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Minimal Length Effects on Schwinger Mechanism*

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Abstract *In this paper, we investigate effects of the minimal length on the Schwinger mechanism using the quantum field theory (QFT) incorporating the minimal length. We first study the Schwinger mechanism for scalar fields in both usual QFT and the deformed QFT. The same calculations are then performed in the case of Dirac particles. Finally, we discuss how our results imply for the corrections to the Unruh temperature and the Hawking temperature due to the minimal length.*

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1 Introduction

Using the proper-time method, Schwinger[1] calculated the effective action of a charged particle in an external electromagnetic field. He found that the action has an imaginary part for a uniform electric field, which leads to the vacuum decay through pair production. Due to its purely non-perturbative nature, this quantum field theoretical prediction is of fundamental importance. The Schwinger mechanism sheds lights on topics as diverse as the string breaking rate in QCD[2−3] and on black hole physics.[4]

On the other hand, various theories of quantum gravity, such as string theory, loop quantum gravity and quantum geometry, predict the existence of a minimal length.[5−7] The generalized uncertainty principle (GUP)[8] is a simply way to realize this minimal length. A lot of fruits have been achieved in the GUP models.[9−15] An effective model of the GUP in one-dimensional quantum mechanics is given by[16−17]

$$k(p) = \frac{1}{\sqrt{\beta}}\tanh\big(\sqrt{\beta}p\big)\,, \tag{1}$$

$$\omega(E) = \frac{1}{\sqrt{\beta}}\tanh\big(\sqrt{\beta}E\big)\,, \tag{2}$$

where the generators of the translations in space and time are the wave vector k and the frequency ω, $\beta = \beta_0/m_p^2$, m_p is the Planck mass and β_0 is a dimensionless parameter marking quantum gravity effects. We set $c = \hbar = G = 1$ in the paper. The quantization in position representation $\hat{x} = x$ leads to

$$k = -\mathrm{i}\partial_x\,, \quad \omega = \mathrm{i}\partial_t\,. \tag{3}$$

Therefore, the low energy limit $p \ll m_p$ including order of p^3/M_f^3 gives

$$p = -\mathrm{i}\partial_x\Big(1 - \frac{\beta}{3}\partial_x^2\Big)\,, \tag{4}$$

$$E = \mathrm{i}\partial_t\Big(1 - \frac{\beta}{3}\partial_t^2\Big)\,. \tag{5}$$

From Eq. (1), it is noted that although one can increase p arbitrarily, k has an upper bound which is $1/\sqrt{\beta}$. The upper bound on k implies that that particles could not possess arbitrarily small Compton wavelengths $\lambda = 2\pi/k$ and that there exists a minimal length $\sim \sqrt{\beta}$.

In this paper, we investigate scalars and fermions pair production from a static classical electric field using the deformed QFT which incorporates the minimal length via Eqs. (4) and (5). The organization of this paper is as follows. In Sec. 2, based on the usual and the minimal length modified field theoretic considerations, Schwinger's mechanism is derived in the case of spinless particles. We then show how the same calculations can be performed in the case of Dirac particles in Sec. 3. Section 4 is devoted to our discussions and conclusions.

2 Scalar Pair Production

In this section, we first derive the formula for the scalar pair production rate in the framework of QFT. We then use the formula to calculate pair creation rate in usual QFT and the minimal length modified QFT. As shown

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in Ref. [18], the scalar field theoretic vacuum to vacuum amplitude can be written as

$$\langle \text{vac} \to \text{vac} \rangle \propto \int [\mathrm{d}\phi] \exp\Big(\mathrm{i} \int \mathrm{d}x^4 \mathcal{L}\Big)\,, \tag{6}$$

where $\mathcal{L}$ is the Lagrangian density and ϕ is the corresponding scalar field. Assume $\mathcal{L}$ is given by $\mathcal{L} = \phi^+ \mathcal{O}_s \phi$ where $\mathcal{O}_s$ is some differential operator. Defining eigenfunctions ϕ_n with eigenvalues λ_n,

