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The Role of One Single Lambda Hyperon on Binding Energy Difference of Hypernuclear Mirror Pair

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Abstract *Investigation on isospin symmetry in light Lambda hypernuclei is one of the most important issues in hypernuclear physics. In order to know the influences introduced by a single Lambda hyperon, we study the binding energy difference of mirror hypernuclear pair with mass $A = 16$, 18, 28, 40, and 42 using a time-odd triaxial relativistic mean field theory. Effects as the spin-orbit interaction, the time-odd component of vector fields, the core polarization, the proton-neutron mass difference, and the center-of-mass energy correction are self-consistently considered. Compared to the reported results of ordinary nuclei, the binding energy difference of mirror hypernuclei shows trivial change. With core polarization modified by an impurity hyperon, the isospin nonconserving effect between proton and neutron is hardly reduced for nuclei in study.*

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1 Introduction

Binding energy difference BED for ordinary mirror nuclei mainly comes from Coulomb effects and other electromagnetic effects.[1–2] The contribution to BED from Coulomb effects was early pointed out[3] to include effects of i) proton-neutron mass difference,[4–5] ii) finite size of the proton, iii) polarization of the core by the valence particle,[6–7] iv) isospin impurities in the core wave function, v) other corrections coming from the difference between neutron and proton wave functions.[8]

Due to the immersion of a non-charged Λ hyperon, impurity effects in nuclear structure[9–10] and medium effects of baryons probed by hyperons[11] can be observed.[12–14] Compared with the axially deformed relativistic mean field (RMF) calculations, the discrepancy between the calculated and experimental binding energy difference in a time-odd triaxial RMF theory is systematically reduced by the appropriate treatment of the γ deformation degree of freedom, center-of-mass motion, and core polarization effect.[15] To know the hyperon's role in this topic, we study the energy difference of single-Λ hypernuclear mirror pair in a time-odd triaxial RMF theory. Where, effects as the spin-orbit interaction, the time-odd component of vector fields, the core polarization, the proton-neutron mass difference, and the center-of-mass energy correction are self-consistently considered.

2 Theoretical Framework

The relativistic mean field (RMF) theory is one of the successful microscopic models for nuclear structure by incorporating naturally the spin-orbit interaction.[16–17] It has applied well in the hypernuclear structure and electromagnetic properties.[18–19] The effective Lagrangian density here is constructed by the baryon field (ψ_B), two iso-scalar meson fields (σ and ω), the iso-vector meson field (ρ) and the photon field (A), including non-linear self-couplings of σ, ω mesons and a tensor coupling term $(f_{\omega BB}/4m_B)\bar{\psi}_B\sigma^{\mu\nu}\Omega_{\mu\nu}\psi_B$.[4]

$$
\begin{aligned}
\mathcal{L} = {} & \bar{\psi}_B(\not{p}_B - m_B - g_{\sigma B}\sigma - g_{\omega B}\not{\omega} - g_{\rho B}\vec{\not{\rho}}\vec{\tau})\psi_B \\
& + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 \\
& - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu + \frac{1}{4}c_3(\omega_\mu\omega^\mu)^2 \\
& - \frac{1}{4}\vec{R}_{\mu\nu}\vec{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\vec{\rho}_\mu\vec{\rho}^\mu + \frac{f_{\omega BB}}{4m_B}\bar{\psi}_B\sigma^{\mu\nu}\Omega_{\mu\nu}\psi_B \\
& - \bar{\psi}_B\frac{(1-\tau_3)e\not{A}}{2}\psi_B - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \,.
\end{aligned} \tag{1}
$$

