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Flow of a Jeffery-Six Constant Fluid Between Coaxial Cylinders with Heat Transfer Analysis

Muhammad Yousaf Malik,* Azad Hussain, and Sohail Nadeem

Department of Mathematics, Quaid-i-Azam University 45320, Islamabad 44000, Pakistan

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Abstract *In the present investigation we have discussed the flow of a Jeffrey-six constant incompressible fluid between two infinite coaxial cylinders in the presence of heat transfer analysis. The governing equations of Jeffrey-six constant fluid along with energy equation have been derived in cylindrical coordinates. The highly nonlinear equations are simplified with the help of non-dimensional parameters and then solved analytically with the help of homotopy analysis method (HAM) for two fundamental flows namely Couette and Generalized Couette flow. The effects of emerging parameters are discussed through graphs. The convergence of the HAM solution has been discussed by plotting h-curves.*

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Key words: Jeffery-six constant fluid, coaxial cylinders, heat transfer analysis, HAM solutions

1 Introduction

A large class of real fluids does not exhibit the linear relationship between stress and the rate of strain. Because of the non-linear dependence, the analysis of the behavior of the fluid motion of the non-Newtonian fluids tend to be much more complicated and subtle in comparison with that of the Newtonian fluids. In the literature, the mechanics of non-linear fluids presents special challenges to engineers, physicists and mathematicians since the non-linearity can manifest itself in a variety of ways. One of the simplest way in which the viscoelastic fluids have been classified is the methodology given by Rivlin and Ericksen[1] and Truesdell and Noll,[2] who presents constitutive relations for the stress tensor as a function of the symmetric part of the velocity gradient and its higher (total) derivatives. In recent years there have been several studies[3−12] on flows of non-Newtonian fluids, not only because of their technological significance but also in the interesting mathematical features presented by the equations governing the flow. On the other hand, it is well known that the rheological properties of many fluids are not well modelled by the Navier–Stokes equations.[13] It is not possible to obtain a single equation exhibting all properties of all non-Newtonian fluids from available literature. That is why several models of non-Newtonian fluids are proposed. Jeffery-six constant fluid is one of these models. Literature survey indicates that very less attention has been given to the flows of a Jeffrey-six constant fluid. Recently, Nadeem and Akbar[14] studied the effects of temperature dependent viscosity on peristaltic flow of a Jeffrey-six constant fluid in a uniform vertical tube. In another study, Nadeem and Akbar[15] examined influence of heat and mass transfer on a peristaltic motion of a Jeffrey-six constant fluid in an annulus. In continuation, Nadeem *et al.*[16] considered numerical solutions of peristaltic flow of a Jeffrey-six constant fluid with variable MHD. Further, Akbar *et al.*[17] discussed simulation of heat transfer on the peristaltic flow of a Jeffrey-six constant fluid in a diverging tube.

The primary objective of this investigation is to model and analyze the flow a Jeffrey-six constant incompressible fluid between two infinite coaxial cylinders. Two types of fundamental flows namely the Couette and Generalized Couette flow are considered. The heat transfer is also carried out. Analytical expressions of velocity and temperature are developed in each case by using a powerful technique namely the homotopy analysis method (HAM).[18−28] Convergence of the series solution is carefully checked. Finally, the influence of the various parameters intrinsic to the problems is discussed by plotting graphs.

2 Mathematical Model

Let us consider the exact formulation of the problem of a viscoelastic fluid flow between coaxial cylinders under the action of a pressure gradient. The equations of motion in terms of extrastresses have the form[30]

$$\rho \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\bigtriangledown \boldsymbol{p} + \bigtriangledown \cdot \boldsymbol{T}\,, \tag{1}$$

where ρ is the density, $\boldsymbol{v}$ is the velocity vector, p is the mechanical pressure and $\boldsymbol{T}$ is the extrastresses tensor. The fluid is assumed to be incompressible, therefore

$$\bigtriangledown \cdot \boldsymbol{v} = 0\,. \tag{2}$$

The Jeffreys equation with the most general associated time derivative symmetry has the form[29]

$$T + \lambda_1 F_{abc} T = \mu (D + \lambda_2 F_{abc} D)\,, \tag{3}$$

or is written in an expanded from as

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*Corresponding author, E-mail: drmymalik@hotmail.com