$$\mathcal{O}_s \phi_n = \lambda_n \phi_n\,, \tag{7}$$

we expand

$$\phi = \sum_n a_n \phi_n\,. \tag{8}$$

Using orthogonality of ϕ_n, we find

$$\begin{aligned} \mathrm{e}^{-\xi_s} &\equiv \langle \text{vac} \to \text{vac} \rangle \propto \int \mathrm{d}a_n \mathrm{d}a_n^* \exp\Big(\mathrm{i} \int \mathrm{d}x^4 \lambda_n |a_n|^2\Big) \\ &\propto \prod_n \frac{1}{\lambda_n} = \frac{1}{\det \mathcal{O}_s} = \exp(-\mathrm{Tr}(\ln \mathcal{O}_s)) \\ &= \exp\Big(-\sum_n \ln \lambda_n\Big)\,. \end{aligned} \tag{9}$$

Using the integral

$$\ln a = -\int_0^{+\infty} \mathrm{d}s \frac{\mathrm{e}^{-as}}{s} + \text{const.}\,, \tag{10}$$

one has

$$\xi_s = \xi_s^E + C\,, \tag{11}$$

where C is a constant and we define

$$\xi_s^E \equiv -\sum_n \int_0^{+\infty} \frac{\mathrm{d}s}{s} \mathrm{e}^{-\lambda_n s}\,. \tag{12}$$

The imaginary part of ξ_s is always infinite due to vacuum energy shift. Here, we are only interested in the real part of ξ^s which gives the vacuum decay. As shown later in this section, the electric field E only appears in λ_n and C is independent of E. To obtain C we consider the case without the electric field. When there is no electric field, the vacuum is stable and no pairs are produced. In this case, the vacuum to vacuum amplitude is

$$\mathrm{e}^{\mathrm{i}\alpha} = \mathrm{e}^{-\xi_s} = \exp(-\xi_s^0 + C)\,,$$

where α is a phase irrelevant to the vacuum decay and ξ_s^0 is ξ_s^E with $E = 0$. Thus, one has

$$C = \xi_s^0 + \mathrm{i}\alpha\,. \tag{13}$$

Plugging Eq. (13) into Eq. (11), we find that the real part of ξ_s is

$$\gamma_s \equiv \mathrm{Re}\xi_s = \mathrm{Re}(\xi_s^E - \xi_s^0)\,. \tag{14}$$

Squaring γ_s, we observe that the total pair production rate per unit volume is simply

$$\frac{\text{prob}_{\text{pair}}}{VT} = \frac{1}{VT}(1 - \mathrm{e}^{-2\gamma_s}) \approx \frac{2\gamma_s}{VT}\,. \tag{15}$$

2.1 *Usual Scalar Field*

For the case of a scalar field ϕ of mass m and charge e in the presence of an external electromagnetic interaction described by vector potential A_μ, the Lagrangian is given by

$$\mathcal{L}_s = \phi^+ \mathcal{O}_s^{(0)} \phi\,, \tag{16}$$

where $\mathcal{O}_s^{(0)} = (\partial + \mathrm{i}eA)^2 + m^2$. If there is a uniform electric field $\boldsymbol{E} = E\boldsymbol{e}_z$, we can utilize the gauge $\boldsymbol{A} = -Et\boldsymbol{e}_z$. The operator $\mathcal{O}_s^{(0)}$ then becomes

$$\mathcal{O}_s^{(0)} = \partial_t^2 - \partial_\perp^2 - (\partial_z + \mathrm{i}eEt)^2 + m^2\,, \tag{17}$$

where $\partial_\perp^2 = \partial_x^2 + \partial_y^2$. Assume that eigenfunction of $\mathcal{O}_s^{(0)}$ takes the form as $\psi = \exp(\mathrm{i}\boldsymbol{k}_\perp \cdot \boldsymbol{r}_\perp + \mathrm{i}k_z z)\sigma(t)$ where $\boldsymbol{r}_\perp = x\boldsymbol{e}_x + y\boldsymbol{e}_y$ and $\sigma(t)$ satisfies

$$\Big[\Big(\frac{\mathrm{d}}{\mathrm{d}t}\Big)^2 + k_\perp^2 + (k_z + eEt)^2 + m^2\Big]\sigma(t) = \lambda\sigma(t)\,. \tag{18}$$

