Commun. Theor. Phys. **70** (2018) 459–464 Vol. 70, No. 4, October 1, 2018

P-V Criticality of Born-Infeld AdS Black Holes Surrounded by Quintessence*

Chen-Hao Wu (吴晨昊),[1,†] De-Cheng Zou (邹德成),[1,2,‡] and Yue Wang (王越)[1]

[1]Center for Gravitation and Cosmology, College of Physical Science and Technology, Yangzhou University, Yangzhou 225009, China

[2]Institute of Basic Sciences and Department of Computer Simulation, Inje University, Gimhae 50834, Korea

(Received April 9, 2018; revised manuscript received May 30, 2018)

Abstract *We discuss the P-V criticality and phase transition in the extended phase space of Born-Infeld AdS (BI-AdS) black hole surrounded by quintessence dark energy, where the cosmological constant* Λ *is identified with the thermodynamical pressure* P*. Comparing with Van der Waals(VdW)-like SBH/LBH phase transition of Born-Infeld AdS (BI-AdS) black hole, we find that the BI-AdS black hole surrounded by quintessence dark energy possesses lower critical temperature because of parameter* $a > 0$*, even disappears since the parameter a taking enough large values leads to* $T_c \leq 0$*. Moreover, the interesting thermodynamic phenomenon of reentrant phase transition (RPT) are also observed, and the quintessence dark energy plays a similar role in this RPT.*

DOI: 10.1088/0253-6102/70/4/459

Key words: general relativity, quintessence, black hole thermodynamics

1 Introduction

Black hole thermodynamics has been an intriguing subject of discussions for many years. In view of AdS/CFT correspondence, the black hole thermodynamics in the presence of a negative cosmological constant become more interesting nowadays. Actually, phase transitions in asymptotically AdS black holes allow for a dual interpretation in the thermal conformal field theory (CFT) living on the AdS boundary the principal example being the well known radiation/Schwarzschild-AdS (SAdS) black hole Hawking-Page transition.[1] Then, the discussion of thermodynamics in AdS black holes has been generalized to the extended phase space, where the cosmological constant is identified with thermodynamic pressure and its variations are included in the first law of black hole thermodynamics.[2–3] In the extended phase space with cosmological constant and volume as thermodynamic variables, it was interestingly found that the system admits a more direct and precise coincidence between the first order Van der Waals(VdW)-like small black hole/large black hole (SBH/LBH) phase transition and the liquid-gas change of phase occurring in fluids.[4] In Ref. [5], it recovered the intermediate/small/large phase transitions in the four-dimensional Born-Infeld AdS (BI-AdS) black hole, which is reminiscent of reentrant phase transitions (RPTs) observed for multicomponent fluid systems, ferroelectrics, gels, liquid crystals, and binary gases, e.g., Ref. [6]. A system undergoes an RPT if a monotonic variation of any thermodynamic quantity results in two (or more) phase transitions such that the final state is macroscopically similar to the initial state. Moreover, this RPT also appears in the higher-dimensional rotating AdS black holes,[7–8] five-dimensional hairy AdS black hole,[9] higher-dimensional Gauss-Bonnet AdS black hole,[10–11] and higher-dimensional AdS black hole in dRGT massive gravity.[12]

In addition, it is widely believed that our universe is expanding with acceleration due to some unknown dark energy with negative pressure. It is also supported by some modern observational results[13–14] and may be explained by the existence of dark energy. Among the various dark energy model, we only concern with the existence of quintessence. Kislev first investigated black holes surrounded by quintessence.[15] More discussions in this direction can be found as well in quasinormal modes[16–18] and thermodynamic properties[19–20] of black holes, exact black hole solutions in Lovelock gravity[21–22] surrounded by quintessence matter. The generalization of the spherical quintessential solution to the axially symmetric case was also addressed.[23] It would be of interest to probe whether dark energy influences the thermodynamics of black holes in the extended phase space. The charged AdS black holes affected by quintessence was firstly calculated in Ref. [24]. Later, *P-V* criticality of Kerr-Newman AdS black hole with a quintessence field also was discussed in Ref. [25]. In this paper, we will discuss the *P-V* criticality and thermodynamics of Born-Infeld AdS black hole affected by quintessence.