$$T+\lambda_1\Big[\frac{\mathrm{d}T}{\mathrm{d}t}-W\cdot T+T\cdot W+a(T\cdot D+D\cdot T)+bT:DI+cD\mathrm{tr}T\Big]=2\mu\Big[D+\lambda_2\Big(\frac{\mathrm{d}D}{\mathrm{d}t}-W\cdot D+D.W+2aD\cdot D+bD:DI\Big)\Big]\,, \tag{4}$$

where $\bigtriangledown \boldsymbol{v}=\boldsymbol{D}+W$ is the velocity gradient, $D=(\bigtriangledown \boldsymbol{v}^t+\bigtriangledown \boldsymbol{v})/2$ is the symmetric part of $\bigtriangledown \boldsymbol{v}$, $W=(\bigtriangledown \boldsymbol{v}^t-\bigtriangledown \boldsymbol{v})/2$ is the antisymmetric part of $\bigtriangledown\ \boldsymbol{v}$, μ is the viscosity, λ_1 is the relaxation time and λ_2 is the delay time.

We seek the velocity of the form

$$\boldsymbol{v}=(0,\ 0,v_z(r))\,. \tag{5}$$

Using Eqs. (4) and (5), one can write the dimensionless problem describing the flow as

$$\frac{\mu}{r}\frac{\mathrm{d}v}{\mathrm{d}r}+\frac{\mu}{r}(A+u+2zt)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^3+\frac{\mu}{r}(zt^2+s)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^5+\mu\frac{\mathrm{d}^2v}{\mathrm{d}r^2}+\mu(zt^2+5s)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4$$
$$+\mu(3A+3u+2zt)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2-B-Bt^2\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4-2B\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2=0\,, \tag{6}$$

whence

$$r=\frac{\bar r}{R},\quad s=\frac{A_1^*V_0^4}{R^4},\quad u=\frac{A_2^*V_0^2}{R^2},\quad C_1=\frac{\partial p}{\partial z},\quad t=\frac{A_3^*V_0^2}{R^2},\quad B=\frac{C_1R^2}{\mu_*V_0},\quad v=\frac{\bar v}{v_0},\quad \mu=\frac{\bar\mu}{\mu_*},\quad A=\frac{A_4^*V_0^2}{R^2}\,, \tag{7}$$

$$z=\frac{\lambda_2}{\lambda_1}\,,$$
$$A_4^*=\frac{-\lambda_1\lambda_2}{2}(-1+a+b)(1+a+c)\Big[1-\frac{\lambda_2}{\lambda_1}\Big]\,,$$
$$A_3^*=\frac{\lambda_1}{2}[\lambda_1(1-a-c)(1+a+b)-bc\lambda_1-\lambda_2(1+a+c)(-1+a+b)]\,,$$
$$A_2^*=\frac{\lambda_1}{2}\Big[1-\frac{\lambda_2}{\lambda_1}\Big][\lambda_1(1-a-c)(1+a+b)-bc\lambda_1-\lambda_2(1+a+c)(-1+a+b)]\,,$$
$$A_1^*=\frac{-\lambda_1^2\lambda_2}{4}(-1+a+b)(1+a+c)\left[\Big[1-\frac{\lambda_2}{\lambda_1}\Big]\left[\begin{array}{c}\lambda_1(1-a-c)(1+a+b)\\ -bc\lambda_1-\lambda_2(1+a+c)(-1+a+b)\end{array}\right]\right]\,, \tag{8}$$

where μ_*, R, and V_0 are respectively the reference viscosity, radius of inner cylinder and reference velocity. Moreover, a, b, and c are material parameters of the fluid and are assumed constants.

2.1 *Energy Equation*

In order to take heat transfer analysis into account, the dimensionless form of energy equation can be written as

$$\mu\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2+\mu(A+zt)\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4-\frac{1}{r}\frac{\mathrm{d}\theta'}{\mathrm{d}r}-\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}-\frac{t}{r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2-t\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2=0\,, \tag{9}$$

where

$$\Gamma=\frac{V_0^2\ \mu_*}{k(\theta_m-\theta_w)}\,.$$

3 Analytical Solutions

3.1 *Couette Flow*

Consider an incompressible and thermodynamic fluid between two infinite coaxial cylinders. The flow is induced by motion of an inner cylinder. The outer cylinder is kept fixed. The heat transfer analysis is also taken into account. The dimensionless problem, which can describe the flow is defined as

