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Gaussian Beam Propagation in a Kerr Type Metamaterial Medium Using ABCD Matrix Method

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Abstract *In this paper, a split step ABCD matrix method is suggested to investigate Gaussian beam propagation in a Kerr type metamaterial medium. This method is based on dividing the medium interval into subsequent steps. Meanwhile, Gaussian beam profile in every step is obtained by finding the ABCD matrix of that particular step, and is used to find the ABCD matrix of the next step. Results of the suggested matrix method have been compared with the results of numerical split-step Fourier method for a Kerr medium, which indicates a good agreement. Then, we use the ABCD matrix to investigate Gaussian beams propagation in a Kerr type metamaterial, which is also in agreement with pervious results by other methods.*

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Key words: ABCD matrix method, Kerr type metamaterials, paraxial beam, beam propagation

1 Introduction

Kerr effect is a phenomenon in which the refractive index of a material changes because of an applied electrical field. The changing of refractive index is proportional to the square of the applied electric field.[1] The Effects of laser beam propagation in nonlinear Kerr media have been investigated both theoretical and experimentally for many years.[2−5] Different methods such as beam propagation method,[6−7] variational method,[8−12] finite-different time-domain,[13−14] ABCD transfer matrix method[15−16] and etc. have been used for this matter. However, ABCD matrix method is the most convenient and useful tools in calculating different beam profiles in optical elements, which is extensively used to describe beam propagation in free space.[17−18] On the other hand, metamaterials are engineered structures which have attracted many interests in recent years due to their unique properties and applications.[19−24] When light beams propagate through left-handed media (LHM), some interesting effects like large negative lateral shifts,[25] negative refraction of electromagnetic energy,[26] beam focusing and phase compensation[27] have been reported. A class of metamaterials is those with permittivity and permeability simultaneously less than zero.[28−30] These LHMs are so called because their E, H and k vectors form a left-handed coordinate set with the consequence that their wave propagation vector is anti-parallel to the direction of energy flow. It shows that a simple ABCD matrix formalism can be used to investigate beam propagation in linear LHM slab systems.[31] Moreover, metamaterials with nonlinearity can be made by embedding the metallic structure of arrays of wires and split-ring resonators into a dielectric with a nonlinear permittivity that depends on the intensity of the electric field.[32] These structures show quite different properties rather than linear metamaterials or conventional nonlinear materials, which have been investigated by both analytical and numerical methods previously.[33] In this paper, a split step matrix method is suggested to investigate laser beam self-focusing under propagation in a Kerr medium, and results are compared to those of numerical simulation. Then, the suggested matrix is used to study Gaussian beam propagation through a Kerr LHM slab. This method is based on dividing the investigation interval into subsequent steps. Beam profile in every step is obtained by ABCD matrix of that step and beam profile of the last step. It should be noted that dividing the nonlinear material to subsequent steps adds a significant accuracy to the previous ABCD matrix method. Comparing results of this method for beam propagation in Kerr LHM slab with numerical split-step Fourier method shows a good agreement as well. Consequently, the suggested ABCD matrix method can be used as a convenient and promising way to study beam propagation even in long length left handed nonlinear medium.

2 Physical Model

The amplitude of electric field, under the slowly varying amplitude approximation, can be written as:

$$u(r,q) = u_0 \frac{\mathrm{i} z_0}{q} \exp\left(\frac{-\mathrm{i} k r^2}{2q}\right), \qquad (1)$$

where is the amplitude constant, $z_0 = n k_0 w_0^2/2$ is the Rayleigh length, and w_0 is the waist size of a Gaussian envelope. The complex parameter q is defined by

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$1/q = 1/R - \mathrm{i}\lambda/\pi w^2$. Evolution of Gaussian beam complex number can be described by transfer matrix, as

$$q_2 = \frac{Aq_1 + C}{Bq_1 + D}\,, \tag{2}$$

where A, B, C, and D are elements of the matrix. The corresponding field distribution with q_2 is

$$u(r,z) = u_0\Big(\frac{q_1}{Aq_1 + B}\Big)\exp\Big(-\frac{\mathrm{i}kr^2}{2q_2}\Big)\,. \tag{3}$$

Changes in refractive index of Kerr media are described as

$$n = n_L + \frac{1}{2}n_2u^2 = n_L + \frac{n_2}{n_L c\varepsilon_0}I = n_L + \gamma_2 I\,, \tag{4}$$

where n_L is the linear part of refractive index, n_2 and γ_2 are the electric field and the nonlinear refractive index coefficient, respectively. I is the laser intensity, ε_0 is the electric permittivity in vacuum, and c is the speed of light in vacuum.

