Commun. Theor. Phys. **68** (2017) 111–116 Vol. 68, No. 1, July 1, 2017

Modified Potential Around a Moving Test Charge in Strongly Coupled Dusty Plasma

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(Received June 30, 2016; revised manuscript received December 3, 2016)

Abstract *The theory of dynamical (wake) potential behind a moving test charge in a weakly coupled dusty plasma is extended to that including of strong interaction between dust grains. Such strong interaction is included in the dielectric response function by a generalized hydrodynamic (GH) fluid model. It is shown that the strong interaction between dusts including the lattice spacing correction has a significant effect on the wake potential in dusty plasma. This may be used to investigate basic features of phase transition and possibility of lattice formation of dusty plasma.*

PACS numbers: 52.25.Mq, 52.27.Lw, 52.27.Gr **DOI:** 10.1088/0253-6102/68/1/111

Key words: dusty plasma, strongly coupled plasma, generalized hydrodynamic fluid model

1 Introduction

The interaction potential between the dust grains as a fundamental subject in dusty plasma systems has been attracted a great deal of attention during the last decades.[1−13] The interaction potential between the dust grains under the special assumptions can be estimated by Debye shielded Coulomb (DSC) interaction.[1] But, the assumption that the interaction potential is in the form of the DSC interaction is not always realistic. The influence of streaming of plasma species,[5−6,12] external magnetic field,[6,12,14−15] dust charge fluctuation,[4,16−17] quantum effects,[8−9,13,18] and non-Maxwellian components[10−11,19] on the interaction potential have been examined.

In the presence of a static or slowly moving test charge particulate, the plasma medium may be polarized which leads to the shielding of test charge. Thus, the potential around the test charge particle can be divided to a short-range Debye–Hückle potential and also a long-range far-field one. Montgomery *et al.*[2] have studied the far-field potential of a moving test charge in a collisionless plasma. They showed that this potential fall-off as the inverse cube of the distance. On the other hand, the resonances behavior of the test charge through the interaction with a plasma wave, giving rise to an oscillatory wake-field behind the test particle. Such wake potential has been introduced first by Nambu and Akama.[3] They showed that the dust grains with same polarity can attract each other due to the collective interactions involving the dust acoustic waves. Such attractive interaction can provide the possibility of lattice formation[20−22] in Coulomb system.[23−24] Shukla and Rao[25] have discussed the possibility of Coulomb crystallization due to the such attractive force in a colloidal plasmas with streaming ions. Shukla and Malandsø[26] have investigated the potential of a test particle in the presence of electro-acoustic waves in a uniform unmagnetized plasma comprising fixed ions and two-distinct groups of electrons. They found also a wake potential behind the test particle. Lemons *et al.*[27] have investigated a two-dimensional wake potential in planar and cylindrical geometry. They analyzed the wakefield structures in the limit of large downstream coordinate to obtain an analytical expression for the wakefields. The influence of external magnetic field has been also examined on the wake potential.[14−15] Ali[28] extended the work to study the Debye and wake potentials of a test charge in a multi-electron plasma to incorporate the impact of hot electron current in the perspective of thermal equilibrium. Recently, Ali[29] has studied the potential distribution around a test charge in a positive dust-electron plasma. They showed that by imposing the certain conditions on the velocity of the test charge, the electrostatic potential can be decomposed into the Debye–Hückel, wake-field and far-field potentials that are significantly modified in the limit of a large dust-charge relaxation rate. Very recently, Eliasson and Akbari–Moghanjoughi[13] have investigated the shielding potential around a test charge in a finite temperature plasma. They showed that an attractive force may be appeared between the shielded ions in an arbitrary degenerate plasma below a critical temperature and density.

One of the most interesting and fascinating aspects of a laboratory dusty plasma is that dust grains are strongly coupled (i.e. $\Gamma \gg 1$, where Γ is a coupling parameter defined as $\Gamma = q_d^2 \exp(-a_d/\lambda_d)/a_d k_B T_d$ in which q_d, a_d, T_d and λ_d are, respectively, the dust grain charge, the inter-particle distance, the dust kinetic temperature, and the dust Debye length. The physics of strongly coupled plasma is of considerable interest because of its possible application to white dwarf matter, plasmas produced by

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laser compression of matter or in nuclear explosions, interior of heavy planets and non ideal plasmas for industrial applications. The influence of strong correlations between the dust particles on the low frequency collective modes has been investigated theoretically[30−38] and experimentally.[39]

The early research on the interaction potential mainly concentrated on the plasma systems at low Γ. But, in the past years motivated by the application of strongly coupled dusty plasma, the authors have studied some aspects of such high value Γplasma. However, the wake potential effects are less well studied. In this paper, we adopt the viscoelastic description to study the dynamical (wake) potential in a dusty plasma including the strong interaction between dust particles.

