A Cellular Automaton Model for Heterogeneous and Incosistent Driver Behavior in Urban Traffic*

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Abstract *In this paper a cellular automaton model is proposed to describe driver behavior at a single-lane urban roundabout. Driver behavior has been considered as heterogeneous and inconsistent. Most traffic papers in the literature just discussed heterogeneous driver behavior, to our best knowledge. Two truncated Gaussian distributions are used to model heterogeneous and inconsistent driver behavior, respectively. The physical meanings of two truncated distributions are indicated. This method may help enhance a better understanding of driver behavior at roundabout traffic, and even possibly provide references for roundabout design and management.*

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1 Introduction

In urban traffic, roundabouts have been considered as an alternative traffic facility that can improve safety and operational efficiency, compared to un-signalized intersection controls.[1] Most of roundabouts are governed by the yield-at-entry rule (i.e., vehicles from the secondary roads give way to the vehicles on the circulatory road).

Empirical and theoretical methods have been proposed to measure roundabout capacity and performance such as delay and queue length.[1] With regard to both methods, gap-acceptance criteria are commonly used.[2–3] Gap acceptance is a process by which a minor-street vehicle accepts an available gap to maneuver.[4] Gap-acceptance models are, however, generally unrealistic in assuming that drivers are consistent and homogenous.[5] A consistent driver would be expected to behave in the same way in all similar situations, while in a homogenous population, all drivers have the same critical gap (the minimum time interval, between two major-stream vehicles, required by one minor-stream vehicle to pass through) and are expected to behave uniformly. As a matter of fact, driver behavior should be heterogeneous and inconsistent. In Ref. [6], Akcelik indicates that driver behavior is directly related to capacity and performance measurement. More recently, many researchers focus on roundabout management and safety.[7–11] Those studies appeal for good driver behavior. Therefore, driver behavior is of paramount importance in analyzing traffic flow in urban networks. In this paper a cellular automaton (CA) model integrating two truncated Gaussian distributions is proposed to describe heterogeneous and inconsistent driver behavior at a single-lane roundabout. This method may be expected to enhance a better understanding of driver behavior, and even provide references for roundabout design and management.

2 Model

A novel and realistic CA model, based on the Normal Acceptable Space (NAS) method, is proposed to simulate heterogeneous driver behavior and inconsistent driver behavior at entrances of cross traffic, respectively. The NAS method is theoretically based on statistics and practically based on the fine grid (the length of each cell corresponds to 1 m in a real road, and a car is assumed to occupy 5 cells). Statistical theory is used to describe driver behavior; while the fine grid guarantees population distribution have a usual shape. The latter is extremely important in the NAS method. If the size of CA cells is large, the population distribution of driver behavior cannot be described by a normal distribution. If the grid is too small (< 1 m), sample accuracy is enhanced but it provides little help to analyze driver behavior instead increases the computational complexity. Note that the NAS model is a CA-based model, developed for cross traffic at roundabouts. In general, a driver normally decides to leave the roundabout or not according to the distance (i.e., the number of cells) between the current position and the predefined exit. When the distance approaches zero, the driver

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prepares to leave the roundabout. On the other hand, the distance may influence the entrance of a car from the next arm of the roundabout. Therefore, using space gap acceptance rather than time gap acceptance is more appropriate to describe interacted driver behavior in this model.

In this paper, heterogeneous driver behavior and inconsistent driver behavior are modeled within a statistical framework and then the relationship between heterogeneous driver behavior and inconsistent driver behavior is discussed.

2.1 *Modeling of Heterogeneous Driver Behavior*

According to the central limit theorem,[12] distribution of space required for all drivers at entrances of cross traffic is assumed to follow a Gaussian distribution. This population distribution is referred to as GDH (Gaussian Distribution for Heterogeneous driver behavior). There are two pre-conditions required to judge whether a distribution can be roughly viewed as a Gaussian distribution in the central limit theorem: (i) independent sample size N is very large and (ii) the population has a finite variance and a finite mean.

With regard to case (i), it is clear that the number of drivers is large, which meets the requirement of using very large samples in statistics. Although there is still no consensus of standard, on how large the sample size must be in order to apply the central limit theorem, a sample size of 30 or more[12] is usually regarded as sufficient. For a population of drivers, the condition of independent sample size $N \geq 30$ is quite easily satisfied.