Dirac equation and Klein–Gordon equation can be derived from the Lagrangian density as:

$$
\mathrm{i}\gamma_\mu\partial^\mu\psi_B = \Big[\gamma_\mu V_B^\mu + (m_B + S_B) - \frac{f_{\omega BB}}{4m_B}\sigma_{\mu\nu}\Omega^{\mu\nu}\Big]\psi_B \,, \tag{2}
$$

$$
\begin{aligned}
& (\partial^\mu\partial_\mu + m_\omega^2 + c_3\omega_\nu\omega^\nu)\omega_\mu \\
& \quad = \bar{\psi}_B g_\omega\gamma_\mu\psi_B + \frac{f_{\omega BB}}{2m_B}\partial^\mu(\bar{\psi}_B\sigma_{\mu\nu}\psi) \,,
\end{aligned} \tag{3}
$$

where scalar potential S_B and vector potential V_B^μ are given by $S_B = g_{\sigma B}\sigma$, $V_B^\mu = g_{\omega B}\omega^\mu + g_{\rho B}\tau_3\cdot\rho_3^\mu + Q_B A^\mu$. Equations of motion for meson fields σ, ρ and for photon field A can be derived in similar way. For odd-mass nuclei, time-odd (space-like) components of vector fields

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appear and it can modify the nuclear current and the single particle properties.[7,20] Thus, Dirac equations have to be solved separately in two subspaces with different simplex.

Within a framework of the time-odd triaxial RMF approach, the harmonic oscillator basis expansion method has been used to solve motion of equations Eqs. (2), (3). With a binding energy error less than 0.1%, Gauss mesh points $n_{\rm gh} = 16$, major shell numbers $n_f = 12$ for baryons and $n_b = 10$ for mesons are used.[22] Effective meson-baryon interaction PK1-Y2 ($g_{\sigma\Lambda} = 0.705 g_{\sigma N}$, $g_{\omega\Lambda} = f_{\omega\Lambda\Lambda} = 0.772 g_{\omega N}$ for meson-hyperon interaction) with a microscopical description of center-of-mass energy correction is applied.[22] A simple particle occupation with no pairing is used. More numerical details can be found in Ref. [7].

3 Results and Discussion

For odd-A nuclei, the space-like component of vector meson fields and photon field will cause a time-reversal invariance breaking in nuclear states, exhibiting as the energy splitting. In Fig. 1, the baryon single particle energy of mirror hypernuclei is presented. For different valence baryons, there is a mirror symmetry in time reversal conjugate levels for mirror hypernuclei pair. E.g., levels with negative angular momentum projection (T-) is deeper than those with positive angular momentum projection in $^{16}_{\Lambda}$N while it is reversed in $^{16}_{\Lambda}$O. So is the same for $^{40}_{\Lambda}$K and $^{40}_{\Lambda}$Ca. Due to the Coulomb interaction, behaviors of neutron and proton in mirror hypernuclei are distinct. Valence protons are pushed away by the Coulomb force compare to corresponding valence neutrons in mirror nuclei pair, see the cross symbol in Fig. 1. Single particle properties, induced by the electromagnetic spin orbit interaction and the Coulomb part, produce large effects in BED for nuclei in upper sd and fp shells.[23] In the following, we will discuss the energy difference of mirror nuclei.

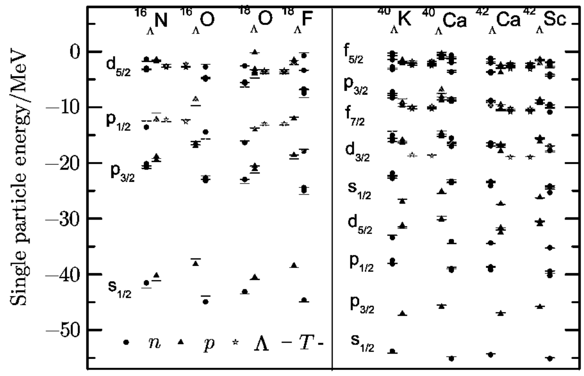

Fig. 1 The baryon single particle levels of mirror hypernuclei calculated in a time-odd triaxial deformed RMF approach with parameter set PK1-Y2.[22] Symbols as triangle, star remark nucleon and hyperon positive angular momentum projection levels, the solid lines present negative angular momentum projection levels. The cross symbol indicates the valence baryon level.