The layout of this paper is as follows: After introduction, the basic equations governing our plasma model are presented in Sec. 2. Then, dynamical potential is derived to in Sec. 3. Finally, a discussion is provided in Sec. 4.

2 Dielectric Response Function

Let us consider a strongly coupled dusty plasma $1 < \Gamma < \Gamma_c$, which shows an intermediate behavior between the fluid and solid sates. There are several theoretical approaches for investigation of such viscoelastic system, such as the generalized hydrodynamic model,[30,32,40] the quasi-localized charge approximation[41] the localized corrections method[42−43] and the fluid[44−46] models. The viscoelastic description of the electrostatic response of strongly coupled dusty plasma medium can be provided by employing the generalized hydrodynamic model (GH). This phenomenological model may be generally remained valid over a wide range of the coupling parameterΓ, as long as the plasma retains its fluid characteristics. The model breaks down in the crystalline state, where convection ceases and the formation of a lattice structure introduces additional symmetries. Typically, the viscoelastic coefficients are functions of the coupling parameter Γ. As for the weakly coupled plasma regime ($\Gamma < 1$), these coefficients simply lead to viscous damping of the collective modes. While in opposite regime ($1 < \Gamma$), the viscoelastic coefficients give the contributions of elasticity effects into the restoring force.

The dielectric response function of dusty plasma for the dust acoustic modes including the strong correlation effect is given by[30]

$$\varepsilon(\omega,k) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pd}^2}{\omega^2 - \gamma_d\mu_d\lambda_d^2k^2 + \mathrm{i}\omega k^2[\eta*/(1-\mathrm{i}\omega\tau_m)]}\,, \tag{1}$$

where $\eta^* = (\zeta + 4\eta/3)/m_d n_{d0}$; ζ, η, τ_m, γ_d, μ_d, λ_d, and λ_D are respectively the bulk and shear viscosity, the relaxation time, adiabatic index, the compressibility, the dust Debye length, and the plasma Debye length. By setting $\varepsilon(\omega,k) = 0$, we can obtain the DA frequency in a strongly coupled dusty plasma as

$$\omega^2 = \gamma_d\mu_d\lambda_d^2k^2 + \frac{k^2\lambda_D^2\omega_{pd}^2}{1+k^2\lambda_D^2} - \mathrm{i}\omega\eta * k^2, \quad \omega\tau_m \ll 1\,, \tag{2a}$$

$$\omega^2 = \gamma_d\mu_d\lambda_d^2k^2 + \frac{k^2\lambda_D^2\omega_{pd}^2}{1+k^2\lambda_D^2} + \frac{\eta*}{\tau_m}k^2, \quad \omega\tau_m \gg 1\,. \tag{2b}$$

In the following we restrict our attention to the limit of $\omega\tau_m \gg 1$ (i.e., kinetic regime), which refers to the dusty plasma toward a viscous liquid/solid like behavior. Note that, the kinetic regime is valid only in the weak coupling regime or in the very strong coupling regime where the condition $\omega\tau_m \gg 1$ can be fulfilled. This comes from this simple fact that τ_m which is proportional to η^* closely follow its behavior and thus it takes high values in the strong coupling as well as the weak coupling regimes. For $1 \le \Gamma < 10$, the relaxation time τ_m is of order unity and thus the essential condition of "kinetic regime" cannot be satisfied.

Accordingly, Eq. (1) reduces to

$$\varepsilon(\omega,k) = 1 + \frac{1}{k^2\lambda_D^2} - \frac{\omega_{pd}^2}{\omega^2 - \gamma_d\mu_d\lambda_d^2k^2 - (\eta*/\tau_m)k^2}\,. \tag{3}$$

Then, by introducing $\chi = \gamma_d\mu_d\lambda_d^2 + \eta^*/\tau_m$, Eq. (3) can be written as

$$\varepsilon(\omega,k) = \frac{\omega_k^2}{\omega_{pd}^2} - \frac{\omega_{pd}^2}{\omega^2 - \chi k^2}\,, \tag{4}$$

where $\omega_k = c_s k/\sqrt{1+k^2\lambda_D^2}$ is the dispersion relation of DA waves and $c_s = \lambda_D\omega_{pd}$ is DA speed.