With regard to case (ii), each driver has his/her own special space criteria, which are normally based on age, gender, driving skill and so on. However, in general, these space criteria fluctuate along a certain range and do not have very unusual values. That is to say, a finite variance and a finite mean exist. Therefore, according to case (i) and (ii), population distribution (heterogeneous driver behavior) can be approximately considered as a Gaussian distribution. The density function of GDH can be written as follows:

$$f(x) = \frac{1}{\sigma_H \sqrt{2\pi}} \mathrm{e}^{(x-\mu_H)^2/2\sigma_H^2}, \tag{1}$$

where x is random variable, μ_H is the mean and σ_H is the deviation of GDH. Thus, the value y_n, defined as the required space to enter a roundabout for a driver n, can be determined by Eq. (2). At the same time y_n represents a type of heterogeneous driver behavior. That is, in our model, there are seven types of heterogeneous driver behavior. Each type of heterogeneous driver behavior corresponds to different space requirements to enter a roundabout. In Gaussian distribution, 99.73% confidence interval is given by $[\mu_H - 3\sigma_H,\ \mu_H + 3\sigma_H]$.[12] In Eq. (2), a confidence interval of $[\mu_H - 3\sigma_H,\ \mu_H + 3\sigma_H]$ is simply extended to be 100%. Meanwhile, μ_H is assumed to fall in [0.4, 0.6]. Therefore, heterogeneous driver behavior for driver n can be described as follows:

$$y_n = \begin{cases} \mu_H - 3\sigma_H\,, & \text{if} \quad 0 \leq p_n \leq 0.0212\,, \\ \mu_H - 2\sigma_H\,, & \text{if} \quad 0.0212 < p_n \leq 0.1586\,, \\ \mu_H - \sigma_H\,, & \text{if} \quad 0.1586 < p_n < 0.4\,, \\ \mu_H\,, & \text{if} \quad 0.4 \leq p_n \leq 0.6\,, \\ \mu_H + \sigma_H\,, & \text{if} \quad 0.6 < p_n \leq 0.8414\,, \\ \mu_H + 2\sigma_H\,, & \text{if} \quad 0.8614 < p_n \leq 0.9788\,, \\ \mu_H + 3\sigma_H\,, & \text{if} \quad 0.9788 < p_n \leq 1\,. \end{cases} \tag{2}$$

2.2 *Modeling of Inconsistent Driver Behavior*

With regard to a single driver n at an entry of a roundabout, the value $d(n)$ can be viewed as the required space to enter (the number of required unoccupied cells in CA models). Therefore, $d(n)$ can be one of a set of data sets in the data records, namely, $d(n) = \{d_1(n), d_2(n), d_3(n), \ldots, d_l(n)\}$, $l = 1, 2, 3, \ldots, L$. L is the number of possible values which is required by a driver to enter the junction. The value $d(n)$ may consist of two independent parts:

$$d(n) = d + \tau_n\,. \tag{3}$$

That is, $d(n)$ is a sum of a constant part d plus a random and independent error component τ_n. Moreover, the error portion can itself be thought of as a sum of m components:[12]

$$\tau_n = g(e_1 + e_2 + e_3 + \cdots + e_m)\,. \tag{4}$$

Here, e_i is a random variable that can only take on one of following two values:

$e_i = 1$, when factor i is valid,

$e_i = 0$, when factor i is not valid.

Thus, the error τ_n can be roughly assumed to follow a Gaussian distribution, i.e., $\tau_n \sim N(0, \sigma_n^2)$, if the inconsistent driver behavior of driver n is sampled many times. In fact, the basic value d stands for the driver's habits under normal driving circumstance, i.e., $d = y_n$. τ_n is influenced by many stochastic factors such as temporary road construction, a pedestrian crossing, bad weather, night visibility, peak hour. However, these stochastic factors normally impact slightly on constant part d. $d(n)$ fluctuates along the basic value d. Therefore, $d(n)$ is also assumed to approximately follow a Gaussian distribution, i.e., $d(n) \sim N(d, \sigma_n^2)$. The possibility density function of $d(n)$ can be written as follows:

$$f_n(x) = \frac{1}{\sigma_n \sqrt{2\pi}} \mathrm{e}^{(x_n - d_n)^2/2\sigma_n^2}. \tag{5}$$

