Instability Analysis of Positron-Acoustic Waves in a Magnetized Multi-Species Plasma

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Abstract *The nonlinear propagation of electrostatic excitations and their multi-dimensional instability in a magnetized, degenerate electron-positron-ion (EPI) plasma system (containing inertial cold positrons, relativistic degenerate electrons and hot positrons, and negatively charged immobile heavy ions) are theoretically investigated. The reductive perturbation method is employed to derive the Zakharov–Kuznetsov equation which admits a localized solitary wave solution for small but finite amplitude limit, and the multi-dimensional instability of the positron acoustic solitary waves (PASWs) is studied by the small-k perturbation expansion method. It is found that the basic characteristics (viz. phase speed, amplitude, width) of the PASWs are significantly affected by the degree of obliqueness, relativistic degeneracy, and plasma particle number densities. The instability criterion and its growth rate, which are depending on the magnetic field and the propagation directions of both the PASWs, and their perturbation modes are discussed. The present analysis can be helpful in understanding the nonlinear phenomenon in dense astrophysical as well as space plasma systems, especially in pulsar environments.*

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1 Introduction

In contrast with the past, there has been continual attention in quantum plasmas in ultra-dense astrophysical systems[1−9] as well as laboratory plasmas.[10−12] The quantum effects become noticeable on the study of electron-positron (EP) plasmas which exist in the pulsar magnetosphere,[13−14] active galactic nuclei,[15] in the early universe,[16] etc., and in laboratory situations.[17−18] In most of the cases, the EP plasma is supposed to exist in relativistic regimes,[19] and most of the theoretical investigations on the nonlinear structures in EP as well as electron-positron-ion (EPI) plasma medium have been done considering the relativistic cases.[20−21] Positrons are created in the interstellar medium when the atoms become interacted by the cosmic ray nuclei.[22] The data obtained during the alpha magnetic spectrometer flight permitting to probe the radiation belts in the Earths innermost magnetosphere provided an evidence of the presence of positrons.[23−24] In order to study the collective behavior of a plasma containing electrons and positron, it is necessary to find the condition to neglect the annihilation process. Many authors have neglected the annihilation process in the ultra-relativistic dense plasmas.[25−26]

Many astrophysical environments (viz. neutron stars, white dwarfs, etc.) where Fermi energy of the plasma species becomes comparable to or higher than their rest mass energy, show an unavoidable dependency on relativistic effects of plasma components.[27−30] The density of such a compact objects turned into so high that prevents gravitational collapse through the presence of degenerate pressure. Perhaps, there is a dramatic increase in energy of the accelerated particles to generate electrons and positrons by virtue of colliding these high-energy particles. For such an interstellar compact object, the degenerate pressure for the cold positron fluid can be given by the following equation,

$$P_{\rm pc} = L_{\rm pc} {n_{\rm pc}}^{\gamma}\,, \tag{1}$$

where

$$\gamma = \frac{5}{3}\,, \quad L_{\rm pc} = \frac{3}{5}\Bigl(\frac{\pi}{3}\Bigr)^{1/3}\frac{\pi\hbar^2}{m} \simeq \frac{3}{5}\Lambda_c\hbar c\,, \tag{2}$$

for the non-relativistic limit (where $\Lambda_c = \pi\hbar/mc = 1.2 \times 10^{-10}$ cm, and $\hbar$ is the Planck constant divided by 2π). While for the electron and hot positron fluids,

$$P_e = L_e {n_e}^{\gamma}\,, \quad P_{\rm ph} = L_{\rm ph} {n_{\rm ph}}^{\gamma}\,, \tag{3}$$

where for the non-relativistic limit[31−35]

$$\gamma = \frac{5}{3}\,, \quad L_{\rm pc} = L_e = L_{\rm ph}\,, \tag{4}$$

and for the ultra-relativistic limit[31−35]

$$\gamma = \frac{4}{3}\,, \quad L_e = L_{\rm ph} = \frac{3}{4}\Bigl(\frac{\pi^2}{9}\Bigr)^{1/3}\hbar c \simeq \frac{3}{4}\hbar c\,. \tag{5}$$

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Presently, a considerable attention has been drawn to study the PA waves in astrophysical dense EPI plasmas containing distinct group of positrons (i.e. cold positrons and/or hot positrons).[36−38] The coexistence of cold and hot positron populations in pulsar magnetosphere has been predicted by Bharuthram.[39] In two temperature EPI plasma the interesting phenomena differing from that of one temperature can exist. Furthermore, in contrast to the case of the pure EP plasma, in two temperature EPI plasma the modulational instability may occur. Therefore, the prediction of existing cold and hot positrons species becomes momemtous to know the novel electrostatic perturbations as well as multi-dimensional instability in magnetized EPI plasmas. Basically, PA waves are the result of two distinct positron components (cold positron and hot positron) at different temperatures. The cold positron mass provides the inertia whereas the hot positrons and hot electrons provide the essential thermal pressure to develop the restoring force for the existence of PA waves. It is typically high frequency waves in comparison with the ion plasma frequency propagating at a phase speed which lies between the hot and cold positron thermal velocities. On the PA wave time scale, the ions are generally assumed stationary forming a neutralizing background. This means that the ion dynamics does not influence the PA waves because the PA wave frequency is much larger than the ion plasma frequency.