The Gaussian laser intensity can be approximated by the first two terms of its Taylor expansion as

$$I = I_0\exp\Big[-2\Big(\frac{r}{w}\Big)^2\Big] \approx I_0\Big[1 - 2\Big(\frac{r}{w}\Big)^2\Big]\,. \tag{5}$$

Then, the intensity-dependent refractive index of Kerr media can be written in the following form

$$n = n_L + \frac{n_2 I_0}{n_L c\varepsilon_0}\Big[1 - 2\Big(\frac{r}{w}\Big)^2\Big] = n_I\Big(1 - \frac{1}{2}\gamma^2 r^2\Big)\,, \tag{6}$$

where

$$n_I = n_L + \frac{n_2 I_0}{n_L c\varepsilon_0}\,, \tag{7}$$

$$\gamma = \Big(\frac{4n_2 I_0}{n_L c\varepsilon_0 n_I}\Big)^{1/2}\frac{1}{w} = \Big(\frac{8n_2 P}{n_L c\varepsilon_0 n_I \pi}\Big)^{1/2}\frac{1}{w^2}\,, \tag{8}$$

$$P = \pi w^2 I_0/2\,. \tag{9}$$

So, the ABCD matrix for this media is

$$A = \begin{pmatrix} \cos(\gamma l) & \sin(\gamma l)/(n_I\gamma) \\ -n_I\gamma\sin(\gamma l) & \cos(\gamma l) \end{pmatrix}\,, \tag{10}$$

where l is the length of the media.

Now, we divide the distance of beam propagation in Kerr LHM to N subsequent steps. Initial laser beam power is used to find matrix elements in the first step, complex parameter and beam width after passing the first step that can be obtained from Eq. (2). Then, calculated beam profile is used to find the next step ABCD matrix and therefore beam parameters at the end of this step. Obviously, the number of enough steps depends on length of media and the laser beam profile. Also, it should be noted that γ is calculated in the beginning of every step and will be assumed to be constant through every step, as well as the beam waist. To find high accuracy and optimum number of steps, we must increase the number of steps until there is no significant change in Gaussian beam behavior under propagation. Another way to investigate beam propagation is to use a numerical method like split-step Fourier method to solve nonlinear Schrödinger equation, which is used to show the accuracy of the suggested matrix method. Split step Fourier method is a computational technique in electromagnetic, which is used to solve paraxial wave equation under slowly varying envelope approximation, for linear and nonlinear equations. More details of this method can be found in Refs. [33–34].

Nonlinear Schrodinger wave equation in a Kerr metamaterial slab can be written as:[35]

$$2\mathrm{i}k\frac{\partial u}{\partial z} + \Big(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\Big)u + \frac{2k^2\mu}{n_L}C_{\mathrm{NL}}|u^2|u = 0\,, \tag{11}$$

where $k = n_L 2\pi/\lambda$, n_L is the linear refraction index of the LHM slab, u is the complex envelop of the electric field, $C_{\mathrm{NL}} = \chi/2n_L$ is the nonlinear coefficient, χ is the cubic susceptibility, μ is the magnetic permeability of the LHM slab. By solving the wave Eq. (11) numerically, the beam propagation in a Kerr metamaterial slab can be simulated.