In general, a driver may accept a value which is less than $d(n)$, due to a long waiting time or other urgent conditions, while a driver may also accept a value which is larger than $d(n)$, due to bad weather, night visibility or other factors. Let $x_{\min}$ represents the number of minimum acceptable cells and $x_{\max}$ stands for the number of maximum acceptable cells for a driver to interact with other drivers. If $x > x_{\max}$, a car can pass the intersection without delay and there is no interaction. Values less than $x_{\min}$ are rejected, due to safety factors and values larger than $x_{\max}$ do not need to be considered, due to no interaction involved (free flow). Therefore, the resulting model can be viewed as a truncated Gaussian distribution,[12] where the left and right parts have been cut off. Thus, the distribution of space required for a single driver can be assumed to follow a truncated Gaussian distribution, which is referred to as a Gaussian Distribution for Inconsistent driver behavior (GDI for short). It can be mathematically written in Eq. (6). Therefore, the value z_n of inconsistent driver behavior for the n-th driver can be expressed by Eq. (7). According to Eqs. (6) and (7), the value of z_n can be determined by Eq. (8). Therefore, inconsistent driver behavior for driver n can be described as follows:

$$f_n(x) = \frac{1}{\sigma_n\sqrt{2\pi}}\,\mathrm{e}^{(x_n-\mu_n)^2/2\sigma_n^2}, \quad x_{\min} \leqslant x \leqslant x_{\max}\,, \tag{6}$$

$$z_n = \begin{cases} \mu_n - 3\sigma_n, & \text{if} \quad 0 \le p'_n \le 0.0212\,, \\ \mu_n - 2\sigma_n, & \text{if} \quad 0.0212 \le p'_n \le 0.1586\,, \\ \mu_n - \sigma_n, & \text{if} \quad 0.1586 < p'_n < 0.4\,, \\ \mu_n, & \text{if} \quad 0.4 \le p'_n \le 0.6\,, \\ \mu_n + \sigma_n, & \text{if} \quad 0.6 < p'_n \le 0.8414\,, \\ \mu_n + 2\sigma_n, & \text{if} \quad 0.8614 < p'_n \le 0.9788\,, \\ \mu_n + 3\sigma_n, & \text{if} \quad 0.9788 < p'_n \le 1\,, \end{cases} \tag{7}$$

$$z_n = \begin{cases} \max(z_n, x_{\min}), & \text{if} \quad z_n < x_{\min}\,, \\ \min(z_n, x_{\max}), & \text{if} \quad z_n < x_{\max}\,. \end{cases} \tag{8}$$

The relationship between GDH and GDI is that for the n-th driver the value of y_n in Eq. (2) is equal to the value of μ_n in Eq. (7). In other words, $z_n = y_n + k\sigma_n$. The coefficient k takes an integer value from $\{-3, -2, -1, 0, 1, 2, 3\}$, which is determined by a probability p'_n in Eq. (7). In this way, the heterogeneous driver behavior and inconsistent driver behavior can be systematically simulated by the NAS method.

2.3 *Updating Rules on a Roundabout*

The update rules for vehicles on the roundabout are as follows. Let $s_n(t)$ denotes the number of unoccupied cells in front of vehicle n at time t, $v_n(t)$ denotes the current speed of vehicle n, and $e_n(t)$ denotes the number of cells between current position and the exit of vehicle n. The update rules of vehicle n on the roundabout can thus be summarized as follows: If $v_n(t) < \min\{s_n(t), e_n(t)\}$, then vehicle n keeps current speed (does not accelerate on the roundabout); Otherwise, $v_n(t) < \min\{s_n(t), e_n(t)\} - 1$.

The assumption that vehicles do not accelerate on a roundabout is based on the following considerations: (i) Accelerating around a roundabout provides little help to shorten travel time. A roundabout's geometry limits the opportunity to speed up since the circulatory road is a circle, not a straight line road; (ii) A car will decelerate to avoid collision in cross traffic and (iii) A car normally approaches a destination exit at low speed.