3 Dynamical Potential

In order to obtain the electrostatic potential around a test dust particle in the presence of electrostatic modes in a strongly coupled dusty plasma, we use the Γ corrected dielectric response function (which introduced by Eq. (1)), to find[47]

$$\Phi(r,t) = \int \frac{q_t}{2\pi^2k^2}\frac{\delta(\omega - k\cdot v)}{\varepsilon(\omega,k)}\exp[\mathrm{i}k\cdot(r - vt)]\,\mathrm{d}k\,\mathrm{d}\omega\,, \tag{5}$$

where q_t and v_t are respectively the charge and the velocity vector of the test dust particle. The reciprocal of the dielectric function associated with the DA oscillations can be written from Eq. (4) of the form

$$\frac{1}{\varepsilon(\omega,k)} = \frac{k^2\lambda_D^2}{1+k^2\lambda_D^2}\Big[1 + \frac{\omega_k^2}{\omega^2 - \chi k^2 - \omega_k^2}\Big]\,. \tag{6}$$

Then, by substituting of expression (6) into Eq. (5), the consequence integral can be decomposed into the Debye–

Hückel ($\Phi_{\rm DH}$) and wake-field (Φ_W) potentials. The first one leads to the well-known Debye–Hückel screening potential, and the latter potential term causing due to the DA oscillations through the motion of test charge, is a wake potential, given by

$$\Phi_W = \frac{q_t}{2\pi^2}\int \frac{\lambda_D^2}{1+k^2\lambda_D^2}\frac{\omega_k^2}{\omega^2-\chi k^2-\omega_k^2}\exp[\mathrm{i}k\cdot(r-vt)]\delta(\omega-k\cdot v)\,\mathrm{d}k\,\mathrm{d}\omega\,. \tag{7}$$

In order to find an analytical form for the wake-field potential, let us consider Eq. (8) in the cylindrical coordinate. Thus, Eq. (7) reduces to

$$\Phi_W = \frac{q_t}{\pi}\int k_\perp\,\mathrm{d}k_\perp\,\mathrm{d}k_{||}\,\mathrm{d}\omega\frac{\lambda_D^2}{1+k^2\lambda_D^2}\frac{\omega_k^2}{\omega^2-\omega_k'^2}J_0(k_\perp\rho)\exp(\mathrm{i}k_{||}z-\mathrm{i}\omega t)\delta(\omega-k_{||}v)\,, \tag{8}$$

where z, ρ, $k_{||}$, $k_\perp$, and J_0, are respectively the cylindrical coordinates of the field point, the wave number component parallel and perpendicular to the z axis, and the zeroth order Bessel function, also $\omega_k' = (\chi k^2+\omega_k^2)^{1/2}$. Then, by performing $k_{||}$ integration, the following expression is obtained from Eq. (8)

$$\Phi_W = \frac{q_t\lambda_D^2}{\pi v}\int J_0(k_\perp\rho)k_\perp\,\mathrm{d}k_\perp\int\frac{1}{1+k_\perp^2\lambda_D^2+\omega^2\lambda_D^2/v^2}\frac{\omega_k^2}{(\omega-\omega_k')(\omega+\omega_k')}\exp(\mathrm{i}\omega z/v-\mathrm{i}\omega t)\,\mathrm{d}\omega\,. \tag{9}$$

Here, we keep in mind that both the characteristics frequencies ω_k and ω_k' are changing through the performing of integration. Equation (9) is include of two types of singularities as $\omega_{\rm I} = \pm\mathrm{i}v(1+k_\perp^2\lambda_D^2)^{1/2}/\lambda_D$ and $\omega_{\rm II} = \pm\omega_k'$. The first type of singularity leads to the Colombian part of Φ_W, while the oscillating behavior (wake-field potential) comes from the latter one. Since we are interested now in the oscillating contribution to the collective potential, we restricted our integration to the residues at the poles at $\omega_{\rm II} = \pm\omega_k'$. On carrying out the ω integrations, we obtain from Eq. (9)

$$\Phi_W = \frac{2q_t\lambda_D^2}{v}\int\frac{1}{1+k_\perp^2\lambda_D^2+\omega_k'^2\lambda_D^2/v^2}\frac{\omega_k^2}{\omega_k'}J_0(k_\perp\rho)\sin\Big[\frac{\omega_k'}{v}(z-vt)\Big]k_\perp\,\mathrm{d}k_\perp\,. \tag{10}$$