Table 1 The binding energy $-E$, the binding energy difference ΔE and the microscope center of mass energy $E_{\rm c.m}$ (in MeV) for mirror hypernuclei with A = 16, 18, 40, 42. The experimental data of nuclear core $E^{\rm Exp.}_{\rm core}$ are taken from Ref. [24]. With time-odd fields switching on and off, results are marked by $T = 1$ and $T = 0$. Data in brackets are the corresponding ones for ordinary nuclei from Ref. [15].

Nucl.	$-E^{\rm Exp.}_{\rm core}$	$\Delta E^{\rm Exp.}_{\rm core}$	$-E^{T=1}_{\rm tot}$	$\Delta E^{T=1}_{\rm tot}$	$-E^{T=0}_{\rm tot}$	$\Delta E^{T=0}_{\rm tot}$	$E^{T=1}_{\rm c.m}$	$\Delta E^{T=1}_{\rm c.m}$	$E^{T=0}_{\rm c.m}$	$\Delta E^{T=0}_{\rm c.m}$
$^{16}_{\Lambda}$N	115.49	3.53	130.60(116.91)	3.70(3.71)	130.15	3.72	12.37(11.11)	0.06(0.05)	12.28	0.06
$^{16}_{\Lambda}$O	111.96		126.90(113.20)		126.43		12.31(11.06)		12.22	
$^{18}_{\Lambda}$O	131.76	3.54	147.19(133.03)	3.86(3.83)	146.89	3.83	11.62(10.66)	0.08(0.08)	11.60	0.09
$^{18}_{\Lambda}$F	128.22		143.33(129.20)		143.06		11.54(10.58)		11.51	
$^{28}_{\Lambda}$Al	224.95	5.59	241.23(222.18)	5.89(5.64)	241.23	7.13	12.05(10.60)	0.08(0.07)	12.05	1.34
$^{28}_{\Lambda}$Si	219.36		235.34(216.53)		234.10		11.97(10.53)		10.71	
$^{40}_{\Lambda}$K	333.72	7.31	352.15(333.05)	7.32(7.30)	351.89	7.32	9.00(8.65)	0.05(0.03)	8.97	0.05
$^{40}_{\Lambda}$Ca	326.41		344.83(325.76)		344.57		8.95(8.62)		8.92	
$^{42}_{\Lambda}$Ca	350.41	7.27	370.11(350.76)	7.35(7.33)	369.88	7.32	8.90(8.57)	0.05(0.04)	8.89	0.05
$^{42}_{\Lambda}$Sc	343.14		362.76(343.43)		362.56		8.85(8.53)		8.84	

In Table 1, energies and energy differences for mirror hypernuclei (mass number A = 16, 18, 28, 40, 42) calculated with PK1-Y2 are shown. Results in ordinary mirror nuclei with parameter set PK1 from Ref. [15] are listed in brackets. They are obtained in the same way as this work. From $\Delta E^{T=1}_{\rm tot}$ in Table 1, we can see the results are consistent with the ordinary calculation in Ref. [15]. By switching off the time-odd fields ($T = 0$), the nuclear binding energy is slightly weaken while the BED nearly unchanged. As the spin-orbit splitting for Λ is in one order of the magnitude for nucleon, the single particle behaviors of baryon do not affect the BED for nuclei in study. Since the mean field approximation breaks the translational symmetry, treatment of center of mass motion may possibly effect the nuclear binding energy. A standard microscopic recipe for the center of mass correction is adopted by expanding the correction in orders of moments $\langle \hat{P}^{2n}_{\rm c.m} \rangle$ and stopping at 1st. order.[25] From Table 1, the magnitude of $\Delta E^{\rm micr}_{\rm c.m}$ for each mirror pair is about tens of keV, whose contribution to BED is only 1% ∼ 2%. The discrepancy between $\Delta E^{T=1}_{\rm tot}$ and $\Delta E^{T=0}_{\rm tot}$ for $A = 28$ can be merged by the difference of $\Delta E^{T=1}_{\rm c.m}$ and $\Delta E^{T=0}_{\rm c.m}$.