2.4 *Exiting Rules from a Roundabout*

Drivers clearly have their own destinations in mind, so that the destination exits of each vehicle can be assigned before entering. Characterizing a given exit for each vehicle before entry is clearly more realistic than assuming that such a decision is made once entry is affected. In other words, the sum of possibilities to all exits is equal to 1 for each entrance. Each exit is randomly assigned an integer number. This number is equal to the number of the cells that a vehicle needs to pass in order to arrive at its destination exit. Therefore, the position of a car on the roundabout is the number of cells between current position and its destination exit.

3 Results and Discussion

The NAS method is applied to a case study. Experiments were implemented for 36000 time steps (equivalent to 10 hours) for a street length of 1000 cells on all approaches. Each cell in the simulation is equal to 1 m in the real world and a unit velocity is assumed as 3.6 km/h. Each car occupies 5 cells. Each time step is 1 second. The average values of capacity, delay and queue length are used to validate the NAS method. The distribution of driver behavior in the NAS method is assumed to follow three different Gaussian distribution, which are listed in Table 1. All driver behavior ranges within [$x_{\min}$, $x_{\max}$], where $x_{\min}$, $x_{\max}$ are different in NAS 1, 2, and 3. The standard deviation of each truncated Gaussian distribution is assumed to be 2 cells. Table 1 shows assumed different distributions of driver behavior in the NAS method. The standard deviations in GDI for all drivers are assumed to have the same value, i.e. $\sigma_I = 1$.

Table 1 Different driver behavior distribution.

NAS	μ_H in GDH	σ_H in GDH	σ_I in GDI	$x_{\min}$	$x_{\max}$
NAS 1	10	2	1	6	16
NAS 2	14	2	1	8	18
NAS 3	18	2	1	12	24

Table 2 shows the numerical results of capacity, delay and queue length under different driver behavior distributions. Figure 1 shows illustration of capacity, delay and queue length under different driver behavior distributions. To make it clear, we assume that driver behavior in NAS 1, 2, and 3 represents aggressive, normal and conservative, respectively.

Table 2 Capacity, delay and 95% queue length with different driver behavior distribution. [C] = capacity; [D] = delay; [Q] = 95% queue length.

	NAS 1			NAS 2			NAS 3		
Road	[C]	[D]	[Q]	[C]	[D]	[Q]	[C]	[D]	[Q]
Arm 1	782	22.4	9.2	735	23	9.4	770	24.8	9.7
Arm 2	883	12	6.5	880	14	6.63	868	18	8.2
Arm 3	856	8	2.5	848	8.2	2.74	842	9	2.81
Arm 4	988	15	8.6	971	18	9.6	966	20	10

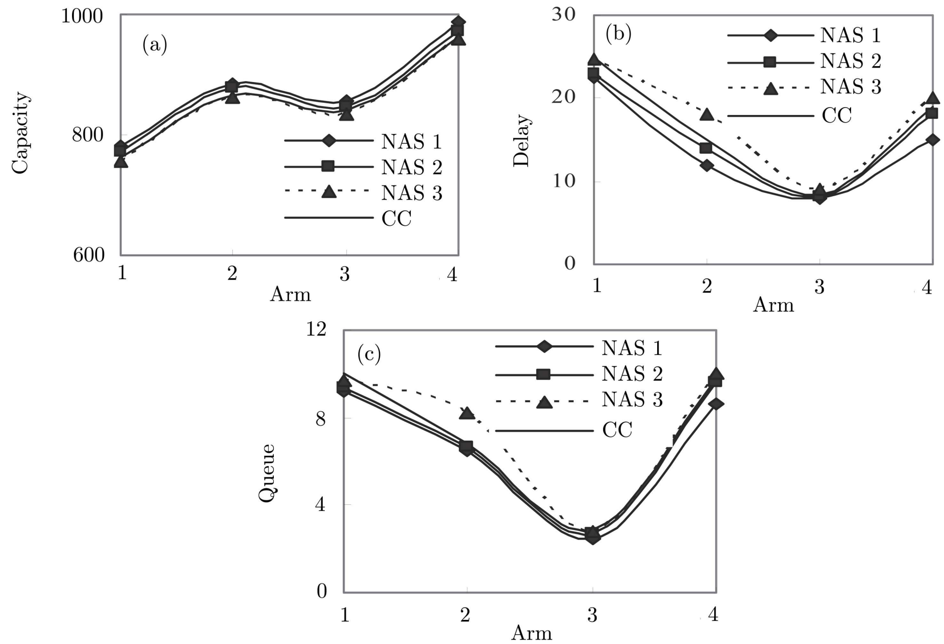

Fig. 1 Comparison of capacity, delay and 95% queue length between NAS method and conventional computational (CC) method. Driver behavior in NAS 1, 2, and 3 see Table 1.