The nonlinear propagation of IA, PA and Electron-acoustic waves for the relativistic plasma has been theoretically studied by a number of authors.[38,40−52] El-Shamy *et al.*[38] numerically investigated the influences of the cold/hot positron parameters on the phase shifts in a plasma system consisting of cold positrons, immobile ions, electrons, and hot positrons. Sahu[44] examined the planar and nonplanar PA shock waves in an unmagnetized EPI plasma system (containing mobile cold positrons, stationary positive ions, and Boltzmann-distributed electrons and hot positrons), and showed the effects of the ion kinematic viscosity, and Boltzmann-distributed electrons and hot positrons on PA shock waves. In a relativistically degenerate magnetoplasma, Ali and Rahman[53] investigated IA waves and numerically shown that the IA solitons and shocks are significantly influenced by various parameters of plasma system.

However, all of these theoretical investigations[38,40−45] on the PA waves are accomplished in an unmagnetized EPI plasma system, and the effect of magnetic field or obliqueness is ignored. The Zakharov–Kuznetsov (ZK) equation and it's solitary wave solution have been derived using the reductive perturbation method in a magnetized EPI plasma (consisting of inertial cold positrons, negatively charged immobile heavy ions, degenerate electrons, and hot positrons). Their multi-dimensional instability are also studied by the small-k (long-wavelength plane wave) perturbation expansion method. The effects of relativistic electrons/hot positrons degenerate pressure, obliqueness, etc. on the width, amplitude, and phase speed of the PASWs have been studied.

The article is organized in the following way. The normalized governing equations are presented in Sec. 2. The ZK equation and it's stationary solitary wave solution are derived on Sec. 3. The instability of the PASWs (linear wave analysis) has been analyzed in Secs. 4 and 5. The parametric investigations of our plasma systems is given in Sec. 6. Finally, this paper ends up with the conclusion which is given in Sec. 7.

2 Governing Equations

We consider the propagation of low-frequency PASWs in a four components magnetized EPI plasma (containing inertial cold positrons, negatively charged immobile heavy ions, degenerate electrons, and hot positrons). At equilibrium, the quasi-neutrality condition reads $n_{\rm pc0} + n_{\rm hp0} = n_{e0} + Z_i n_{i0}$, where $n_{\rm pc0}$, n_{hp0}, n_{e0}, and n_{i0} are the equilibrium densities of the cold positrons, hot positrons, electrons and ions, respectively and Z_i is the ion-charged state. In our present work, we have considered the effects that come from degenerate pressure only; not any quantum effects related to any particular components. For this reason, quantum effect terms i.e., Bohm potential, quantum spin effects, etc. are ignored in Eqs. (6) and (7). In some other works,[54−55] authors have taken into account Bohm potential associated with particular plasma components such as electrons and positrons. Authors may consider typical parameters instead of the normalized parameters but that is beyond the scope of the present work. It is notable that the mass of cold positron is exactly the same as that of hot positron and electron, but their temperatures are different. The temperature of the cold positron is assumed to be very small, and in our present work it is neglected compared to the temperatures of hot positrons and electrons. So, we consider inertial cold positron, and define the waves associated with the inertial cold positron fluid as the PASWs. The model, what we have considered, is magnetized and since cold positron gives the inertia of this system so that we provide the magnetized equation for the cold positron only. Furthermore, the magnetized cold ion or cold positron or cold electron is common in Refs. [56–59].

The phase velocity of the PASWs is assumed to be much larger than the cold positrons thermal velocity and much less than the electrons/hot positrons thermal velocities, i.e. $v_{T_{\rm pc}} \ll \omega/\kappa \ll v_{T_{\rm ph}}/v_{T_e}$. In addition, electrons and hot positrons are considered to be inertialess and move almost parallel to the external magnetic field direction and negatively charged immobile heavy ions participate only to maintain the quasi-neutrality conditon. Under these situations, the basic set of nonlinear dynamic equations for magnetized cold positrons are governed as follows:

$$\frac{\partial n_{\rm pc}}{\partial t} + \nabla\cdot(n_{\rm pc}\cdot \boldsymbol{u}_{\rm pc}) = 0\,, \tag{6}$$

$$\frac{\partial \boldsymbol{u}_{\rm pc}}{\partial t} + (\boldsymbol{u}_{\rm pc}\cdot\nabla)\boldsymbol{u}_{\rm pc} = -\nabla\phi + \omega_{\rm cpc}(\boldsymbol{u}_{\rm pc}\times\hat{z})\,. \tag{7}$$

The momentum equations for inertialess degenerate electrons and hot positrons are given by

$$j\nabla\phi - \frac{K_s}{n_s}\nabla {n_s}^{\gamma} = 0\,. \tag{8}$$

The Poisson equation is written as

$$\nabla^2\phi = \alpha n_e - n_{\rm pc} - \sigma n_{\rm ph} + \mu\,, \tag{9}$$

where $n_{\rm pc}$ (n_s) is the cold positron (hot positron/electron) number density normalized by its equilibrium value $n_{\rm pc0}$ ($n_{\rm ph0}/n_{e0}$); $u_{\rm pc}$ is the cold positron fluid speed normalized by $C_{\rm pc} = (m_s c^2/m_{\rm pc})^{1/2}$, m_s($m_{\rm pc}$ and m_e) being the rest mass of species (cold positron and electron); j represents the charge state of plasma species (i.e. $j=1$ for electron and $j=-1$ for hot positron); c being the speed of light in vacuum; ϕ is the electrostatic wave potential normalized by $m_e c^2/e$, e being the magnitude of the charge of an electron; the time variable t is normalized by ${\omega_{\rm pc}}^{-1} = (m_{\rm pc}/4\pi e^2 n_{\rm pc0})^{1/2}$, and the space variable is normalized by $\lambda_D = (m_e c^2/4\pi e^2 n_{\rm pc0})^{1/2}$, respectively. Here $\alpha = n_{e0}/n_{\rm pc0}$, $\sigma = n_{\rm ph0}/n_{\rm pc0}$, $\mu = n_{i0}/n_{\rm pc0}$, and $\omega_{\rm cpc}$ being the cold positron cyclotron frequency ($eB_0/m_{\rm pc}c$) normalized by ${\omega_{\rm pc}}^{-1}$, and $K_s = n_{s0}^{\gamma-1}L_s/m_s c^2$, respectively.