3 Simulation and Discussion

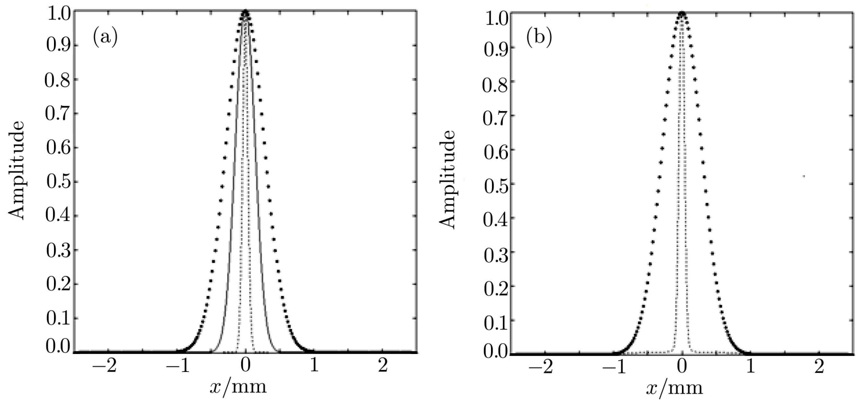

Fig. 1 Beam propagation behavior in Kerr media with $l = 4$ cm (a) Initial beam (dotted line), propagated beam by ABCD matrix with 1 step (solid line) and with 50 steps (dashed line). (b) Initial beam (dotted line) and propagated beam by split step Fourier method (dashed line).

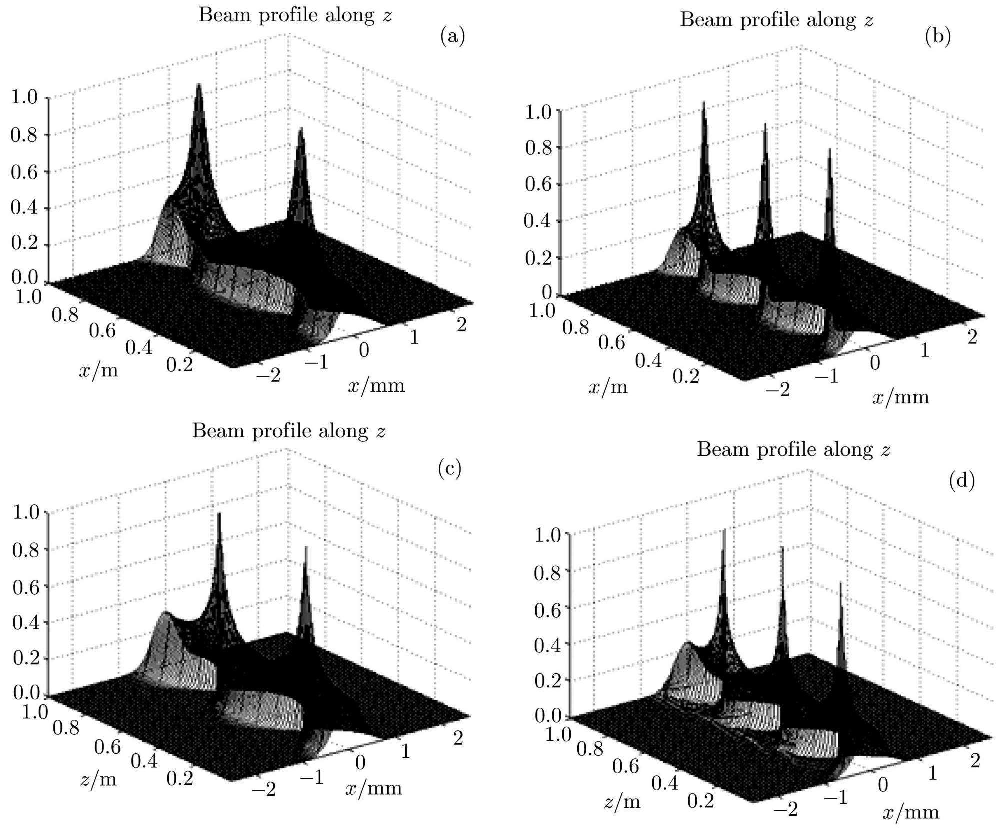

Fig. 2 Gaussian beam propagation in a Kerr media with $l = 4$ cm by ABCD matrix method (a) for incident and critical power ratio of 4 and 300 steps, (b) for incident and critical power ratio of 8 and 800 steps. The same counterpart by split step Fourier method (c) for incident and critical power ratio of 4 and (d) for incident and critical power ratio of 8.