Performing the Wick Rotation $t \to -\mathrm{i}\tau$ and $E \to -\mathrm{i}\tilde{E}$, we have

$$\begin{aligned} &\Big[-\Big(\frac{\mathrm{d}}{\mathrm{d}\tau}\Big)^2 + e^2\tilde{E}^2\Big(\tau - \frac{k_z}{e\tilde{E}}\Big)^2\Big]\sigma(t) \\ &\quad = (\lambda - k_\perp^2 - m^2)\sigma(t)\,. \end{aligned} \tag{19}$$

Obviously, Eq. (19) describes a one-dimensional harmonic oscillator with its well centered at $k_z/e\tilde{E}$ and a resonant frequency $2e\tilde{E}$. One can express $\mathrm{d}/\mathrm{d}\tau$ and τ in terms of ladder operators, a and a^+, as

$$\tau - \frac{k_z}{e\tilde{E}} = \frac{1}{\sqrt{2e\tilde{E}}}(a^+ + a)\,, \tag{20}$$

$$\frac{\mathrm{d}}{\mathrm{d}\tau} = -\sqrt{\frac{e\tilde{E}}{2}}(a^+ - a)\,. \tag{21}$$

Thus, the energy levels are quantized as

$$\lambda_{n,k_\perp}^{s(0)} = k_\perp^2 + m^2 + (2n+1)e\tilde{E}\,, \tag{22}$$

where $n = 0, 1, 2, \ldots$ Note that k_x and k_y range over all values from $-\infty$ to ∞, but k_z is constrained to be in the range $0 < k_z < e\tilde{E}\mathrm{i}T$ in order that the entire range of time is included as k_z is varied, where T is a total interaction time. Thus, the corresponding degeneracy is $eETV\,\mathrm{d}k_\perp/(2\pi)^3$ where V is the volume. After switching back to t and E, Eqs. (12) and (22) yield

$$\xi_s^E = -\frac{eEVT}{(2\pi)^3} \int \mathrm{d}k_\perp \int_0^{+\infty} \frac{\mathrm{d}s}{s} \exp[-(k_\perp^2 + m^2)s] \sum_{n=0}^{\infty} \exp[-\mathrm{i}(2n+1)eEs]\,. \tag{23}$$

With the help of Dirac comb

$$\pi \sum_{k=-\infty}^{\infty} \delta(t - k\pi) = \sum_{n=-\infty}^{\infty} \exp(-\mathrm{i}2nt)\,, \tag{24}$$

one easily gets

$$\mathrm{Re}\,\xi_s^E = -\frac{eEVT}{2(2\pi)^3}\int \mathrm{d}k_\perp \int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp[-(k_\perp^2+m^2)s]\exp(-\mathrm{i}eEs)\pi\sum_{k=-\infty}^{\infty}\delta(eEs-k\pi)\,. \tag{25}$$

When E approaches zero, we have for $\mathrm{Re}\,\xi_s^0$

$$\mathrm{Re}\,\xi_s^0 = -\frac{VT}{16\pi^2}\int \mathrm{d}k_\perp \int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp[-(k_\perp^2+m^2)s]\delta(s)\,. \tag{26}$$

Thus, we find

$$\begin{aligned}\gamma_s^{(0)} &= \mathrm{Re}(\xi_s^E-\xi_s^0) = -\frac{VT}{16\pi^2}\int \mathrm{d}k_\perp \int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp[-(k_\perp^2+m^2)s]\exp(-\mathrm{i}eEs)\Big[\sum_{k=-\infty}^{\infty}\delta\Big(s-\frac{k\pi}{eE}\Big)-\delta(s)\Big]\\ &= \frac{e^2E^2VT}{16\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}\exp\Big(-m^2\frac{k\pi}{eE}\Big)\,.\end{aligned} \tag{27}$$