3 Derivation of Z-K Equation

To derive the ZK equation for describing the nonlinear propagation of the PASWs in the EPI plasma under consideration, we use Eqs. (6)–(9), and employ the reductive perturbation technique.[60] We first introduce the stretched coordinates as

$$X = \epsilon^{1/2}x\,, \tag{10}$$

$$Y = \epsilon^{1/2}y\,, \tag{11}$$

$$Z = \epsilon^{1/2}(z - V_p t)\,, \tag{12}$$

$$\tau = \epsilon^{3/2}t\,, \tag{13}$$

where ϵ is a small parameter measuring the weakness of the dispersion, V_p is the linear phase speed normalized by the PASWs speed ($C_{\rm pc}$). It may be noted here that X, Y, and Z are all normalized by the Debye radius (λ_D), and τ is normalized by the ion plasma period ($\omega_{\rm pc}^{-1}$). We next expand the quantities about their equilibrium values in a power series of ϵ as[60–61]

$$n_{\rm pc} = 1 + \epsilon n_{\rm pc}^{(1)} + \epsilon^2 n_{\rm pc}^{(2)} + \cdots\,, \tag{14}$$

$$n_s = 1 + \epsilon n_s^{(1)} + \epsilon^2 n_s^{(2)} + \cdots\,, \tag{15}$$

$$u_{\rm pcx} = \epsilon^{3/2}u_{\rm pcx}^{(1)} + \epsilon^2 u_{\rm pcx}^{(2)} + \cdots\,, \tag{16}$$

$$u_{\rm pcy} = \epsilon^{3/2}u_{\rm pcy}^{(1)} + \epsilon^2 u_{\rm pcy}^{(2)} + \cdots\,, \tag{17}$$

$$u_{\rm pcz} = \epsilon u_{\rm pcz}^{(1)} + \epsilon^2 u_{\rm pcz}^{(2)} + \cdot\,, \tag{18}$$

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \cdots \tag{19}$$

Now, using Eqs. (10)–(19) into Eqs. (6)–(9), taking the lowest order coefficient of ϵ, we can write

$$n_{\rm pc}^{(1)} = \frac{\phi^{(1)}}{V_p^2}\,, \tag{20}$$

$$u_{\rm pcz}^{(1)} = \frac{\phi^{(1)}}{V_p}\,, \tag{21}$$

$$n_s^{(1)} = \frac{j\phi^{(1)}}{K'}\,, \tag{22}$$

$$V_p = \Big[\frac{K'}{j(\alpha-\sigma)}\Big]^{1/2}\,, \tag{23}$$

where $K' = \gamma K$ and Eq. (23) is the phase speed of the PA waves propagating in the magnetized EPI plasma under consideration. It is seen that V_p decreases (increases) with the increase of the electron number density (cold positron number density). The first order X and Y-components of Eq. (7) can be written as

$$u_{\rm pcx}^{(1)} = -\frac{1}{\omega_{\rm cpc}}\frac{\partial\phi^{(1)}}{\partial Y}\,, \tag{24}$$

$$u_{\rm pcy}^{(1)} = \frac{1}{\omega_{\rm cpc}}\frac{\partial\phi^{(1)}}{\partial X}\,. \tag{25}$$

Equations (24) and (25) represent the X and Y components of the cold positron electric field drifts. These equations are also satisfied by the second order continuity equation. Again, using Eqs. (10)–(19) into Eqs. (6)–(9), and eliminating $u_{\rm pcx}^{(1)}$ and $u_{\rm pcy}^{(1)}$, the next higher order X and Y components can be found as

$$u_{\rm pcx}^{(2)} = \frac{V_p}{\omega_{\rm cpc}^2}\frac{\partial^2\phi^{(1)}}{\partial Z\partial X}\,, \tag{26}$$

$$u_{\rm pcy}^{(2)} = \frac{V_p}{\omega_{\rm cpc}^2}\frac{\partial^2\phi^{(1)}}{\partial Z\partial Y}\,, \tag{27}$$

$$\frac{\partial^2\phi^{(1)}}{\partial X^2} + \frac{\partial^2\phi^{(1)}}{\partial Y^2} + \frac{\partial^2\phi^{(1)}}{\partial Z^2} = \alpha {n_e}^{(2)} - {n_{\rm pc}}^{(2)} - \sigma {n_{\rm ph}}^{(2)}\,. \tag{28}$$