This paper is organized as follows. In Sec. 2, the thermodynamics of Born-Infeld AdS black hole affected by quintessence will be introduced. In Sec. 3, we study the critical behaviors of Born-Infeld black hole affected by

*Supported by the National Natural Science Foundation of China under Grant No. 1160515, and Natural Science Foundation of Jiangsu Province under Grant No. BK20160452

†E-mail: chenhao_wu@outlook.com

‡Corresponding author, E-mail: dczou@yzu.edu.cn

quintessence. We end the paper with closing remarks in Sec. 4.

2 Solution of Born-Infeld AdS Black Holes Surrounded by Quintessence

We assume that the ansatz takes the following form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \quad (1)$$

which satisfies Einstein's field equation

$$G^{\mu}{}_{\nu} + \Lambda\delta^{\mu}{}_{\nu} = 8\pi T^{\mu}{}_{\nu} + 8\pi\tilde{T}^{\mu}{}_{\nu}\,, \quad (2)$$

$$\partial_\mu\Big(\frac{\sqrt{-g}F^{\mu\nu}}{\sqrt{1+F^2/2b^2}}\Big) = 0\,, \quad (3)$$

where $G^{\mu}{}_{\nu} = R^{\mu}{}_{\nu} - (1/2)R\delta^{\mu}{}_{\nu}$ is the Einstein tensor, $T^{\mu}{}_{\nu}$ denotes the quintessence matter (see Ref. [15] for further details) with

$$T^{t}{}_{t} = T^{r}{}_{r} = -\rho_q\,, \quad T^{\theta}{}_{\theta} = T^{\varphi}{}_{\varphi} = -\frac{1}{2}\rho_q(3\omega_q + 1)\,, \quad (4)$$

and the energy-momentum tensor $\tilde{T}^{\mu}_{\nu}$ of Born-Ineld field reads as

$$\tilde{T}^{\mu}_{\nu} = \frac{2F^{\mu\lambda}F_{\nu}{}^{\lambda}}{\sqrt{1+F_{\mu\nu}F^{\mu\nu}/2b^2}} + \frac{1}{2}\delta^{\mu}_{\nu}\mathcal{L}(\mathcal{F})\,,$$

$$\mathcal{L}(\mathcal{F}) = 4b^2\Big(1 - \sqrt{1 + \frac{F^{\mu\nu}F_{\mu\nu}}{2b^2}}\Big)\,.$$

Here $F_{\mu\nu}$ is the electromagnetic tensor, b is the Born-Infeld parameter, and the parameter ω_q describes the equation of state with $\omega_q = p/\rho$, where p and ρ are the pressure and energy density of the quintessence respectively, in which ω_q will not equal $-1/3$, -1 if $-1 < \omega_q < -1/3$ can explain the universe accelerating expansion.[15]

From Eqs. (2)–(3), we can easily obtain the black hole solution $f(r)$

$$f(r) = 1 - \frac{2M}{r} + \frac{2b^2r^2}{3}\Big(1 - \sqrt{1 + \frac{Q^2}{b^2r^4}}\Big) + \frac{4Q^2}{3r^2}\,{}_2F_1\Big[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{b^2r_+^4}\Big] - \frac{a}{r^{3w_q+1}} - \frac{\Lambda r^2}{3}\,, \quad (5)$$

where the parameters Q and M represent charge and mass of black hole, respectively. When $a = 0$, $f(r)$ reduces to the BI-AdS black hole solution. The normalization factor a is related to the density of quintessence ρ as

$$\rho = -\frac{a}{2}\frac{3w_q}{r^{3(w_q+1)}}\,. \quad (6)$$

In general, for the usual quintessence, the energy density ρ is positive and the state parameter w_q is negative, which means that the constant a must be positive.