Now, in the limit of long wavelength, the value of ω_k and ω_k' in Eq. (10) are given by $\omega_k^2 = k_\perp^2c_s^2/[1-(\chi+c_s^2)/v^2]$ and $\omega_k'^2 = (\chi+c_s^2)k_\perp^2/[1-(\chi+c_s^2)/v^2]$. Then, we consider the $k_\perp$ dependence of both the characteristics frequencies ω_k and ω_k' in Eq. (10), in the long wavelength limit we obtain

$$\Phi_W = \frac{2q_t\lambda_D^2}{v}\frac{c_s^2}{\sqrt{(\chi+c_s^2)[1-(\chi+c_s^2)/v^2]}}\int J_0(k_\perp\rho)\sin\Big[\frac{\sqrt{\chi+c_s^2}}{\sqrt{v^2-(\chi+c_s^2)}}k_\perp(z-vt)\Big]k_\perp^2\,\mathrm{d}k_\perp\,. \tag{11}$$

The main term of integral expression (11) leads us to the following closed form for the wake-field potential

$$\Phi_W = \frac{2q_tc_s^2}{(\chi+c_s^2)|z-vt|}\cos\Big[\frac{(z-vt)}{L_D}\Big]\,, \tag{12}$$

where $L_D = \lambda_D\sqrt{v^2-(\chi+c_s^2)}/\sqrt{\chi+c_s^2}$ represents new lattice spacing in the present strongly coupled dusty plasma. The wake potential (13) shows an attractive behavior for negative values of its oscillatory part, i.e., $\cos[(z-vt)/L_D] < 0$. In the limit $\chi\to 0$, the dynamical potential (12) turns to the usual wake-potential with $\Phi_W = 2q_t\cos[c_s(z-vt)/\lambda_D\sqrt{v^2-c_s^2}]/|z-vt|$.

Here we need to estimate the new lattice spacing L_D via the parameters η^*, τ_m, and μ_d. However these parameters are difficult to obtain from basic theories, but in certain circumstances it is possible to obtain approximate to expressions from simulation.[48–49] From Table V of Ref. [49], the normalized shear viscosity η^*has the typical values $\eta^*\approx 1.04$ for $\Gamma = 1$, $\eta^*\approx 0.08$ for $\Gamma = 10$, and $\eta^*\approx 0.3$ for $\Gamma = 160$. However, approximate expressions can be constructed from the results of theories and numerical simulations like MD. For example, for the range $10\le\Gamma\le 200$, Xie and Chen[50] have shown that one can obtain from Table V and Fig. 33 of Ref. [49] the functional relation of $\eta^* = 0.02\sqrt{\Gamma}$ from a one component plasma theory. Also, the excess internal energy of the system has been estimated for $10\le\Gamma\le 200$ as $u(\Gamma)\cong -0.90\Gamma+0.95\Gamma^{1/4}+0.19\Gamma^{-1/4}-0.81$[51] and for $\Gamma < 1$ as $u(\Gamma)\cong -\sqrt{3}\Gamma^{3/2}/2$.[52] Thus, based on the above expressions the relaxation time $\tau_m = \eta^*/\lambda_d^2[1-\gamma_d\mu_d+4u(\Gamma)/15]$ and the compressibility $\mu_d = 1+u(\Gamma)/3+\Gamma\partial_\Gamma u(\Gamma)/9$, can be estimated for different coupling regime. Therefore, the lattice spacing and the dynamical potential as a function of coupling parameter take the following forms

$$L_D = \lambda_D^2\sqrt{v^2/c_s^2-[\lambda_d^2+4\lambda_d^2u(\Gamma)/15+\lambda_D^2]/\lambda_D^2}\Big/\sqrt{\lambda_d^2+4\lambda_d^2u(\Gamma)/15+\lambda_D^2}\,, \tag{13}$$

$$\Phi_W = \frac{2q_t\lambda_D^2}{[\lambda_d^2+4\lambda_d^2u(\Gamma)/15+\lambda_D^2]|z-vt|}\cos\Big[\frac{\lambda_D(z-vt)}{\sqrt{\lambda_d^2+4\lambda_d^2u(\Gamma)/15+\lambda_D^2}}\Big]\,. \tag{14}$$