Equations (26) and (27) represent the X and Y components of the cold positron polarization drifts. Now, following the same procedure one can obtain the next higher order continuity equation, and Z component of the momentum equation. Using these new higher order equations along with Eqs. (20)–(28), one can eliminate $n_{\rm pc}^{(2)}$, $u_{\rm pcz}^{(2)}$, and $\phi^{(2)}$, and finally obtain

$$\frac{\partial\phi^{(1)}}{\partial\tau} + AB\phi^{(1)}\frac{\partial\phi^{(1)}}{\partial Z} + \frac{A}{2}\frac{\partial}{\partial Z} \times\Big[\frac{\partial^2}{\partial Z^2} + D\Big(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\Big)\Big]\phi^{(1)} = 0\,, \tag{29}$$

where

$$A = V_p^3\,, \tag{30}$$

$$B=\frac{1}{2}\Big[\frac{\alpha(\gamma-2)}{K'^2}+\frac{3}{V_p^4}-\frac{\sigma(\gamma-2)}{K'^2}\Big]\,, \tag{31}$$

$$D=1+\frac{1}{\omega_{\rm cpc}^2}\,. \tag{32}$$

Equation (29) is the ZK equation describing the nonlinear propagation of the PASWs in a magnetized EPI plasma with degenerate relativistic electron and hot positron fluids.

The stationary SWs solution of the ZK equation can be written as

$$\phi_0(Z)=\psi_m{\rm sech}^2(\kappa Z)\,, \tag{33}$$

where $\psi_m=3U_0/\delta_1$ is the amplitude, and $\kappa=\sqrt{U_o/4\delta_2}$ is the inverse of the width of the solitary waves. The PASWs with positive (negative) potential $\psi_m>0$ ($\psi_m<0$) is found for $B>0$ ($B<0$) for the permissible value of any parameter.

4 Instability of the SWs

The instability of the obliquely propagating PASWs is studied by adopting the method of small-k perturbation expansion.[62−66] We first assume that

$$\phi^{(1)}=\phi_0(\mathcal{Z})+\psi(\mathcal{Z},\zeta,\eta,t)\,, \tag{34}$$

for a long-wavelength plane wave perturbation in a direction with direction cosines (l_ζ,l_η,l_ξ), ψ can be written as

$$\psi=\varphi(\mathcal{Z})\,{\rm e}^{\,{\rm i}[k(l_\zeta\zeta+l_\eta\eta+l_\xi\mathcal{Z})-\omega t]}\,, \tag{35}$$

in which $l_\zeta^2+l_\eta^2+l_\xi^2=1$, and for small k, we can expand $\varphi(\mathcal{Z})$ and ω as follows

$$\varphi(\mathcal{Z})=\varphi_0(\mathcal{Z})+k\varphi_1(\mathcal{Z})+k^2\varphi_2(\mathcal{Z})+\cdots\,, \tag{36}$$

$$\omega=k\omega_1+k^2\omega_2+\cdots\,. \tag{37}$$

After some algebric calculation the linearized ZK equation can be expressed as

$$\frac{\partial\psi}{\partial t}-U_0\frac{\partial\psi}{\partial\mathcal{Z}}+\delta_1\phi_0\frac{\partial\psi}{\partial\mathcal{Z}}+\delta_1\psi\frac{\partial\phi_0}{\partial\mathcal{Z}}+\delta_2\frac{\partial^3\psi}{\partial\mathcal{Z}^3}+\delta_3\phi_0\frac{\partial\psi}{\partial\zeta}+\delta_4\frac{\partial^3\psi}{\partial\zeta^3}+\delta_5\frac{\partial^3\psi}{\partial\mathcal{Z}^2\partial\zeta}+\delta_6\frac{\partial^3\psi}{\partial\mathcal{Z}\partial\zeta^2}+\delta_7\frac{\partial^3\psi}{\partial\mathcal{Z}\partial\eta^2}+\delta_8\frac{\partial^3\psi}{\partial\zeta\partial\eta^2}=0\,. \tag{38}$$

We have to find the expression of ω_1 by solving the zeroth, first, and second-order equations obtained from Eqs. (35)–(38). After integration, we can write the zeroth-order equation as

$$(-U_0+\delta_1\phi_0)\varphi_0+\delta_2\frac{{\rm d}^2\varphi_0}{{\rm d}\mathcal{Z}^2}=C\,, \tag{39}$$

where C is an integration constant. The solutions of the homogeneous part of the Eq. (38) can be written as

$$f=\frac{{\rm d}\phi_0}{{\rm d}\mathcal{Z}}\,,\quad g=f\int^{\mathcal{Z}}\frac{{\rm d}\mathcal{Z}}{f^2}\,. \tag{40}$$

So, the general solution of this zeroth-order equation is as

$$\varphi_0=C_1f+C_2g-Cf\int^{\mathcal{Z}}\frac{g}{\delta_2}{\rm d}\mathcal{Z}+Cg\int^{\mathcal{Z}}\frac{f}{\delta_2}{\rm d}\mathcal{Z}\,, \tag{41}$$

where C_1 and C_2 are two integration constants. Now, evaluating all integrals, the general solution of this zeroth-order equation, for φ_0 not tending to $\pm\infty$ as $\mathcal{Z}\to\pm\infty$, can finally be simplified to

$$\varphi_0=C_1f\,. \tag{42}$$

The first-order equation, i.e. the equation with terms linear in k, obtained from Eqs. (35)–(38), and (42), after integration, it can be expressed as

$$(-U_0+\delta_1\phi_0)\varphi_1+\delta_2\frac{{\rm d}^2\varphi_1}{{\rm d}\mathcal{Z}^2}={\rm i}C_1(\alpha_1+\beta_1\tanh^2\kappa\mathcal{Z})\phi_0+K, \tag{43}$$

where K is another integration constant, and α_1 and β_1 are given by

$$\alpha_1=(\omega_1+l_\xi U_0)-\frac{1}{2}\phi_m\mu_1+2\kappa^2\mu_2\,,$$
$$\beta_1=\frac{1}{2}\phi_m\mu_1-6\kappa^2\mu_2\,,\quad \mu_1=\delta_1l_\xi+\delta_3l_\zeta\,,$$
$$\mu_2=3\delta_2l_\xi+\delta_5l_\zeta\,. \tag{44}$$