2.2 *Minimal Length Modified Scalar Field*

In the presence of an external electromagnetic potential A_μ, Eqs. (4) and (5) can be generalized to

$$p_i = -\mathrm{i}D_i\Big(1-\frac{\beta}{3}D_i^2\Big)\,, \tag{28}$$

$$E = \mathrm{i}D_t\Big(1-\frac{\beta}{3}D_t^2\Big)\,, \tag{29}$$

where $D_\mu = \partial_\mu + \mathrm{i}eA_\mu$. For a scalar field ϕ of mass m and charge e in the external electromagnetic potential A_μ, the Lagrangian incorporating Eqs. (28) and (29) can be written as

$$\mathcal{L}_s = \eta^{\mu\nu}(p_\mu\phi)^+(p_\nu\phi)+m^2, \tag{30}$$

where $p_\mu = (E, p_i)$. After integrating by part, we have

$$\mathcal{L}_s = \phi^+(\mathcal{O}_s^{(0)}+\delta\mathcal{O}_s)\phi\,, \tag{31}$$

where we define

$$\delta\mathcal{O}_s = -\frac{2}{3}\beta[D_t^4-(D_x^4+D_y^4+D_z^4)]+\mathcal{O}(\beta^2)\,. \tag{32}$$

In order to get eigenfunctions and eigenvalues of $\mathcal{O}_s^{(0)}+\delta\mathcal{O}_s$, we write the eigenfunctions in the form $\phi = \exp(\mathrm{i}\boldsymbol{k}_\perp\cdot\boldsymbol{r}_\perp+\mathrm{i}k_z z)\sigma(t)$. For $\sigma(t)$, $\delta\mathcal{O}_s$ becomes

$$\delta\mathcal{O}_s = -\frac{2}{3}\beta[\partial_t^4-k_\perp^4-(k_z+eEt)^4]+\mathcal{O}(\beta^2)\,. \tag{33}$$

Rotating to imaginary time τ and $\tilde{E}$, we can use Eqs. (20) and (21) to write $\delta\mathcal{O}_s$ in terms of ladder operators

$$\delta\mathcal{O}_s = -\frac{1}{6}\beta e^2\tilde{E}^2\Big[(a^+-a)^4-\frac{4k_\perp^4}{e^2\tilde{E}^2}-(a^++a)^4\Big] + \mathcal{O}(\beta^2)\,. \tag{34}$$

Treating $\delta\mathcal{O}_s$ as perturbations, we find the first-order correction to $\lambda_{n,k_\perp}^{s(0)}$

$$\delta\lambda_{n,k_\perp}^s = \langle n|\delta\mathcal{O}_s|n\rangle = \frac{2\beta k_\perp^4}{3}\,, \tag{35}$$

where $|n\rangle$ is the n-th eigenstate for the one-dimensional harmonic oscillator. The corresponding degeneracy stays same, namely $eETV\,\mathrm{d}k_\perp/(2\pi)^3$. Therefore, Eq. (12) gives to $\mathcal{O}(\beta)$

$$\xi_s^E \approx -\frac{eEVT}{(2\pi)^3}\int \mathrm{d}k_\perp\int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp\Big[-\Big(k_\perp^2+m^2+\frac{2\beta k_\perp^4}{3}\Big)s\Big]\sum_{n=0}^{\infty}\exp[-\mathrm{i}(2n+1)eEs]\,. \tag{36}$$

Note that the above equation is the same as Eq. (23) except the prefactor $\exp(-(2\beta k_\perp^4/3)s)$. Following calculations for $\gamma_s^{(0)}$, one obtains to $\mathcal{O}(\beta)$

$$\begin{aligned}\gamma_s &= \mathrm{Re}(\xi_s^E-\xi_s^0) \approx \frac{eEVT}{16\pi^3}\int \mathrm{d}k_\perp\sum_{k=1}^{\infty}(-1)^{k+1}\frac{1}{k}\exp\Big[-\Big(k_\perp^2+m^2+\frac{2\beta k_\perp^4}{3}\Big)\frac{k\pi}{eE}\Big]\\ &\approx \frac{e^2E^2VT}{16\pi^3}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k^2}\exp\Big(-\frac{m^2\pi k}{eE}-\frac{4\beta eE}{3\pi k}\Big)\,.\end{aligned} \tag{37}$$