In order to see how the effect of strong coupling affects the dynamical potential, we have depicted the wake potential (14) for different values of coupling parameter Γ, in Fig. 1. We have used the typical parameters:[50] $n_i \sim 10^9\ \mathrm{cm}^{-3}$; $n_d \sim 10^4\ \mathrm{cm}^{-3}$; $Z_d \sim 1$–5×10^3, $T_e \sim 1$ eV, $T_i \sim 0.1T_e$, $T_d \sim 0.01$–$0.02T_i$, which yield $\lambda_d/\lambda_D \sim 0.02$–$0.2$ and $\omega\tau_m \gg 1$. This figure shows that an increase in the coupling parameter Γ increases the extension of the wake potential in the space domain. This is an expected result because of increase of lattice spacing by increase of coupling parameter Γ. Furthermore, for the higher values of the dust Debye length (Fig. 1(a)), the increase of the coupling parameter leads to a decrease in the oscillating behavior of the wake potential in comparison with the higher values of the dust Debye length (Fig. 1(a)). For example, for $\Gamma = 100$ and $\lambda_d = 0.2\lambda_D$, the wake potential experiences a space extension with respect to the case of $\lambda_d = 0.1\lambda_D$, due to increase of lattice spacing by increase of λ_d. This turns out that the viscous damping behavior of plasma increases by increasing λ_d and Γ. On the other hand, the competition between the Debye–Hückel potential and wake potential has been plotted in Figs. 2(a) and 2(b), respectively for $\lambda_d = 0.1\lambda_D$ and $\lambda_d = 0.2\lambda_D$. The wake potential dominates over the Debye–Hückel potential outside the Debye sphere.

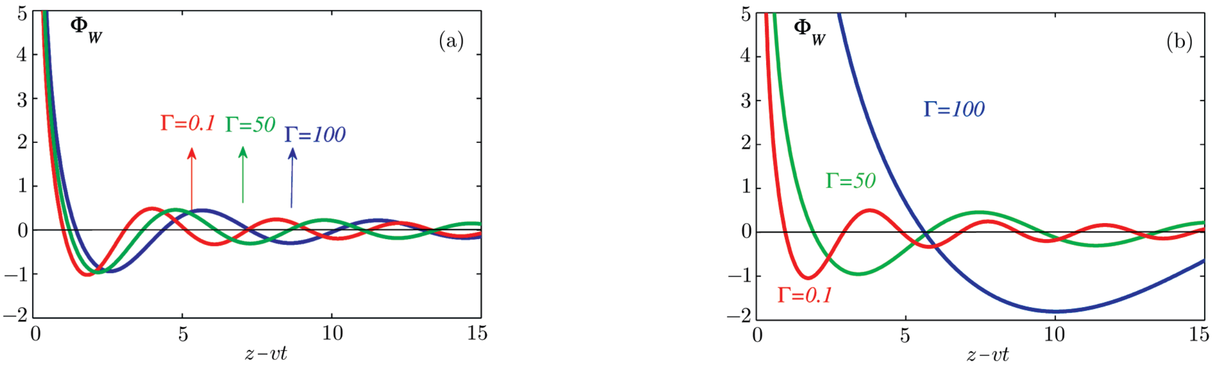

Fig. 1 The spatial variation of the wake potential along the z-axis for different values of coupling parameter Γ, for (a) $\lambda_d/\lambda_D = 0.1$ and (b) $\lambda_d/\lambda_D = 0.2$, with $v = 1.2c_s$. The typical parameters are[50] $n_i \sim 10^9\ \mathrm{cm}^{-3}$; $n_d \sim 10^4\ \mathrm{cm}^{-3}$; $Z_d \sim 1$–5×10^3, $T_e \sim 1$ eV, $T_i \sim 0.1T_e$, $T_d \sim 0.01$–$0.02T_i$.

