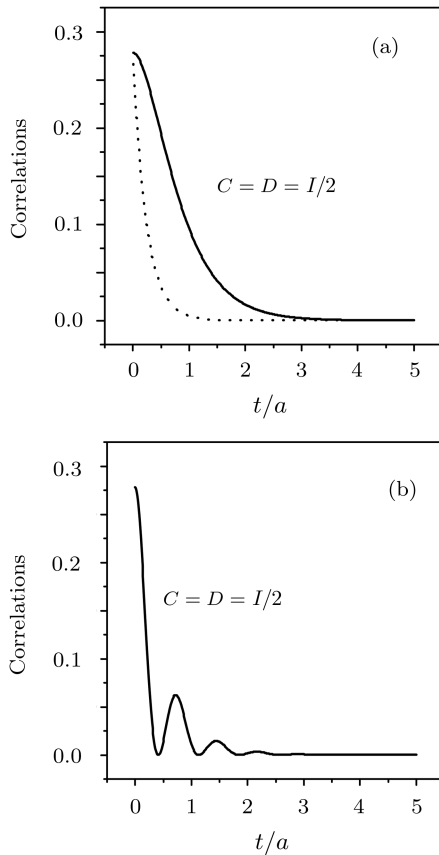


Fig. 1 The dynamics of correlations are plotted when the initial state is chosen to be $c_x = -c_z = c_y^{1/2} = 0.6$. (a) The parameters of the kernel $A = a = \gamma$; (b) The kernel satisfies $A = 10a \gg \gamma$. The solid lines denote quantum and classical correlations and the dotted line represents the dynamics of correlations in the Markovian case.

If another one kind of initial states are chosen to be $c_x = \pm c_y$ and $c_z = \mp 1$, the quantum mutual information is obtained by

$$I(\rho^{AB}) = \sum_{j=1,2} \frac{[1 + (-1)^j p(t) c_x]}{2} \log_2 [1 + (-1)^j p(t) c_x] + \sum_{j=1,2} \frac{[1 + (-1)^j p(t)]}{2} \log_2 [1 + (-1)^j p(t)]. \quad (10)$$

The second term of the above equation always coincides with the classical correlation $C(\rho^{AB})$. It is found that the change rate of quantum correlation is proportional to that of classical correlation decided by the decaying rate of the environment $p(t)$. This synchronous phenomenon of quantum and classical correlations is plotted in Fig. 2. It is clearly seen that the values of classical correlations are always larger than those of quantum correlations. The decreasing and increasing of C and D simultaneously happen.

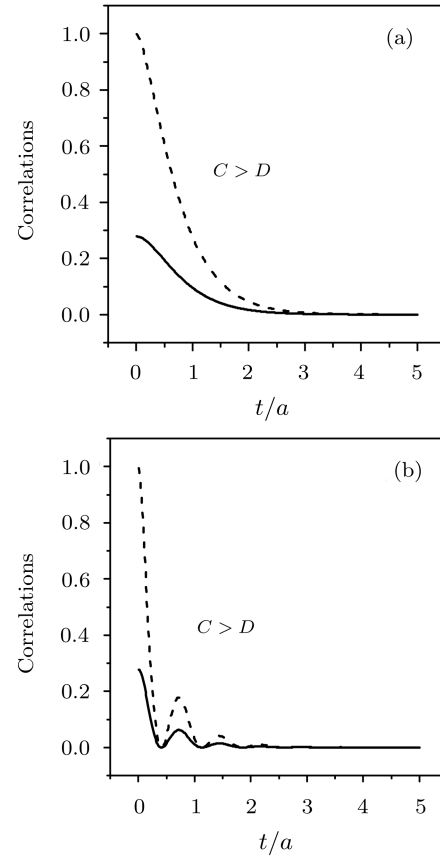


Fig. 2 The dynamics of correlations are plotted when the initial state is chosen to be $c_x = c_y = 0.6$ and $c_z = -1$. (a) The parameters of the kernel $A = a = \gamma$; (b) The kernel satisfies $A = 10a \gg \gamma$. The solid lines denote quantum correlations and the dashed lines represent classical correlations.

The initial states of $c_x = \pm c_z$ are used to reveal the sudden changes of quantum and classical correlations. From Fig. 3, it is shown that there exists one certain characteristic time t_c where the decaying rates of both quantum and classical correlations vary suddenly. To study the sudden changes, we analytically derive the classical correlation. It is found that

$$C(\rho^{AB}) = \sum_{j=1,2} \frac{[1 + (-1)^j p(t)c_x]}{2} \log_2[1 + (-1)^j p(t)c_x], \quad |p(t)| < |p(t_c)|,$$

$$C(\rho^{AB}) = \sum_{j=1,2} \frac{[1 + (-1)^j p^2(t)c_y]}{2} \log_2[1 + (-1)^j p^2(t)c_y], \quad |p(t)| > |p(t_c)|, \quad (11)$$

where $|p(t_c)| = |c_x|/|c_y|$. According to the expressions of $C(\rho^{AB})$, it is clearly shown that the decay of the classical correlation suddenly changes at the characteristic time t_c .