Now, following the same procedure, the general solution of this first-order equation, for φ_1 not tending to $\pm\infty$ as $\mathcal{Z}\to\pm\infty$, can be written as

$$\varphi_1=K_1f+\frac{{\rm i}C_1}{8\delta_2\kappa^2}\Big[(\alpha_1+\beta_1)\mathcal{Z}f+\frac{2}{3}(3\alpha_1+\beta_1)\phi_0\Big]\,. \tag{45}$$

The second-order equation, i.e. the equation with terms involving k^2, obtained from Eq. (38) after substituting Eqs. (35)–(37), can be written as

$$\Big[-U_0\frac{\rm d}{{\rm d}\mathcal{Z}}+\delta_1\frac{\rm d}{{\rm d}\mathcal{Z}}\psi_0+\delta_2\frac{{\rm d}^3}{{\rm d}\mathcal{Z}^3}\Big]\varphi_2={\rm i}\omega_2\varphi_0+{\rm i}(\omega_1+l_\xi U_0)\varphi_1-{\rm i}\mu_1\psi_0\varphi_1+\mu_3\frac{{\rm d}\varphi_0}{{\rm d}\mathcal{Z}}-{\rm i}\mu_2\frac{{\rm d}^2\varphi_1}{{\rm d}\mathcal{Z}^2}\,, \tag{46}$$

where

$$\mu_3=3\delta_2l_\xi^2+2\delta_5l_\zeta l_\xi+\delta_6l_\zeta^2+\delta_7l_\eta^2\,. \tag{47}$$

The solution of this second-order equation exists if the right-hand side is orthogonal to a kernel of the operator adjoint to the operator

$$-U_0\frac{\rm d}{{\rm d}\mathcal{Z}}+\delta_1\frac{\rm d}{{\rm d}\mathcal{Z}}\phi_0+\delta_2\frac{{\rm d}^3}{{\rm d}\mathcal{Z}^3}\,. \tag{48}$$

This kernel, which must tend to zero as $\mathcal{Z}\to\pm\infty$, is $\phi_0=\phi_m{\rm sech}^2(\kappa\mathcal{Z})$. Thus we can write the following equation determining ω_1 as follows

$$\int_{-\infty}^{\infty}\phi_0\Big[{\rm i}\omega_2\varphi_0+{\rm i}(\omega_1+l_\xi U_0)\varphi_1-{\rm i}\mu_1\phi_0\varphi_1+\mu_3\frac{{\rm d}\varphi_0}{{\rm d}\mathcal{Z}}-{\rm i}\mu_2\frac{{\rm d}^2\varphi_1}{d\mathcal{Z}^2}\Big]{\rm d}\mathcal{Z}=0\,. \tag{49}$$

Now, substituting the expressions for φ_0 and φ_1 given by Eqs. (48) and (51), and after integration, we arrive at the following dispersion relation

$$\omega_1 = \Omega - l_\xi U_0 + (\Omega^2 - \Upsilon)^{1/2}\,, \tag{50}$$

where

$$\Omega = \frac{2}{3}(\phi_m \mu_1 - 2\mu_2 \kappa^2)\,, \tag{51}$$

$$\Upsilon = \frac{16}{45}(\phi_m^2 \mu_1^2 - 3\phi_m \mu_1 \mu_2 \kappa^2 - 3\mu_2^2 \kappa^4 + 12\delta_2 \mu_3 \kappa^4)\,. \tag{52}$$

It is clear from the dispersion relation (50) that there is always instability if $(\Upsilon - \Omega^2) > 0$. Thus, using Eqs. (32), (36), (44), (47), (51), and (52), one can express the instability criterion as[67–68]

$$S_i > 0\,, \tag{53}$$

where S_i can be expressed as

$$S_i = l_\eta^2(\omega_{\rm cpc}^2 + \sin^2\delta) + l_\zeta^2\Big[\omega_{\rm cpc}^2\Big(1 - \frac{5}{3}\tan^2\delta\Big) - \frac{5}{3}\Big]\,. \tag{54}$$

If this instability criterion $S_i > 0$ is satisfied, the growth rate $\Gamma = \sqrt{(\Upsilon - \Omega^2)}$ of the unstable perturbation of these PASWs is given by

$$\Gamma = \frac{2U_0}{15^{1/2}}\frac{\sqrt{S_i(1 + \omega_{\rm cpc}^2)}}{\sin^2\delta + \omega_{\rm cpc}^2}\,. \tag{55}$$

Equation (55) represents that the growth rate (Γ) of the unstable perturbation is a linear function of PASWs speed (U_0), but a nonlinear function of the propagating angle (δ), cold positron-cyclotron frequency ($\omega_{\rm cpc}$) and direction cosines (l_ζ and l_η).