$$\frac{\mu}{r}(A+u+2zt)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^3+\mu(3A+3u+2zt)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2$$
$$+\mu\frac{\mathrm{d}^2v}{\mathrm{d}r^2}+\frac{\mu}{r}(zt^2+s)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^5+\mu(zt^2+5s)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4+\frac{\mu}{r}\frac{\mathrm{d}v}{\mathrm{d}r}=0\,, \tag{10}$$

$$\mu\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2+\mu(A+zt)\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4-\frac{1}{r}\frac{\mathrm{d}\theta}{\mathrm{d}r}-\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}-\frac{t}{r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2-t\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2=0\,, \tag{11}$$

$$v(r)=1,\quad \theta(r)=1,\quad r=1\,, \tag{12}$$

$$v(r)=0,\quad \theta(r)=0,\quad r=b\,. \tag{13}$$

For HAM solution, we choose the following initial guesses

$$v_0(r) = \frac{(r-b)}{(1-b)}\,, \tag{14}$$

$$\theta_0(r) = \frac{(r-b)}{(1-b)}\,. \tag{15}$$

The auxiliary linear operators are defined as

$$\pounds_{vr}(v) = v''\,, \tag{16}$$

$$\pounds_{\theta r}(\theta) = \theta''\,, \tag{17}$$

which satisfy

$$\mathcal{L}_{vr}(A_1 + B_1 r) = 0\,, \tag{18}$$

$$\mathcal{L}_{\theta r}(A_2 + B_2 r) = 0\,, \tag{19}$$

where A_1, A_2, B_1, and B_2 are the constants.

If p $\in [0,1]$ is an embedding parameter and $\hbar_v$ and $\hbar_\theta$ are auxiliary parameters then the problems at the zero and m-th order are respectively given by

$$(1-p)\mathcal{L}_v[\bar{v}(r,p) - v_0(r)] = p\hbar_v N_v[\bar{v}(r,p), \bar{\theta}(r,p)]\,, \tag{20}$$

$$(1-p)\mathcal{L}_\theta[\bar{\theta}(r,p) - \theta_0(r)] = p\hbar_\theta N_\theta[\bar{v}(r,p), \bar{\theta}(r,p)]\,, \tag{21}$$

$$\mathcal{L}_v[v_m(r) - \chi_m v_{m-1}(r)] = \hbar_v R_v(r)\,, \tag{22}$$

$$\mathcal{L}_\theta[\theta_m(r) - \chi_m \theta_{m-1}(r)] = \hbar_\theta R_\theta(r)\,, \tag{23}$$

$$\bar{v}(r,p) = 1, \quad \bar{\theta}(r,p) = 1, \quad r = 1\,, \tag{24}$$

$$\bar{v}(r,p) = 0, \quad \bar{\theta}(r,p) = 0, \quad r = b\,. \tag{25}$$

$$\bar{v}_m(r,p) = 0, \quad \bar{\theta}_m(r,p) = 0, \quad r = 1\,, \tag{26}$$

$$\bar{v}_m(r,p) = 0, \quad \bar{\theta}_m(r,p) = 0, \quad r = b\,, \tag{27}$$

where

$$N_v[\bar{v}(r,p)] = \frac{\mu}{r}\frac{\mathrm{d}v}{\mathrm{d}r} + \frac{\mu}{r}(A + u + 2zt)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^3 + \frac{\mu}{r}(zt^2 + s)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^5 + \mu\frac{\mathrm{d}^2v}{\mathrm{d}r^2} + \mu(zt^2 + 5s)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4 + \mu(3A + 3u + 2zt)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2. \tag{28}$$

$$N_\theta[\bar{v}(r,p), \bar{\theta}(r,p)] = \mu\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2 + \mu(A + zt)\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4 - \frac{1}{r}\frac{\mathrm{d}\theta}{\mathrm{d}r} - \frac{\mathrm{d}^2\theta}{\mathrm{d}^2r} - \frac{t}{r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2 - t\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2, \tag{29}$$