3 Fermion Pair Production

The procedure in Sec. 2 can also be applied to calculations for a fermion field. However, two differences should be noted. First, the Grassmann numbers are used in fermion case. Second, instead of $\mathcal{O}_f$, we usually calculate eigenvalues of $\mathcal{O}_f\tilde{\mathcal{O}}_f$, where $\tilde{\mathcal{O}}_f$ is defined as follows. In even dimension, there exist two charge conjugate operators C_+ and C_-[19] such that

$$C_\pm\gamma^\mu C_\pm^{-1} = \pm\gamma^{\mu\mathrm{T}}, \tag{38}$$

where γ^μ are Gamma matrices. We then define

$$\tilde{\mathcal{O}}_f \equiv C_-^{-1}(\gamma^0(C_+\mathcal{O}_f C_+^{-1})^*\gamma^0)^{\mathrm{T}}C_-\,. \tag{39}$$

Since $\tilde{\mathcal{O}}_f$ is Hermitian, one finds

$$\det\tilde{\mathcal{O}}_f = \det\mathcal{O}_f^* = \det\mathcal{O}_f^+ = \det\mathcal{O}_f\,. \tag{40}$$

Assume the Lagrangian for a fermion field ψ is given in the form of $\mathcal{L}=\bar{\psi}\mathcal{O}_f\psi$. Defining eigenfunctions ψ_n with eigenvalues λ'_n,

$$\mathcal{O}_f\psi_n=\lambda'_n\psi_n\,, \tag{41}$$

we expand

$$\psi=\sum_n\xi_n\psi_n\,. \tag{42}$$

Thus, the vacuum to vacuum amplitude for ψ is

$$\begin{aligned}\mathrm{e}^{-\xi_f}&\equiv\langle\mathrm{vac}\to\mathrm{vac}\rangle\propto\int\mathrm{d}\xi_n\,\mathrm{d}\xi_n^*\exp\Big(\mathrm{i}\int\mathrm{d}x^4\lambda'_n|\xi_n|^2\Big)\\&\propto\prod_n\lambda'_n=\det\mathcal{O}_f=(\det\mathcal{O}_f\tilde{\mathcal{O}}_f)^{1/2}\\&=\exp\Big[\frac{1}{2}\mathrm{Tr}(\ln\mathcal{O}_f\tilde{\mathcal{O}}_f)\Big]=\exp\Big(\frac{1}{2}\sum_n\ln\lambda_n\Big)\,,\end{aligned} \tag{43}$$

where λ_n are eigenvalues of $\mathcal{O}_f\tilde{\mathcal{O}}_f$. The real part of ξ_f is given by

$$\gamma_f=\mathrm{Re}\xi_f=\mathrm{Re}(\xi_f^E-\xi_f^0)\,, \tag{44}$$

where ξ_f^0 is ξ_f^E with $E=0$ and we define

$$\xi_f^E\equiv\frac{1}{2}\sum_n\int_0^{+\infty}\frac{\mathrm{d}s}{s}\,\mathrm{e}^{-\lambda_n s}. \tag{45}$$

Squaring γ_f, we observe that the total pair production rate per unit volume is simply

$$\frac{\mathrm{prob_{pair}}}{VT}=\frac{1}{VT}(1-\mathrm{e}^{-2\gamma_f})\approx\frac{2\gamma_f}{VT}\,. \tag{46}$$

3.1 *Usual Fermion Field*

The Lagrangian for a charged spinor of mass m and charge e is

$$\mathcal{L}=\mathrm{i}\bar{\psi}D^\mu\gamma_\mu\psi-m\bar{\psi}\psi=\bar{\psi}\mathcal{O}_f^{(0)}\psi\,, \tag{47}$$

where $\mathcal{O}_f^{(0)}=\mathrm{i}D^\mu\gamma_\mu-m$. Using Eq. (39), one finds

$$\mathcal{O}_f^{(0)}\tilde{\mathcal{O}}_f^{(0)}=\kappa_1+\kappa_2\,, \tag{48}$$

where $\sigma^{\mu\nu}=(\mathrm{i}/2)[\gamma^\mu,\gamma^\nu]$, $[D_\mu,D_\nu]=\mathrm{i}eF_{\mu\nu}$, $\kappa_1\equiv D^2+m^2$ and $\kappa_2=(e/2)F_{\mu\nu}\sigma^{\mu\nu}$. Here the eigenvalues of κ_1 in the case of a constant electric field are determined in Sec. 2 and are given by