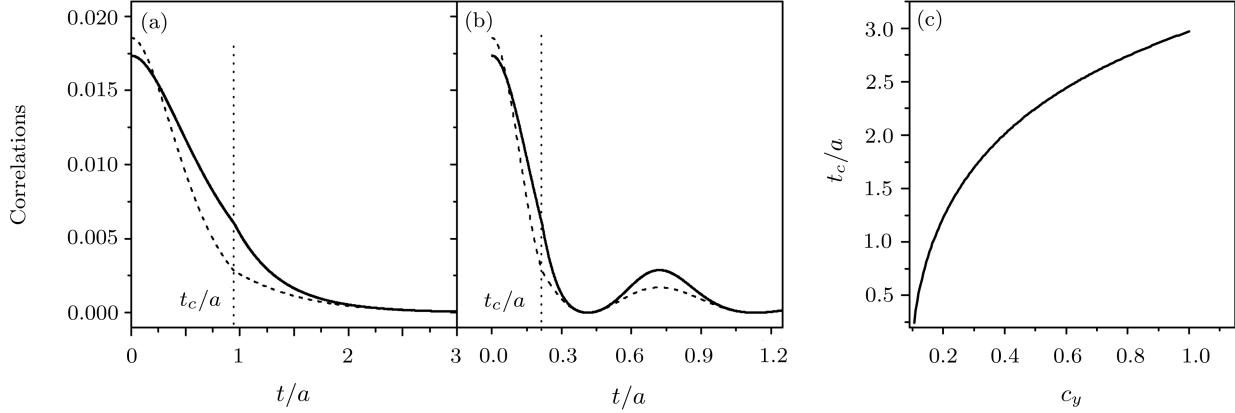


Fig. 3 The dynamics of correlations are plotted when the initial state is chosen to be $c_x = c_z = 0.1$ and $c_y = 0.16$ for the kernel satisfying $A = a = \gamma$; (b) The dynamics of correlations are plotted when the initial state is chosen to be $c_x = c_z = 0.1$ and $c_y = 0.16$ for the kernel satisfying $A = 10a \gg \gamma$; (c) The characteristic time t_c/a is plotted with the change of c_y when $c_x = c_z = 0.1$ and $A = a = \gamma$.

It is known that the sudden changes of quantum and classical correlations are caused by the variation of the inherent properties of the states. The distances from a given state to the closest state are drawn on to understand the physical origin of the sudden changes. According to Ref. [13], the distance between the state ρ^{AB} and the closest classical state ρ_{cl}^{AB} can quantify the quantum correlations defined by the relative entropy

$$D(\rho^{AB} || \rho_{cl}^{AB}) = \text{Tr}(\rho^{AB} \log_2 \rho^{AB}) - \text{Tr}(\rho^{AB} \log_2 \rho_{cl}^{AB}). \quad (12)$$

For Bell-diagonal states, $D(\rho^{AB} || \rho_{cl}^{AB}) = D(\rho^{AB})$. It is the fact that the sudden change of quantum correlation is closely dependent on the variation of the closest classical state. For $t < t_c$, the closest classical state can be written by

$$\rho_{cl}^{AB} = (\lambda_1 + \lambda_4)(|eg\rangle\langle eg| + |ge\rangle\langle ge|) + (\lambda_2 + \lambda_3)(|ee\rangle\langle ee| + |gg\rangle\langle gg|). \quad (13)$$

While the closest classical state for $t > t_c$,

$$\rho_{cl}^{AB} = \frac{\lambda_1 + \lambda_2}{2}(|\Psi^+\rangle\langle\Psi^+| + |\Phi^+\rangle\langle\Phi^+|) + \frac{\lambda_3 + \lambda_4}{2}(|\Psi^-\rangle\langle\Psi^-| + |\Phi^-\rangle\langle\Phi^-|). \quad (14)$$

Via the control of the kernel, the values of quantum correlations are greater than those of classical correlations in the large time interval from Figs. 3(a) and 3(b). When

the kernel satisfies $A = a = \gamma$, the characteristic time is obtained by

$$t_c = \ln \frac{1 + \sqrt{1 - |c_x|/|c_y|}}{|c_x|/|c_y|}$$

and numerically plotted in Fig. 3(c). It is seen that the characteristic time can be enlarged by the increasing of the ratio $|c_x|/|c_y|$. For the time interval $t < t_c$, quantum correlation declines more slowly than classical correlation.

4 Discussion

The decoherence of quantum and classical correlations is investigated in the condition that two noninteracting qubits are asymmetrically subjected to different non-Markovian Pauli channels. It is found that the destruction of quantum and classical correlations can be delayed via the engineering of the kernel characterizing the environment memory effects. The dynamical behavior of quantum and classical correlations are dependent on the selection of the initial states. For one class of Bell-diagonal mixed states, quantum correlations measured by quantum discord vary synchronously with classical correlations. This phenomena provide us a convenient way to evaluate quantum correlations by the direct observation of classical correlations. Meanwhile, the decaying rates of quantum correlations suddenly change at the characteristic time. For $t < t_c$, quantum correlation decline more

slowly than classical correlation. In a large time interval, classical correlations are less than quantum correlations, which is useful for the realization of quantum information

processing. The physical explanation of sudden changes of quantum correlations is obtained by the distance between the quantum state and the closest classical state.

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