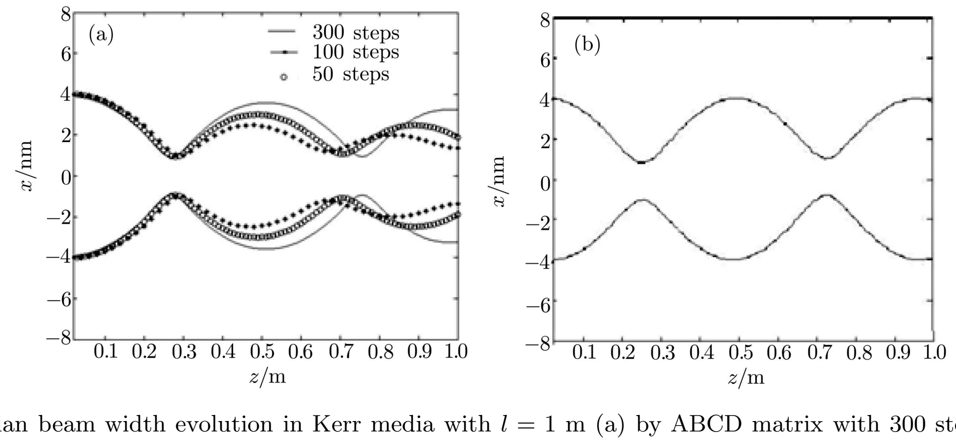

Fig. 3 Gaussian beam width evolution in Kerr media with $l = 1$ m (a) by ABCD matrix with 300 steps (solid line), 100 steps (stars) and 50 steps (circles). (b) by split step Fourier method.

At first, the suggested transfer matrix method is applied to investigate Gaussian beam propagation in a nonlinear Kerr medium. Then, this method will be used for studying the beam propagation behavior in a Kerr LHM slab. We already know, in a nonlinear Kerr medium self-focusing occurs for lasers beams with initial power more than the critical power of Kerr medium. The Gaussian Beam propagation in a Kerr medium is simulated for $l = 4$ cm. Figure 1(a) shows the initial beam (dotted line), propagated beam simulated by ABCD matrix with one step (solid line) and with 50 steps (dashed line). In order to compare this method to numerical method, in Fig. 1(b) we depicted the results of numerical method. Initial beam (dotted line) and propagated beam simulated by split-step Fourier method (dashed line) are shown in this figure.

As a result of Fig. 1, the beam propagation simulated by ABCD matrix with 50 steps is in a good agreement with simulation result of split-step Fourier method. However, there is an obvious difference between ABCD matrix method with 1 and 50 steps.

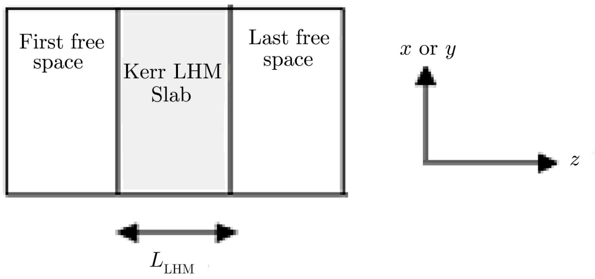

Fig. 4 Schematic of the Kerr LHM structure.

Figure 2 shows the Gaussian beam evolution in a Kerr medium with $l = 1$ m by the ABCD matrix method ((a) and (b)) and the split step Furrier method ((c) and (d)). In Fig. 2(a) the incident and critical power ratio is chosen as 4 for 300 steps and as 8 for 800 steps in Fig. 2(b). The same configuration is simulated numerically in Figs. 2(c) and 2(d) for incident and critical power ratio of 4 and 8 respectively. As it can be seen its in complete agreement with ABCD method.

The effect of step number on beam propagation simulation with ABCD matrix method is investigated in Fig. 3. In this figure Gaussian beam width evolution in Kerr media with $l = 1$ m by ABCD matrix, with 300 steps (solid line), 100 steps (stars) and 50 steps (circles) is simulated. We compared these results with the Gaussian beam width evaluation in Kerr media with the same length split step Fourier method in Fig. 3(b). Clearly, in the beginning of the media there is still acceptable accuracy for less steps, but when the Gaussian beam passes longer distances in concerned media, ABCD matrix loses ability to simulate the propagation behavior. Hence, with enough steps, Gaussian beam evolution can be investigated by ABCD matrix with an acceptable accuracy. As a matter of fact, for special Gaussian beam parameter and Kerr media properties there is an optimum number of steps to obtain acceptable results and more step numbers make no difference in the results.