$$\lambda_1=k_\perp^2+m^2+(2n+1)e\tilde{E}\,. \tag{49}$$

In chiral representation, we have

$$\kappa_2=\mathrm{i}eE\begin{pmatrix}-\sigma^3&0\\0&\sigma^3\end{pmatrix}\,, \tag{50}$$

where $\boldsymbol{A}=-Et\boldsymbol{e}_z$ for a uniform electric field $\boldsymbol{E}=E\boldsymbol{e}_z$. Temporarily rotating to imaginary time as before, we find that the eigenvalues of $\mathcal{O}_f^{(0)}\tilde{\mathcal{O}}_f^{(0)}$ are

$$\lambda_{n,k_\perp}^{f(0)}=k_\perp^2+m^2+2ne\tilde{E}\ \text{for}\ n=0,1,2,\ldots\,, \tag{51}$$

and the corresponding degeneracies are

$$\frac{2g_neETV\,\mathrm{d}k_\perp}{(2\pi)^3}\,, \tag{52}$$

where $g_0=1$ and $g_{n>0}=2$. Thus one finds

$$\xi_f^E=\frac{eEVT}{(2\pi)^3}\int\mathrm{d}k_\perp\int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp[-(k_\perp^2+m^2)s]\sum_{n=0}^{\infty}g_n\exp(-2\mathrm{i}neEs)\,. \tag{53}$$

Using Dirac comb, we get

$$\mathrm{Re}\xi_f^E=\frac{eEVT}{(2\pi)^3}\int\mathrm{d}k_\perp\int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp[-(k_\perp^2+m^2)s]\pi\sum_{k=-\infty}^{\infty}\delta(eEs-k\pi)\,, \tag{54}$$

$$\mathrm{Re}\xi_f^0=\frac{VT}{(2\pi)^3}\int\mathrm{d}k_\perp\int_0^{+\infty}\frac{\mathrm{d}s}{s}\exp[-(k_\perp^2+m^2)s]\pi\delta(s)\,. \tag{55}$$

Therefore, we have

$$\gamma_f^{(0)}=\mathrm{Re}(\xi_f^E-\xi_f^0)=\frac{e^2E^2VT}{8\pi^3}\sum_{k=1}^{\infty}\frac{1}{k^2}\exp\Big(-m^2\frac{k\pi}{eE}\Big)\,. \tag{56}$$

3.2 *Minimal Length Modified Fermion Field*

For this case, the Lagrangian incorporating Eqs. (28) and (29) for a charged spinor ψ of mass m and charge e can be written as

$$\mathcal{L}_f=\bar{\psi}(\mathrm{i}p_\mu\gamma^\mu-m)\psi\,, \tag{57}$$

where $p_\mu=(E,p_i)$. We then find

$$\mathcal{L}_f=\bar{\psi}\mathcal{O}_f\psi=\bar{\psi}(\mathcal{O}_f^{(0)}+\delta\mathcal{O}'_f)\psi\,, \tag{58}$$

where we define

$$\delta\mathcal{O}'_f=-\frac{\mathrm{i}}{3}\beta D_\mu^3\gamma^\mu+\mathcal{O}(\beta^2)\,. \tag{59}$$

Equations (39) and (59) give the product of $\mathcal{O}_f$ and $\tilde{\mathcal{O}}_f$ to $\mathcal{O}(\beta)$

$$\mathcal{O}_f\tilde{\mathcal{O}}_f=\mathcal{O}_f^{(0)}\tilde{\mathcal{O}}_f^{(0)}+\delta\mathcal{O}_f\,, \tag{60}$$

where we find

$$\delta\mathcal{O}_f=\delta\mathcal{O}_s-e\beta F_{\mu\nu}D_\mu^2\sigma^{\mu\nu}\,. \tag{61}$$