$$\begin{aligned} R_v = {} & \frac{\mu}{r}v_{m-1}^{'} + \frac{\mu}{r}(A + u + 2zt)\sum_{k=0}^{m-1}\sum_{l=0}^{k} v'_{m-1-k}v'_{k-l}v'_l \\ & + \frac{\mu}{r}(zt^2 + s)\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l}\sum_{j=0}^{s} v'_{m-1-k}v'_{k-l}v'_{l-s}v'_{s-j}v'_j + \mu v''_{m-1} \\ & + \mu(zt^2 + 5s)\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l}\sum_{j=0}^{s} v'_{m-1-k}v'_{k-l}v'_{l-s}v'_{s-j}v''_j \\ & + \mu(3A + 3u + 2zt)\sum_{k=0}^{m-1}\sum_{l=0}^{k} v'_{m-1-k}v'_{k-l}v''_l, \end{aligned} \tag{30}$$

$$\begin{aligned} R_\theta = {} & \mu\Gamma\sum_{k=0}^{m-1} v'_{m-1-k}v'_k + \mu(A + zt)\Gamma\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l} v'_{m-1-k}v'_{k-l}v'_{l-s}v'_s \\ & - \frac{1}{r}\theta'_{m-1} - \theta''_{m-1} - \frac{t}{r}\sum_{k=0}^{m-1} v'_{m-1-k}v'_k - t\sum_{k=0}^{m-1}\sum_{l=0}^{k} v'_{m-1-k}v'_{k-l}\theta''_l. \end{aligned} \tag{31}$$

By *Mathematica* the solutions of Eqs. (30) and (31) can be written as

$$v_m(r) = \sum_{n=0}^{m+1} a_{m,n}r^n\,, \quad \theta_m(r) = \sum_{n=0}^{m+2} d_{m,n}r^n\,, \quad m \geq 0\,, \tag{32}$$

where $a_{m,n}$ and $d_{m,n}$ are constants, which can be determined on substituting Eq. (32) into Eqs. (22) and (23).

3.2 *Generalized Couette Flow*

Consider an incompressible and thermodynamic third grade fluid between two infinite coaxial cylinders. The flow is induced by a constant pressure gradient and motion of an inner cylinder. The outer cylinder is kept fixed. The heat transfer analysis is also carried out. The dimensionless problem, which can describe the flow is

$$\frac{\mu}{r}(A + u + 2zt)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^3 + \frac{\mu}{r}(zt^2 + s)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^5 + \mu\frac{\mathrm{d}^2v}{\mathrm{d}r^2} + \mu(zt^2 + 5s)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4 + \frac{\mu\mathrm{d}v}{r\mathrm{d}r} + \mu(3A + 3u + 2zt)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2 - B - Bt^2\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4 - 2B\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2 = 0\,. \tag{33}$$

$$\mu\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2+\mu(A+zt)\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4-\frac{1}{r}\frac{\mathrm{d}\theta}{\mathrm{d}r}-\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}-\frac{t}{r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2-t\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2=0\,, \tag{34}$$

$$v(r)=1,\quad \theta(r)=1,\quad r=1\,, \tag{35}$$

$$v(r)=0,\quad \theta(r)=0,\quad r=b\,. \tag{36}$$

For HAM solution, we choose the following initial guesses

$$v_0(r)=\frac{(r-b)}{(1-b)}\,, \tag{37}$$

$$\theta_0(r)=\frac{(r-b)}{(1-b)}. \tag{38}$$

The auxiliary linear operators are in the form

$$\pounds_{vr}(v)=v''\,, \tag{39}$$

$$\pounds_{\theta r}(\theta)=\theta''\,, \tag{40}$$

which satisfy

$$\mathcal{L}_{vr}(A_1+B_1r)=0\,, \tag{41}$$

$$\mathcal{L}_{\theta r}(A_2+B_2r)=0\,, \tag{42}$$

where A_1, A_2, B_1, and B_2 are the constants.

If p $\in[0,1]$ is an embedding parameter and $\hbar_v$ and $\hbar_\theta$ are auxiliary parameters then the problems at the zero and m-th order are respectively given by

$$(1-p)\mathcal{L}_v[\bar{v}(r,p)-v_0(r)] = p\hbar_v N_v[\bar{v}(r,p),\bar{\theta}(r,p)]\,, \tag{43}$$

$$(1-p)\mathcal{L}_\theta[\bar{\theta}(r,p)-\theta_0(r)] = p\hbar_\theta N_\theta[\bar{v}(r,p),\bar{\theta}(r,p)]\,, \tag{44}$$