5 Linear Wave Analysis

We have derived the linear dispersion relation for PASWs to evaluate the characteristics of the linear waves. By linearizing equation (29), we can write

$$\frac{\partial \phi^{(1)}}{\partial t} + \frac{A}{2}\frac{\partial}{\partial Z}\Big[\frac{\partial^2}{\partial Z^2} + D\Big(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\Big)\Big]\phi^{(1)} = 0\,. \tag{56}$$

We assume that the variation of the dispersion relation (ω/k) in the transverse dimensions (the X and Y directions) is much slower than that of the Z direction. Afterward, we can neglect the transverse dimensions, i.e., $\partial/\partial X = \partial/\partial Y \longrightarrow 0$. Then from Eq. (61),

$$\frac{\partial \phi^{(1)}}{\partial t} + \frac{A}{2}\frac{\partial}{\partial Z}\Big[\frac{\partial^2 \phi^{(1)}}{\partial Z^2}\Big] = 0\,. \tag{57}$$

We first consider that perturbation varying as $\phi^{(1)} \propto {\rm e}^{-({\rm i}\omega + {\rm i}kZ)}$ in the small amplitude limit to derive the dispersion relation. Now, from Eq. (62), the dispersion relation for the linear ZK equation is given by,

$$\omega = \frac{1}{2}Ak^3\,. \tag{58}$$

It is seen from the dispersion relation that PASWs significantly modified by the the ratio of electron to cold positron number density (α), and with the ratio of hot positron to cold positron number density (σ).

6 Parametric Investigations

In this section, we will briefly discuss the effects of the variation of the relative number densities such as the ratio of electron to cold positron number density (α), the ratio of hot positron to cold positron number density (σ), and the obliqueness (δ) of the magnetic field on the basic properties of the PASWs such as the amplitude (ψ_m), the width (Δ), and the instability.

6.1 *Linear Properties*

The dispersion relation which is shown in Eq. (63) is graphically represented in Fig. 1. We find that the curve of the ultra-relativistic case is higher than that of the non-relativistic case. The value of ω increases more rapidly with the increase of the value of k for ultra-relativistic case than the non-relativistic case.

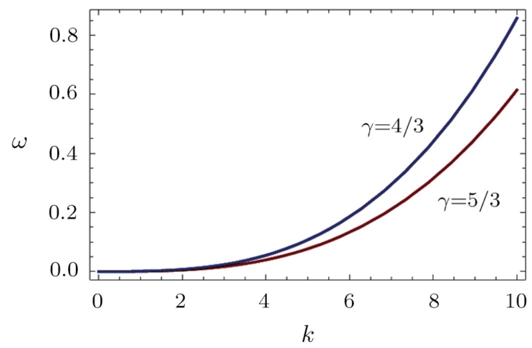

Fig. 1 (Color online) The variation of the angular frequency (ω) with wave number (k) of the PASWs for the non-relativistic (red) and ultra-relativistic (blue) cases.

6.2 *Nonlinear Properties*

(i) Effect of electron to cold positron number density ratio α:

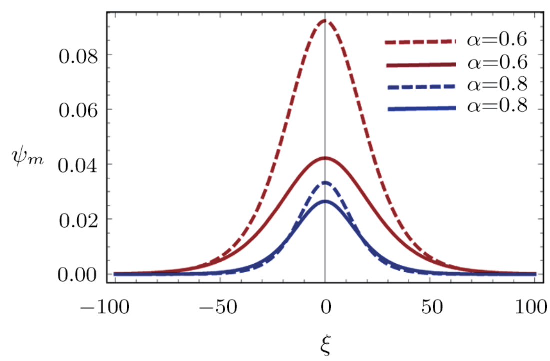

Fig. 2 The variation of amplitude (ψ_m) of the PASWs for different values of α. The dashed curves are for the ultra-relativistic case where the solid curves are for the non-relativistic case.

The effect of electron to cold positron number density ratio (α) on the amplitude (ψ_m) of the PASWs profile is shown in Fig. 2. It is found that the variation of ψ_m for ultra-relativistic ($\gamma = 4/3$) case is higher than the non-relativistic ($\gamma = 5/3$) case. The variation of ψ_m for different values of α is depicted Fig. 2. It is found that the ψ_m decreases (increases) with the increase of the value of electron number density (cold positron number density). The variation of Δ with respect to the variation of δ and α is shown in Figs. 5 and 6. It has been clear from our observation that ψ_m as well as Δ is always greater for the ultra-relativistic case than the non-relativistic case. After reaching a certain value, Δ begins to decrease with the increase of δ, and becomes zero at $\delta = 90°$.

(ii) Effect of hot positron to cold positron number density ratio σ:

The variation of the PASWs profile ψ_m and ξ with σ is shown in Fig. 3 for both non-relativistic and ultra-relativistic cases. It is seen that ψ_m decreases with the increase of the value of σ. The variation of Δ with respect to the variation of δ and σ are depicted in Figs. 7 and 8. The Δ increases with δ and reaches to it is maximum value with increasing of δ. After reaching a certain value it begins to decrease and becomes zero at $\delta = 90°$. It has been clear from our observation that Δ is always greater in ultra-relativistic case than the non-relativistic case.

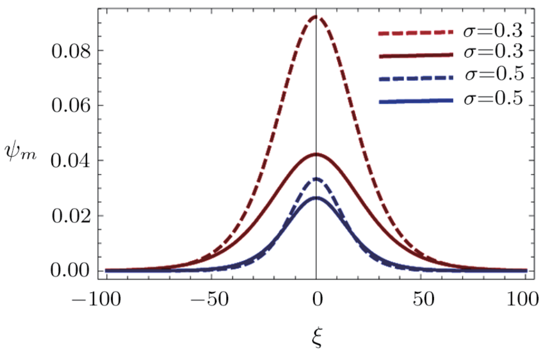

Fig. 3 The variation of amplitude (ψ_m) of the PASWs for different values of σ. The dashed curves are for the ultra-relativistic case where the solid curves are for the non-relativistic case.