The first term in Eq. (61) is just $\delta\mathcal{O}_s$ given in Eq. (33), while one can express the second term in terms of ladder operators after rotating to imaginary time. In fact, using Eqs. (20) and (21) gives

$$F_{\mu\nu}D_\mu^2\sigma^{\mu\nu}=E^2\begin{pmatrix}-\sigma^3&0\\0&\sigma^3\end{pmatrix}(a^{+2}+a^2)\,, \tag{62}$$

which does not contribute to the first-order correction to eigenvalues $\lambda_{n,k_\perp}^{f(0)}$ of the leading operator $\mathcal{O}_f^{(0)}\tilde{\mathcal{O}}_f^{(0)}$ in Eq. (60) since $\langle n|F_{\mu\nu}D_\mu^2\sigma^{\mu\nu}|n\rangle$ is zero. Thus, the eigenvalues of $\mathcal{O}_f\tilde{\mathcal{O}}_f$ to $\mathcal{O}(\beta)$ are

$$\lambda^f_{n,k_\perp} \approx k_\perp^2 + m^2 + 2ne\tilde{E} + \frac{2\beta k_\perp^4}{3} \text{ for } n = 0, 1, 2, \ldots\,, \tag{63}$$

where we use Eq. (35). The corresponding degeneracies are also

$$\frac{2g_n eETV\,\mathrm{d}k_\perp}{(2\pi)^3}\,, \tag{64}$$

where $g_0 = 1$ and $g_{n>0} = 2$. Therefore, one finds for ξ_f^E

$$\xi_f^E \approx \frac{eEVT}{(2\pi)^3}\int \mathrm{d}k_\perp \int_0^{+\infty} \frac{\mathrm{d}s}{s} \exp\Big[-\Big(k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3}\Big)s\Big] \sum_{n=0}^{\infty} g_n \exp(-2ne\tilde{E}s)\,. \tag{65}$$

Using Dirac Comb, we find that

$$\mathrm{Re}\xi_f^E \approx \frac{eEVT}{(2\pi)^3}\int \mathrm{d}k_\perp \int_0^{+\infty} \frac{\mathrm{d}s}{s} \exp\Big[-\Big(k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3}\Big)s\Big]\pi \sum_{k=-\infty}^{\infty} \delta(eEs - k\pi)\,, \tag{66}$$

$$\mathrm{Re}\xi_f^0 \approx \frac{VT}{(2\pi)^3}\int \mathrm{d}k_\perp \int_0^{+\infty} \frac{\mathrm{d}s}{s} \exp\Big[-\Big(k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3}\Big)s\Big]\pi\delta(s)\,. \tag{67}$$

Again, we obtain γ_f to $\mathcal{O}(\beta)$

$$\begin{aligned}\gamma_f &= \mathrm{Re}(\xi_f^E - \xi_f^0)\\ &\approx \frac{eEVT}{8\pi^3}\sum_{k=1}^{\infty}\frac{1}{k}\int \mathrm{d}k_\perp \exp\Big[-\Big(k_\perp^2 + m^2 + \frac{2\beta k_\perp^4}{3}\Big)\frac{k\pi}{eE}\Big]\\ &\approx \frac{e^2E^2VT}{8\pi^3}\sum_{k=1}^{\infty}\frac{1}{k^2}\exp\Big(-m^2\frac{k\pi}{eE} - \frac{4\beta eE}{3\pi k}\Big)\,.\end{aligned} \tag{68}$$

4 Discussion and Conclusion

First, it is noted that the problem of scalar particles pair creation by an electric field in the presence of a minimal length is also studied in Ref. [20]. The authors considered another GUP of form

$$x_i = x_{0i}\,, \tag{69}$$

$$p_i = p_{0i}(1 + \beta p^2)\,, \tag{70}$$

where x_{0i} and p_{0i} satisfy the canonical commutation relations. Using Bogoliubov transformations, they found

$$\gamma_s \sim \exp\Big[-m^2\frac{\pi}{eE}\Big(1 + \frac{\beta m^2}{4}\Big(1 - \frac{e^2E^2}{m^4}\Big)\Big)\Big]\,, \tag{71}$$

where their minimal length corrections depend on the mass of scalar particles while our results do not, at least to $\mathcal{O}(\beta)$.