Notice that black hole event horizon is determined as a larger root of $f(r_+) = 0$. Then, the mass of black hole M can be obtained as

$$M = \frac{r_+}{2}\Big\{1 + \frac{2b^2r_+^2}{3}\Big(1 - \sqrt{1 + \frac{Q^2}{b^2r_+^4}}\Big) + \frac{4Q^2}{3r_+^2}\,{}_2F_1\Big[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{b^2r_+^2}\Big] - \frac{a}{r^{3w_q+1}} - \frac{\Lambda r_+^2}{3}\Big\}\,. \quad (7)$$

The Hawking temperature and entropy can be written as

$$T = \frac{f'(r_+)}{4\pi} = \frac{1}{4\pi}\Big(\frac{1}{r_+} + \frac{3aw_q}{r_+^{2+w_q}} - r_+\Lambda + 2b^2r_+\Big(1 - \sqrt{1 + \frac{Q^2}{b^2r^4}}\Big)\Big)\,, \quad (8)$$

$$S = \int_0^{r_+}\frac{1}{T}\Big(\frac{\partial M}{\partial r_+}\Big)dr_+ = \pi r_+^2\,. \quad (9)$$

In the geometric units $G_N = \hbar = c = k = 1$, the cosmological constant Λ can be interpreted as thermodynamic pressure P in the extended phase space as $P = -\Lambda/8\pi$. Then, its conjugate quantity as thermodynamic volume can be written as

$$V = \Big(\frac{\partial M}{\partial P}\Big)_{(S,Q,a,b)} = \frac{4\pi r_+^3}{3}\,. \quad (10)$$

The black hole mass M can be considered as the enthalpy rather than the internal energy of the gravitational system.[26] With these relations, the first law of black hole thermodynamics in the extended phase space should be expressed as

$$dM = TdS + \Phi dQ + VdP + \mathcal{A}da + \mathcal{B}db\,, \quad (11)$$

where Φ is electric potential, $\mathcal{A}$ is the physical quantity conjugate to the normalization factor a, and $\mathcal{B}$ is the quantity conjugate to the Born-Infeld parameter b called "Born-Infeld vacuum polarization" with

$$\Phi = \Big(\frac{\partial M}{\partial Q}\Big)_{(S,P,a,b)} = \frac{Q}{r_+}\,,$$

$$\mathcal{A} = \Big(\frac{\partial M}{\partial a}\Big)_{(S,P,Q,b)} = -\frac{1}{2r_+^{3w_q}}\,, \quad (12)$$

$$\mathcal{B} = \Big(\frac{\partial M}{\partial b}\Big)_{(S,P,Q,a)} = \frac{2br_+^3}{3}\Big(1 - \sqrt{1 + \frac{Q^2}{b^2r_+^4}}\Big) + \frac{Q^2}{3br_+}\,{}_2F_1\Big[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{b^2r_+^4}\Big]\,. \quad (13)$$

In addition, the Smarr relation with scaling argument can be obtained as

$$M = 2TS + \Phi Q - 2PV + (1 + 3w_q)\mathcal{A}a - \mathcal{B}b\,. \quad (14)$$

3 Phase Transition of Born-Infeld AdS Black Hole Affected by Quintessence

To compare with the Van der Waals fluid equation in four dimensional spacetime, we can translate the "geometric" equation of state to a physical one by identifying the specific volume v of the fluid with the horizon radius of the black hole r_+ as v.[4] With $v = 2r_+$ and $P = -\Lambda/8\pi$, one can obtain the equation of state $P(v, T)$ by solving Eq. (8) as

$$P = \frac{T}{v} - \frac{1}{2\pi v^2} - \frac{8^{w_q} \times 3aw_q}{\pi v^{3(1+w_q)}} - \frac{b^2}{4\pi}\Big(1 - \sqrt{1 + 16\frac{Q^2}{b^2v^4}}\Big)\,. \quad (15)$$

The critical point is an inflection point which occurs when

$$\frac{\partial P}{\partial v}\Big|_{T=T_c,v_+=v_c} = \frac{\partial^2 P}{\partial v^2}\Big|_{T=T_c,v_+=v_c} = 0\,, \tag{16}$$

where T_c and v_c are the critical temperature and the critical specific volume respectively. With Eqs. (15) and (16), the critical temperature T_c can be obtained as

$$T_c = \frac{1}{\pi v_c} + \frac{8^{w_q}\times 9aw_q(1+w_q)}{\pi v_c^{2+3w_q}} - \frac{8Q^2}{\pi v_c^3\sqrt{1+16Q^2/b^2v_c^4}}\,, \tag{17}$$

and the critical specific volume v_c is determined by

$$v_c^2 - \frac{8b^2Q^2\sqrt{1+16Q^2/b^2v_c^4}(16Q^2+3b^2{v_c}^4)v_c^4}{(16Q^2+b^2v_c^4)^2} + \frac{8^{w_q}\times 9aw_q(3w_q+2)(w_q+1)}{v_c^{3w_q-1}} = 0\,. \tag{18}$$