$$\mathcal{L}_v[v_m(r)-\chi_m v_{m-1}(r)]=\hbar_v R_v(r)\,, \tag{45}$$

$$\mathcal{L}_\theta[\theta_m(r)-\chi_m \theta_{m-1}(r)]=\hbar_\theta R_\theta(r)\,, \tag{46}$$

$$\bar{v}(r,p)=1,\quad \bar{\theta}(r,p)=1,\quad r=1\,, \tag{47}$$

$$\bar{v}(r,p)=0,\quad \bar{\theta}(r,p)=0,\quad r=b\,, \tag{48}$$

$$\bar{v}_m(r,p)=0,\quad \bar{\theta}_m(r,p)=0,\quad r=1\,, \tag{49}$$

$$\bar{v}_m(r,p)=0,\quad \bar{\theta}_m(r,p)=0,\quad r=b\,, \tag{50}$$

where

$$\begin{aligned}N_v[\bar{v}(r,p)]={}&\frac{\mu}{r}\frac{\mathrm{d}v}{\mathrm{d}r}+\frac{\mu}{r}(A+u+2zt)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^3+\frac{\mu}{r}(zt^2+s)\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^5+\mu\frac{\mathrm{d}^2v}{\mathrm{d}r^2}+\mu(zt^2+5s)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4\\&+\mu(3A+3u+2zt)\frac{\mathrm{d}^2v}{\mathrm{d}r^2}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2-B-Bt^2\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4-2B\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2,\end{aligned} \tag{51}$$

$$N_\theta[\bar{v}(r,p),\bar{\theta}(r,p)]=\mu\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2+\mu(A+zt)\Gamma\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^4-\frac{1}{r}\frac{\mathrm{d}\theta}{\mathrm{d}r}-\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}-\frac{t}{r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2-t\frac{\mathrm{d}^2\theta}{\mathrm{d}^2r}\Big(\frac{\mathrm{d}v}{\mathrm{d}r}\Big)^2, \tag{52}$$

$$\begin{aligned}R_v={}&\frac{\mu}{r}v'_{m-1}+\frac{\mu}{r}(A+u+2zt)\sum_{k=0}^{m-1}\sum_{l=0}^{k}v'_{m-1-k}v'_{k-l}v'_l\\&+\frac{\mu}{r}(zt^2+s)\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l}\sum_{j=0}^{s}v'_{m-1-k}v'_{k-l}v'_{l-s}v'_{s-j}v'_j\\&+\mu v''_{m-1}+\mu(zt^2+5s)\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l}\sum_{j=0}^{s}v'_{m-1-k}v'_{k-l}v'_{l-s}v'_{s-j}v''_j\\&+\mu(3A+3u+2zt)\sum_{k=0}^{m-1}\sum_{l=0}^{k}v'_{m-1-k}v'_{k-l}v''_l\\&+-B-Bt^2\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l}v'_{m-1-k}v'_{k-l}v'_{l-s}v'_s-2B\sum_{k=0}^{m-1}v'_{m-1-k}v'_k,\end{aligned} \tag{53}$$

$$\begin{aligned}R_\theta={}&\mu\Gamma\sum_{k=0}^{m-1}v'_{m-1-k}v'_k+\mu(A+zt)\Gamma\sum_{k=0}^{m-1}\sum_{l=0}^{k}\sum_{s=0}^{l}v'_{m-1-k}v'_{k-l}v'_{l-s}v'_s\\&-\frac{1}{r}\theta'_{m-1}-\theta''_{m-1}-\frac{t}{r}\sum_{k=0}^{m-1}v'_{m-1-k}v'_k-t\sum_{k=0}^{m-1}\sum_{l=0}^{k}v'_{m-1-k}v'_{k-l}\theta''_l.\end{aligned} \tag{54}$$

By *Mathematica* the solutions of Eqs. (53) and (54) can be written as

$$v_m(r)=\sum_{n=0}^{2m+1}a'_{m,n}r^n\,,\quad \theta_m(r)=\sum_{n=0}^{2m+1}d'_{m,n}r^n\,,\quad m\ge 0\,, \tag{55}$$

where $a'_{m,n}$ and $d'_{m,n}$ are constants, which can be determined on substituting Eq. (55) into Eqs. (43) and (44).