Comparing the usual scalar result Eq. (27) with the usual fermionic result Eq. (56), one might note that there are two differences. The factors "$(-1)^{k+1}$" appear in Eq. (27) because of the sign difference between Eqs. (9) and (43), and an extra $e\tilde{E}$ in Eq. (22) comparing to (51). The fermionic result is a factor of 2 larger than the result for the scalar, as would be expected from the extra spin degree of freedom. The same story happens to the minimal length case if one compares Eq. (37) with Eq. (68). However, it is interesting to note that the minimal length effects on pair production rates for scalars and fermions are the same, the factor $\exp(-4\beta eE/3\pi k)$ in both Eq. (37) with Eq. (68).

In the case of electron-positron pairs, the pair production is irrelevant for laboratory electric fields, let alone the minimal length corrections. However, the Schwinger mechanism has to do with the Unruh effect which predicts that an accelerating observer will observe a thermal spectrum of photons and particle-antiparticle pairs at temperature $T = a/2\pi$, where a is the acceleration.[21] We now investigate how our results imply for the corrections to the Unruh temperature due to the minimal length. Considering a free particle of charge e and mass m moving in a static electric field E, one finds the particle have acceleration $a = em/E$. Keeping only the leading term, the pair production probability per unit volume per unit time is

$$\frac{\mathrm{prob}_{\mathrm{pair}}}{VT} \sim \exp\Big(-m^2\frac{\pi}{eE} - \frac{4\beta eE}{3\pi}\Big)\,. \tag{72}$$

Identifying the reduced mass $m/2 = K$ as the energy associated with the pair production process, we find that the production probability can be written in the form

$$\mathrm{prob}_{\mathrm{pair}} \sim \exp\Big[-\frac{K}{a/2\pi}\Big(1 + \frac{4\beta a^2}{3\pi^2}\Big)\Big]\,. \tag{73}$$

The minimal length modified Unruh temperature can be read from Eq. (73), which gives

$$T_u \sim \frac{a}{2\pi}\Big(1 - \frac{4\beta a^2}{3\pi^2}\Big)\,, \tag{74}$$

where a is the acceleration of the observer.

The Unruh effect can be relevant to the phenomenon of black hole decay due to pair creation, the Hawking radiation.[22] Consider a Schwarzschild black hole with the black hole's mass, M. The event horizon of the Schwarzschild black hole is $r_h = 2M$. Noting that the gravitational acceleration at the event horizon is given by

$$a = \frac{M}{r_h^2} = \frac{1}{4M}\,, \tag{75}$$

one finds from Eq. (74) that the minimal length modified Hawking temperature is

$$T_h \sim \frac{1}{8\pi M}\Big(1 - \frac{\beta}{12\pi^2 M^2}\Big)\,. \tag{76}$$

Using the first law of the black hole thermodynamics, we find the corrected black hole entropy is

$$S = \int \frac{\mathrm{d}M}{T} \sim \frac{A}{4} + \frac{\beta}{3\pi}\ln\Big(\frac{A}{16\pi}\Big)\,, \tag{77}$$

where $A = 4\pi r_h^2 = 16\pi M^2$ is the area of the horizon. The logarithmic term in Eq. (77) is the well known correction from quantum gravity to the classical Bekenstein–Hawking entropy, which have appeared in different studies of GUP modified thermodynamics of black holes.[23–25] A careful reader might not be satisfied with the above heuristic handwaving argument relating the Schwinger mechanism to the Unruh temperature and the Hawking temperature. However, using the minimal length modified Hamilton–Jacobi Method incorporating Eqs. (28) and (29), we found in [26] that the corrected Hawking temperature for the Schwarzschild black is given by

$$T_h \sim \frac{1}{8\pi M}\Big(1 - \frac{\beta}{6M^2}\Big)\,, \tag{78}$$

which is almost same as Eq. (76) except the numerical factor in front of β. Note that the Unruh effect was discussed in a new GUP form.[27] Morover, the effects of the minimal length on Schwinger effect were also discussed in Refs. [28–29] using the hypothesis of path integral duality.

In this paper, incorporating effects of the minimal length, we derive the deformed Schwinger mechanism for both scalars and fermions in a static uniform electric field. Using handwaving argument, the implications of our results for the Unruh temperature and the Hawking temperature are also discussed.

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