Electromagnetic interaction and Coulomb effect play a great role in the binding energy difference BED.[1] The existence of a charge dependent part in the nuclear force would bring in additional corrections to the Coulomb energy difference.[26] In the mean field approximation, there is only ρ_0 exchanges and charge dependent of nuclear interaction introduced by isovector-vector meson $\vec{\rho}$ is absent. Since the effect of the Fock-exchange term of Coulomb interaction is included in fitting process of parameter sets, it is not necessary to calculate the Coulomb exchange term explicitly.[7,27] In Slater approximation, the Coulomb exchange energy is calculated analytically by

$$E^{\rm ex}_{\rm Coul} = \int {\rm d}^3 r \frac{-3}{4}\Big(\frac{3}{4}\Big)^{1/3} e^2 \rho_p^{4/3}(\vec{r})\,,$$

whose contribution is attractive and reduces the anomaly with different proton density.[28] Recently, Slater approximation for the Coulomb exchanges effects has been investigated and the results are improved by 3% ∼ 5% using a relativistic correction to the local energy density approximation.[29] In Table 2, contribution to the energy difference of hypernuclear mirror pair from the direct Coulomb energy are shown. The Coulomb direct energy is obtained by proton current

$$E^{\rm dir}_{\rm Coul} = \frac{e^2}{2} \iint {\rm d}^3 r\, {\rm d}^3 r' \frac{j^\mu_p(\vec{r}) j_{p,\mu}(\vec{r}')}{|\vec{r} - \vec{r}'|}\,.$$

Comparison of the direct Coulomb energy $E^{\rm dir}_{\rm Coul}$ for nuclei ${}^{16,18}_{\Lambda}$O and ${}^{40,42}_{\Lambda}$Ca hints that extra neutron will slightly decrease the $E^{\rm dir}_{\rm Coul}$ in isotopes. There is no difference for the results with and without the time-odd components. Usually a smaller rms radius of charge distribution in atomic nuclei corresponds to a larger Coulomb energy for a given number of protons. Results of r_p in Table 2 show that the shrinkage effect on nuclear radius with hyperon staying at s orbit is not obvious for nuclei in study. By comparison, time-odd effects on the radii show a tiny core polarization.

Table 2 The direct Coulomb energy and the proton rms radii r_p for mirror hypernuclei (${}_Z$A and ${}_{Z-1}$A) with A = 16, 18, 40, 42. Data in brackets are the corresponding ones for ordinary nuclei from Ref. [15].

Nucl.	$E^{{\rm dir},T=1}_{\rm Coul}$	$\Delta E^{{\rm dir},T=1}_{\rm Coul}$	$E^{{\rm dir},T=0}_{\rm Coul}$	$\Delta E^{{\rm dir},T=0}_{\rm Coul}$	$r^{T=1}_p$	$r^{T=0}_p$
${}^{16}_{\Lambda}$N	13.59	3.55	13.57	3.57	2.49(2.49)	2.50
${}^{16}_{\Lambda}$O	17.14		17.13		2.58(2.58)	2.59
${}^{18}_{\Lambda}$O	17.10	3.67	17.10	3.65	2.57(2.57)	2.58
${}^{18}_{\Lambda}$F	20.77		20.75		2.55(2.55)	2.56
${}^{28}_{\Lambda}$Al	39.75	5.43	39.75	5.24	2.93(2.96)	2.93
${}^{28}_{\Lambda}$Si	45.18		45.17		2.99(3.02)	2.99
${}^{40}_{\Lambda}$K	74.29	7.08	74.27	7.09	3.31(3.31)	3.31
${}^{40}_{\Lambda}$Ca	81.37		81.36		3.35(3.35)	3.35
${}^{42}_{\Lambda}$Ca	81.18	7.10	81.17	7.09	3.35(3.35)	3.35
${}^{42}_{\Lambda}$Sc	88.28		88.26		3.40(3.40)	3.40