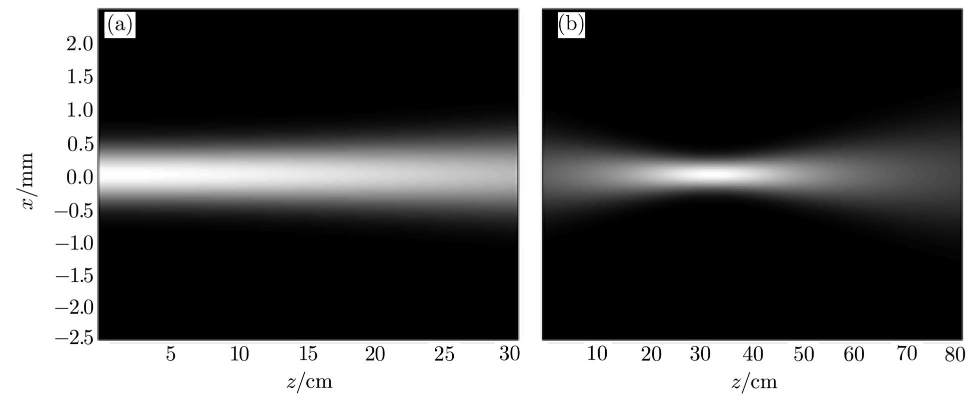

Fig. 5 Gaussian beam propagation with positive susceptibility and initial power more than critical power in (a) Kerr LHM slab, (b) free space after the slab.

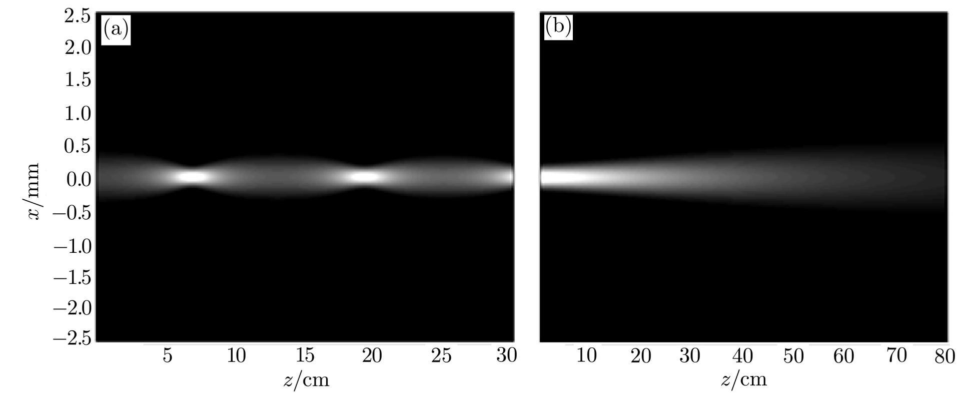

Fig. 6 Gaussian beam propagation with negative susceptibility and initial power more than critical power in (a) Kerr LHM slab, (b) free space after the slab.

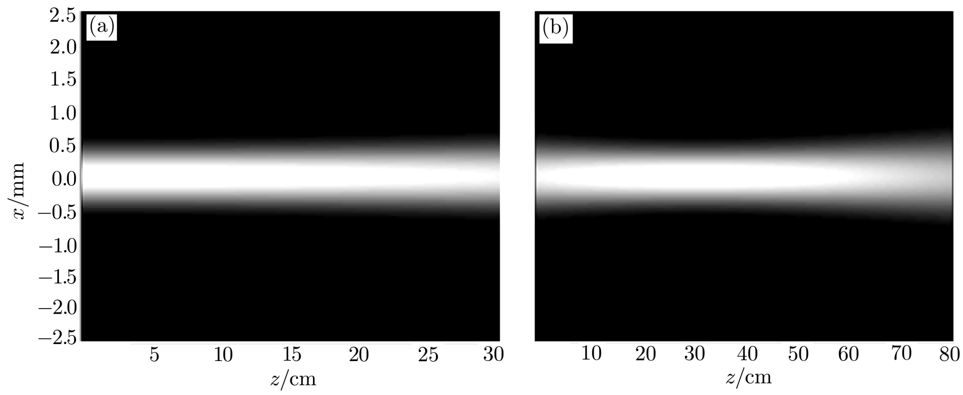

Fig. 7 Gaussian beam propagation with positive susceptibility and initial power less than critical power in (a) Kerr LHM slab, (b) free space after the slab.