4 Graphical Results and Discussion

In order to report the convergence of the obtained series solutions and the effects of sundry parameters in the present investigation we plot Figs. 1–11. Figures 1–4 are prepared to see the convergence region. Figures 1 and 2 correspond to Couette flow, where as Figures 3 and 4 relate to Generalized Couette flow. Figure 5 depicts the velocity variation for Couette flow for different values of u. It is observed that velocity increases as u increases. Figure 6 shows the temperature variation for different values of t for Generalized Couette flow. It is observed that temperature increases as t increases. Figure 7 is plotted in order to see the velocity variation for Generalized Couette flow for different values of B, it is depicted that velocity decreases as B increases. Figure 8 depicts the velocity variation for Couette flow for different values of t. It is seen that velocity increases as t increases. Figure 9 shows the temperature variation for Couette flow for different values of u. It is depicted that temperature first increases and then decreases as u increases. Figure 10 is prepared to observe the velocity variation for Generalized Couette flow for different values of u. It is observed that velocity increases as u increases. Figure 11 is plotted to see the temperature profile for Couette flow for different values of t. It is observed that temperature decreases as t increases.

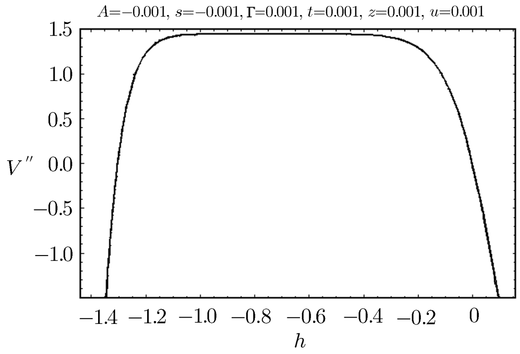

Fig. 1 h-curve for velocity for Couette flow.

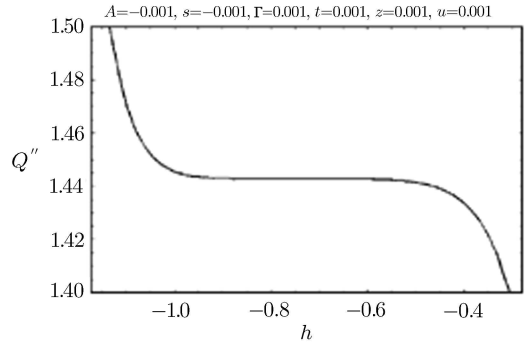

Fig. 2 h-curve for temperature for Couette flow.

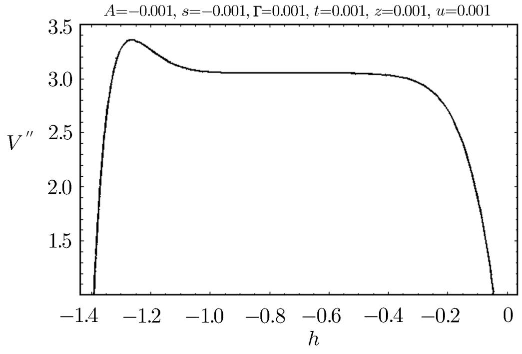

Fig. 3 h-curve for velocity for Generalized Couette flow.

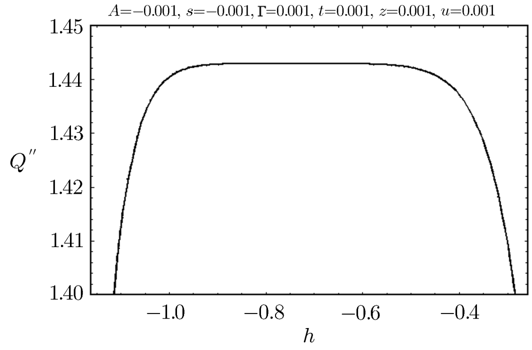

Fig. 4 h-curve for temperature for Generalized Couette flow.

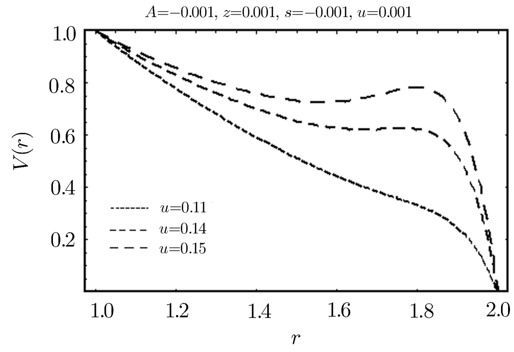

Fig. 5 Velocity profile for Couette flow for different values of u.