Table 3 The Lambda binding energy B_Λ, the energy difference ΔB_Λ in calculation and a parametrization $\Delta B^{\rm para}_\Lambda$ extended from a phenomenological shell model analysis.[31]

	${}^{12}_{\Lambda}$B	${}^{12}_{\Lambda}$C	${}^{16}_{\Lambda}$N	${}^{16}_{\Lambda}$O	${}^{18}_{\Lambda}$O	${}^{18}_{\Lambda}$F	${}^{28}_{\Lambda}$Al	${}^{28}_{\Lambda}$Si	${}^{40}_{\Lambda}$K	${}^{40}_{\Lambda}$Ca	${}^{42}_{\Lambda}$Ca	${}^{42}_{\Lambda}$Sc
B_Λ	11.21	11.19	12.56	12.57	13.20	13.19	17.63	17.49	18.75	18.75	18.98	18.98
ΔB_Λ	0.02		0.01		0.01		0.14		0		0	
$\Delta B^{\rm para}_\Lambda$	0.49		0.94		1.16		1.16		3.63		4.08	

The reduced proton-neutron difference $m_n - m_p$ will cause an enlarged attraction, which can be used to explain the anomaly.[30] We should mention that the proton-neutron difference has already been included in obtaining the parameter set PK1-Y2.[22,27]

Due to ΛN charge symmetry breaking, BED for mirror single-Λ hypernuclei includes an additional hyperon separation energy difference $\Delta B_\Lambda = \Delta M_N - \Delta M_{Hy}$, with $\Delta M = M(Z, A-1) - M(Z-1, A-1)$. Experimental data as $B_\Lambda({}^{12}_{\Lambda}B) = 11.380$ MeV[14] and $B_\Lambda({}^{12}_{\Lambda}C) = 10.76$ MeV[12] show a large $\Delta B_\Lambda = 0.62$ MeV. The data of ΔB_Λ in Table 3 is approximately obtained by difference of hyperon single particle energies. As the attractive interaction provided by Λ hyperon is identical in different nuclei, Λ binding energy difference ΔB_Λ in study is tiny, see Table 3. The impurity effect studied here for BED by including a single Λ can be neglected. In comparison, results from a phenomenological shell model analysis by $\Delta B_{\Lambda}^{\text{para.}} = a(Z-1)+b$ are also listed, where $a = 0.224$ MeV is the charge symmetry part of ΛN interaction and $b = -0.628$ MeV is the charge symmetry breaking one by optimizing to $A = 4, 8, 10, 12$ hypernuclear mirror pair.[31] The tiny ΔB_{Λ} relative to $\Delta B_{\Lambda}^{\text{para.}}$ testifies that the charge dependence of separation energies is small[32] and the phenomenological analysis are not reliable in the upper mass region for ${}^{16}_{\Lambda}\text{O}$.[31]

As hyperon slightly changes the deformation of hypernuclei from its ordinary nuclear core in most cases,[19,33–35] the interplay of pairing and tensor interaction is crucial to derive the deformation.[36] In this work, pairing is missing. To obtain a precise data, a multidimensionally-constrained relativistic mean field model[37] might be useful. More delicate considerations are in progress.

4 Conclusion

In summary, by including proton-nucleon mass difference, center of mass motion, properties for mirror hypernuclei with mass $A = 16, 18, 28, 40$ and 42 are investigated in a time-odd triaxial relativistic mean-field approach. For mirror hypernuclear pair, a mirror symmetry appears in time reversal conjugate levels. With an identical attractive interaction provided by Λ hyperon, the binding energy difference for hypernuclear pair seldom change as compared to that for the ordinary nuclei. The direct Coulomb interaction in isotopes will slightly decrease in isotopes for extra neutrons. Core polarization caused by a Lambda hyperon hardly increases the isospin symmetry effect for mirror hypernuclei studied here.

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