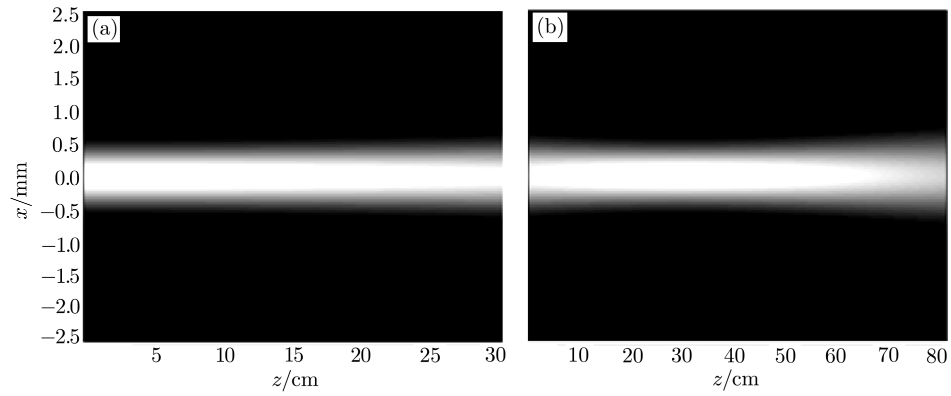

Fig. 8 Gaussian beam propagation with negative susceptibility and initial power less than critical power in (a) Kerr LHM slab, (b) free space after the slab.

It has been shown that self-focusing in Kerr LHM slab depends on the sign of cubic susceptibility and initial beam power.[35] For negative susceptibility and initial beam power more than critical power, self-defocusing have been observed. For other cases including negative and positive susceptibility and initial power more or less than critical power, self-focusing happens in the next free space after the Kerr LHM slab. In order to simulate these behavior, we simulated the Gaussian beam propagation in the Kerr LHM slab with positive and negative susceptibility for different initial beam power with respect to critical power by ABCD split step matrix method. A schematic of such structure is showed in Fig. 4.

Figure 5 shows Gaussian beam propagation with positive susceptibility and initial power more than critical power (a) in the Kerr LHM slab and (b) in free space after the slab. As it can be seen, the Gaussian beam diverges while propagating during LHM slab and focuses after entering the next free space after it. Meanwhile, when the susceptibility is negative the Gaussian beam propagates with a different envelope, as shown in Fig. 6. Figures 6(a) and 6(b) similar to Fig. 5, show the Gaussian beam propagation with negative susceptibility and initial power more than critical power in the Kerr LHM slab and in the free space after the slab respectively. In this case, Gaussian beam has a periodic focusing behavior in Kerr LHM slab and then diverges in the next free space.

Figures 7 and 8 depict the Gaussian beam propagation with positive and negative susceptibility respectively for the initial power less than critical power (a) in Kerr LHM slab, (b) in free space after the slab. As it can be seen, Gaussian beam diverges in the Kerr LHM slab and then diverges in the next free space for these two cases either. These results are in complete agreement with previous result from analytic and numerical simulation.

4 Conclusion

Split step ABCD matrix method is an easy and convenient way to investigate laser beam propagation in nonlinear mediums. Gaussian beam propagation can be investigated by dividing the medium interval to subsequent steps and finding ABCD matrices in every step. In this way, beam propagation can be studied in every point of nonlinear Kerr media. Comparing this method with numerical split-step Fourier method shows a good agreement in results. Therefore, the suggested ABCD matrix method is a reliable way for beam propagation study in a Kerr medium. Also this matrix method can be used for Gaussian beam propagation in nonlinear metamaterials as well. Simulations show that using transfer matrix method is as reliable as other methods. As Gaussian beam propagation in a Kerr LHM slab and the free space after the slab depends on susceptibility sign of media and initial beam power, we simulate and check our method. Definitely, for the cases of negative and positive susceptibility with initial beam power less than the critical power and positive susceptibility with initial beam power more than the critical power, beam focusing occurs at the free space after Kerr LHM slab. While, for negative susceptibility with initial beam power more than the critical power, the beam focusing occurs at the Kerr LHM slab periodically. As a result, one can use this matrix method for beam propagation in a nonlinear left handed metamaterial.

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