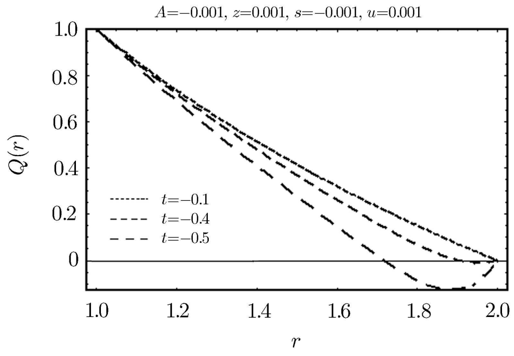

Fig. 6 Temperature profile for Generalized Couette flow for different values of t.

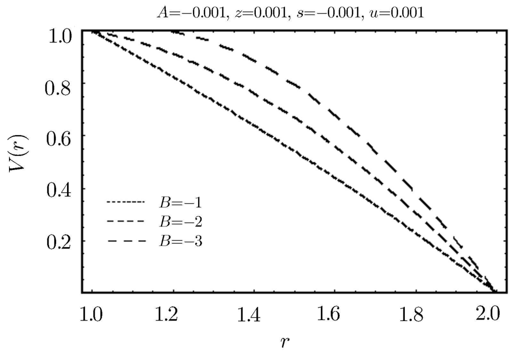

Fig. 7 Velocity profile for Generalized Couette flow for different values of B.

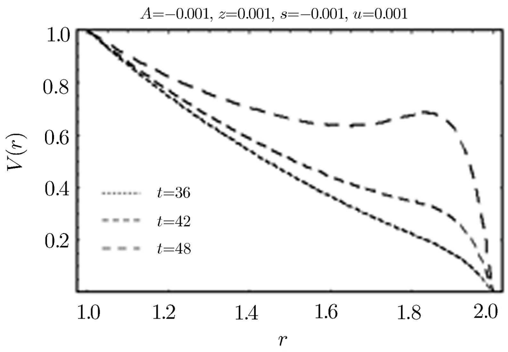

Fig. 8 Velocity profile for Couette flow for different values of t.

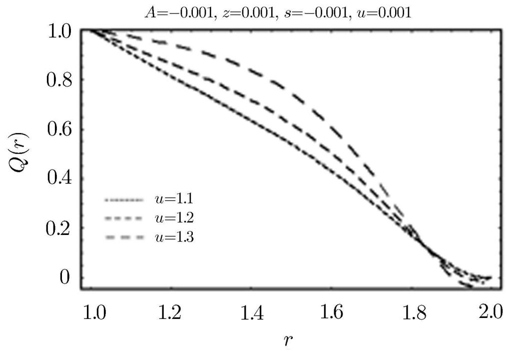

Fig. 9 Temperature profile for Couette flow for different values of u.

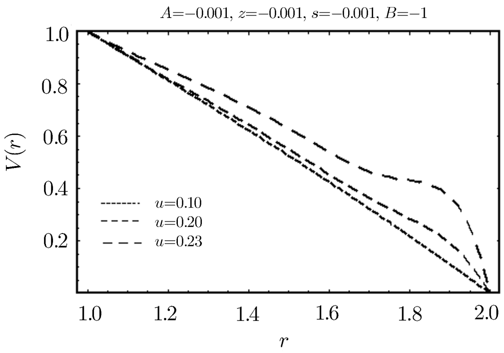

Fig. 10 Velocity profile for Generalized Couette flow for different values of u.

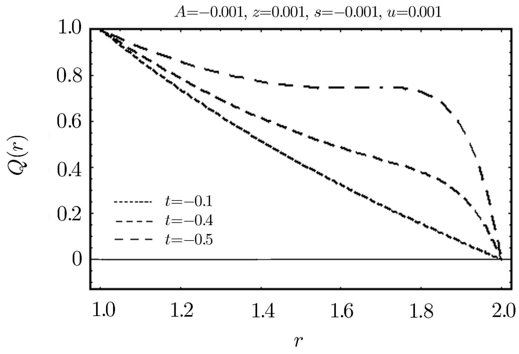

Fig. 11 Temperature profile for Couette flow for different values of t.

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