(iii) Effect of obliqueness parameter δ:

It is assumed that the external magnetic field is directed along the z-axis, i.e., $B_0 = B_0\hat{z}$ and the propagation is in the x–y plane. It is seen that the magnitude of the external magnetic field has no effect on the amplitude of the solitary waves. However, it does have an effect on the width of these solitary waves. The impact of the external magnetic field B_0 through cold positron cyclotron frequency $\omega_{\rm cpc}$ on the width of PASWs has been observed and found that the width of the K-dV soliton increases with the decreasing value of $\omega_{\rm cpc}$ for the non-relativistic and the ultra-relativistic limits. It is shown that, as we increase the magnitude of the magnetic field, the width of these solitary waves decreases, i.e. the external magnetic field makes the solitary structures more spiky. The variation of Δ with δ (α and σ) for non-relativistic and ultra-relativistic case is represented in Fig. 4 (Figs. 5–8). Δ increases with the increase of the value of δ (from $0° - 55°$) but begins to decrease for the values which lies within ($55° - 90°$). It should be mentioned here that the maximum obliqueness i.e. when $\delta \longrightarrow 90°$, the $\Delta \longrightarrow 0$ and ψ_m becomes ∞. So, the assumption that are electrostatic will no longer be valid, and fully electromagnetic theory is needed.

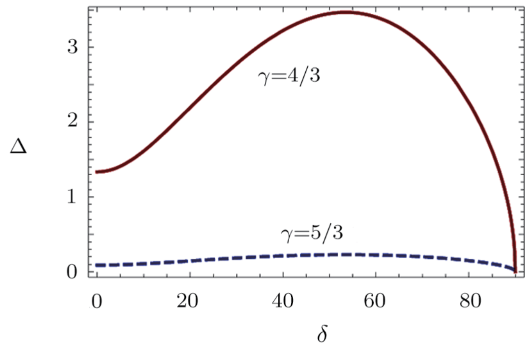

Fig. 4 Comparison between width (Δ) and obliqueness parameter (δ) in considering non-relativistic (dotted curve) and ultra-relativistic (solid curve) cases.

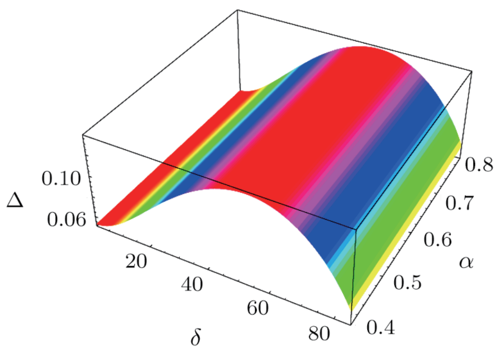

Fig. 5 The variation of width (Δ) of the potential associated with the PASWs with the obliqueness parameter (δ) and the ratio of electron to cold positron number density (α) considering non-relativistic case.

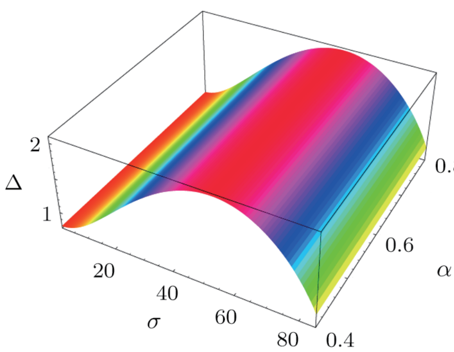

Fig. 6 The variation of width (Δ) of the potential associated with the PASWs with the obliqueness parameter (δ) and the ratio of electron to cold positron number density (α) considering ultra-relativistic case.

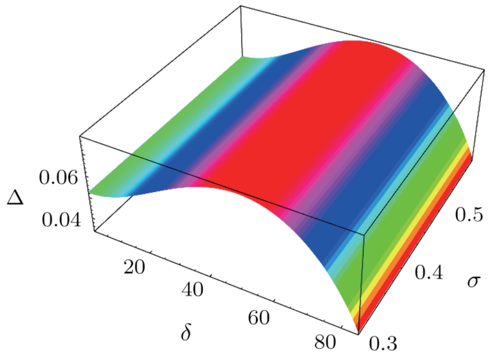

Fig. 7 Showing the profile of the PASWs along with the variation of the width (Δ) and the obliqueness (δ) with σ considering non-relativistic case.

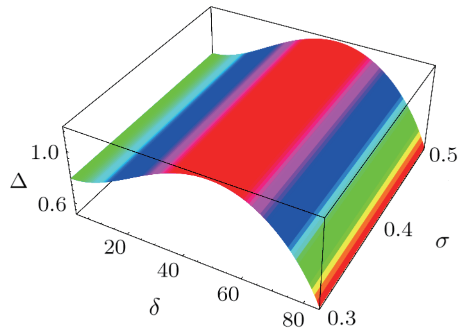

Fig. 8 Showing the profile of the PASWs along with the variation of the width (Δ) and the obliqueness (δ) with σ considering ultra-relativistic case.