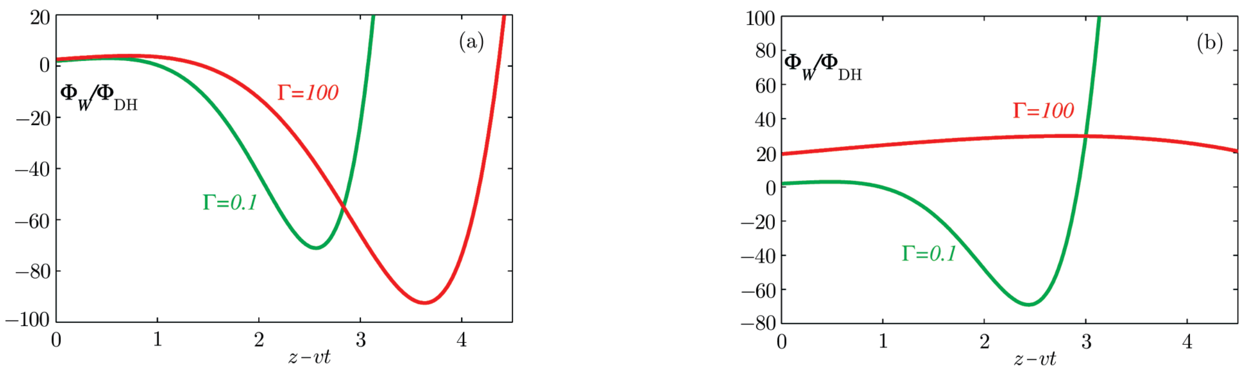

Fig. 2 Competition between wake and Debye–Hückle potentials in weakly and strongly coupled regimes, for (a) $\lambda_d/\lambda_D = 0.1$ and (b) $\lambda_d/\lambda_D = 0.2$, with $v = 1.2c_s$.

It seems appropriate here to add that the GH model closely agrees with the experimental analysis[53] of strongly coupled dusty plasma (particularly with regards to the damping contribution arising from the viscosity effects of strong correlations). But, validity of the GH model is restricted to the coupling parameter far from to the liquid-crystal curve on the phase diagram (i.e., $\Gamma \ll \Gamma_c$). In the limit of $\Gamma \leq \Gamma_c$, the GH model needs to some modifications, which make it applicable for $\Gamma \leq \Gamma_c$. Gozadinos *et al.*[54] have employed the fluid model to study of strongly coupled dusty plasma with coupling parameter close to the liquid-crystal curve on the phase diagram (i.e., $\Gamma \leq \Gamma_c$). They introduced the effective temperature concept which includes the strong correlation between the dust grains, which leads to an additional "dispersive" contributions in the dispersion relation. This is because of the excess pressure contributions (arising from the strong correlations) in the equation of state. As seen earlier this term can play a significant role at large values of Γ and is an important physical manifestation of strong correlation effects.[3−38,55−57]

4 Conclusions

To summarize, we have investigated the influence of strong interaction between dust grains on the dynamical potential in a dusty plasma system. We have adopted the test charge particle approach to derive the interaction potential of a moving test dust grain, taking into account the electric susceptibility that incorporates strong interaction between dust grains in dusty plasma. It is found the dynamical (attractive) potential is significantly affected by the strong interaction effect. Interestingly, it is found that the latter introduces a new lattice spacing L_D. We found that the extension of the wake potential in the space domain increases by increasing coupling parameter Γ. This is an expected result because of increase of lattice spacing by increase of coupling parameter Γ.

Note that the visco-elastic properties of astrophysical media are justifiable under the condition that the astrofluid is strongly coupled (i.e., $1 \ll \Gamma < \Gamma_c$).[58] Indeed, the astrophysical media which show the visco-elastic behavior obey such definition and accounted as large-scale cosmic fluids. What is the origin of attractive forces which can cause coagulation of the charged dust particles in different astrophysics media and thus leads to the formation of strongly coupled systems? The answer is here: the knowledge of wake field structures should help to understand the origin of attractive forces which can cause coagulation of the charged dust particles in the space plasma systems, such as Earth's lower ionosphere (in noctilucent clouds) and molecular clouds.[55] Finally, the present results may be useful for designing the laboratory experiments so that robust dust-Coulomb crystals, also in understanding the underlying physics of dust grain coagulation/agglomeration in space plasmas and astrophysical objects.

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