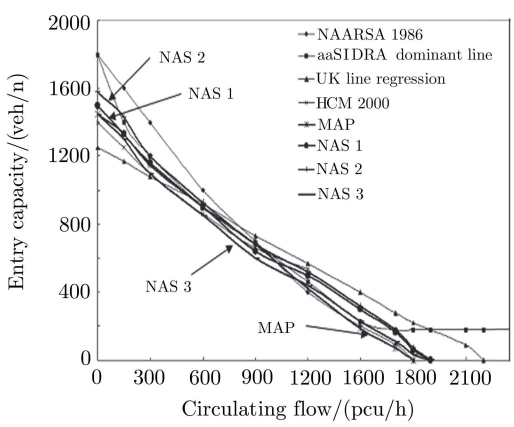

Fig. 2 Comparison of entry capacities estimated by the NAS method and other models.

Figure 2 shows the capacity of a roundabout, obtained from the NAS method and other models. It can be seen that the proposed method is agreed well with the most methodologies. In particular, entry capacity in the NAS 1 is very similar to the Map method, when circulating flow is less than 900 pcu per hour. When circulating flow is larger than 1200 pcu/h, entry capacity in NAS 3 keeps agreement with the MAP method. The pcu (i.e., Passenger Car Units) is usually used to convert heavy vehicles to passenger car equivalence.[4] Normally, car = 1 pcu, heavy vehicle = 2 pcu. However, entry capacity has a few increases, but delay and queue length are slightly decreased in the NAS method, compared to the empirical computational (EC) methods.

4 Summary

In this paper, we propose a novel cellular automaton (CA) model to simulate traffic flow at a single-lane roundabout. Heterogeneous driver behavior and inconsistent driver behavior are modeled using two truncated Gaussian distributions. The relationship between entry capacity and circulating flow is obtained from the NAS method. Comparison with other methods demonstrates that the NAS method agrees well with most methodologies. The numerical results indicate that the performance (delay and queue length) of roundabouts can be described well. Classification of drivers has been modeled by many authors in highway and urban traffic in the literature. These classifications can describe heterogeneous driver behavior, without considering inconsistent driver behavior for the same driver. This paper studies both heterogeneous and inconsistent driver behavior, and builds a natural link between two driver behaviors. The simulation results indicate that the NAS model provides a more realistic description of driver behavior in cross traffic and gives more satisfactory results than other models such CC method and MAP model. This microscopic model will be investigated on a network to further confirm its effectiveness and reported later.

References

[1] R. Akçelik, Mobile Robots **8** (2001) 520.

[2] A. Flannery and T. Datta, Transportation Research Record **1572** (1997) 68.

[3] R.J. Troutbeck, Transportation Research Record **1646** (1998) 54.

[4] Transportation Research Board, *Highway Capacity Manual*, National Research Council, Washington, D.C., U.S.A. (2000).

[5] R. Wang and H.J. Ruskin, Compt. Phys. Commun. **147** (2002) 570.

[6] R. Akcelik, *Roundabout Model Calibration Issues and a Case Study*, *The TRB National Roundabout Conference*, Vail, Colorado, USA (2005) pp. 22–25.

[7] E. Chevallier, L. Ledercg, J. Lelong, and R. Chatagnon, Applied Acoustics **70** (2009) 761.

[8] R. Guo and B. Liu, Journal of Transportation Systems Engineering and Information Technology **10** (2011) 29.

[9] S. Daniels, T. Brijs, E. Nuyts, and G. Wets, Safety Science **49** (2011) 198.

[10] R. Akçelik, Procedia-Social and Behavior Sciences **16** (2011) 686.

[11] A. Azhar and B. Svante, Procedia-Social and Behavior Sciences **16** (2011) 729.

[12] W.L. Hays, *Statistics (the 5th Edition)*, University of Texas at Austin, Harcourt Brace College Publishers (1994).