(iv) Instability of the PASWs:

We have graphically obtained the parametric regimes by the $S_i = 0$ surface plots (Figs. 9–12) above which the PASWs become unstable, and below which the PASWs become stable. These show the variation of the parametric regimes which play an important role for the instability of the PASWs indicating that for the parameters above the surface, the PASWs become unstable. It is seen that $\omega_{\rm cpc}$ increases with the increase of both δ and l_ζ shown in Fig. 9. The increment of the values of δ and l_ζ gives a clear indication that the value of $\omega_{\rm cpc}$ increases for which the PASWs become unstable. $S_i = 0$ surface plot showing the variation of $\omega_{\rm cpc}$ with l_ζ and l_η for $\delta = 15°$ which is represented in Fig. 10. This indicates that as the value of l_ζ and l_η increase, the value of $\omega_{\rm cpc}$ for which the solitary waves become unstable decreases. The nonlinear variations of Γ with l_ζ, l_η, $\omega_{\rm cpc}$, and δ are shown in Figs. 11–12. The variation of Γ with l_ζ, and l_η is depicted in Fig. 11. It is clear from this that the unstable perturbation increases as the increasing of both l_ζ and l_η. With the increasing of the values of $\omega_{\rm cpc}$ and δ, the value of Γ decreases which is depicted in Fig. 12. It is found that α and σ have no any effect on the instability or growth rate of the PASWs.

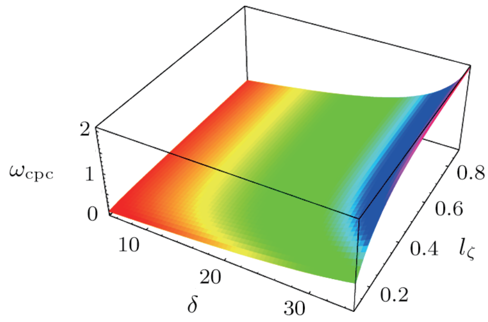

Fig. 9 Plot $S_i = 0$. The variation of $\omega_{\rm cpc}$ with δ and l_ζ for the parameter $l_\eta = 0.10$.

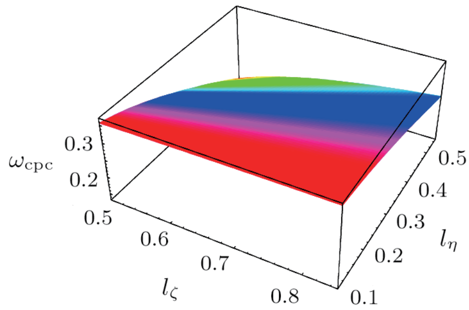

Fig. 10 Plot S_i=0. The variation of $\omega_{\rm cpc}$ with l_ζ and l_η for the parameter $\delta = 15$.

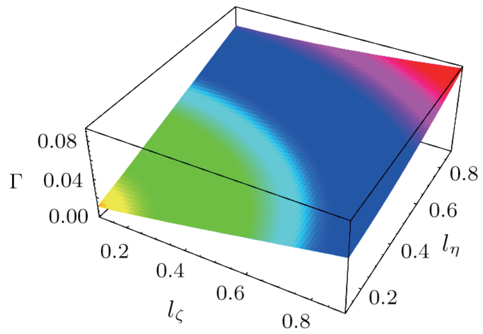

Fig. 11 The variation of Γ with l_ζ and l_η for the parameters U_0=0.1, δ=15, $\omega_{\rm cpc} = 0.80$.

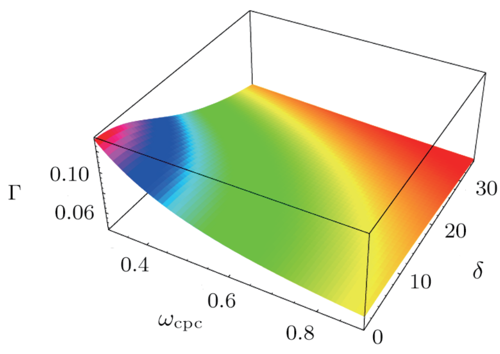

Fig. 12 The variation of Γ with $\omega_{\rm cpc}$ and δ for the parameters U_0=0.1, l_η=0.70, and l_ζ=0.20.

7 Conclusion

To summarize, the propagation of the PASWs in a magnetized EPI plasma (containing inertial cold positrons, relativistic degenerate electrons and hot positrons, and negatively charged immobile heavy ions) has been theoretically investigated. We have studied the basic properties of the PASWs by analyzing the stationary solitary wave solution of the ZK equation, and finally analyzed the instability of these structures by small-k perturbation expansion method.

The solitary like structures, which we have predicted here, are due to balance between the nonlinearity and the dispersion, where the inertia comes from cold positron, and restoring force from the degenerate electron as well as hot positron. In our model, we used the following parameters for our numerical analysis, $U_0 = 0.01 - 0.1$, $\sigma = 0.1 - 0.8$, $n_{\rm so} = 1.0 \times 10^{25} - 10^{30}$, $\delta = 15$, $\alpha = 0.1 - 0.9$, and $\mu = 0.1 - 0.6$[38,44] and for instability analysis $\omega_{\rm cpc} = 0.2$ to 0.9, $l_\zeta = l_\eta = 0.1$ to 0.9, and $\delta = 0^\circ$ to 90°. However, the ranges of the plasma parameters are very wide and relevant to many dense plasma environments. Finally, we hope that our present investigation may help to analyze the formation, and the basic characteristics of PASWs structures in a relativistic degenerate EPI plasma which occurs in space and many astrophysical situations, especially in pulsar environments.[38]

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