Obviously, it is difficult to derive the analytic solution of above equation. In order to further discuss the thermodynamical phase transition of these black holes, we firstly focus on some special values of parameters b and w_q.

When $w_q = -2/3$, Eq. (18) becomes

$$x^3 + px + q = 0\,, \tag{19}$$

where

$$p = -\frac{3b^2}{32Q^2}\,,\quad q = \frac{b^2}{256Q^4}\,,\quad x = \Big(v_c^4 + \frac{16Q^2}{b^2}\Big)^{-1/2}\,.$$

Notice that Eq. (19) is the same as that equation in Ref. [5], so here we do not present the analytical solutions for v_c. With these solutions, the critical temperature and critical pressure can be obtained as

$$T_c = \frac{1}{\pi(-16Q^2/b^2+1/x^2)^{1/4}} - \frac{a}{2\pi} - \frac{8Q^2(-16Q^2/b^2+1/x^2)x^2\sqrt{b^2/(b^2-16Q^2x^2)}}{\pi}\,, \tag{20}$$

$$P_c = \frac{2/\sqrt{-16Q^2/b^2+1/x^2} - 32Q^2x^2\sqrt{b^2/(b^2-16Q^2x^2)} + b^2(-1+\sqrt{b^2/(b^2-16Q^2x^2)})}{4\pi}\,. \tag{21}$$

Comparing with the results of BI-AdS black hole, the critical temperature P_c of BI-AdS black holes surrounded by quintessence also takes the same form, while corresponding critical temperature T_c is affected by the quintessence dark energy on the extended phase space thermodynamics. For instance, the VdW-like SBH/LBH phase transition happens at lower critical temperature than that for BI-AdS black hole because of parameter $a > 0$, even disappears since the parameter a taking enough large values leads to $T_c < 0$. We have calculated some phase space parameters for VdW-like phase transition in BI-AdS black holes and quintessence case, see Table 1.

In addition, it is worth to note that besides the usual SBH/LBH phase transitions, the interesting thermodynamic phenomenon of reentrant phase transition (RPT) is observed in a certain range of temperatures $T \in (T_t, T_z)$ for the BI-AdS black hole when taking $1/\sqrt{8}Q < b < \sqrt{3+2\sqrt{3}}/6Q$. Moreover, this process is also accompanied by a discontinuity in the global minimum of the Gibbs free energy, referred to as a "zeroth-order phase transition". Here we further investigate the influence of quintessence dark energy on the reentrant phase transitions. The quintessence dark energy also causes a lower critical temperature T_t, but does not change the width of region (T_t, T_z) for reentrant phase transition of BI-AdS black hole surrounded by quintessence, comparing with BI-AdS black hole, see Table 2.

Table 1 Phase space parameters at critical point for variable b. Here we set $w_q = -2/3$, $a = 0.1$ and $Q = 1$.

	BI-AdS BH in quintessence			BI-AdS BH		
b	P_c	v_c	T_c	P_c	v_c	T_c
1	0.003 371 91	4.816 18	0.027 713 2	0.003 371 91	4.816 18	0.043 628 7
0.8	0.003 405 19	4.766 23	0.027 899 9	0.003 405 19	4.766 23	0.043 815 4
0.6	0.003 486 51	4.647 96	0.028 335 7	0.003 486 51	4.647 96	0.044 251 2
0.48	0.003 609 16	4.466 93	0.028 980 4	0.003 609 16	4.466 93	0.0442 51 2
0.45	0,003 663 92	4.384 89	0.029 261	0,003 663 92	4.384 89	0.044 895 9
0.42	0.003 739 08	4.269 98	0.029 638 7	0.003 739 08	4.269 98	0.045 176 5
0.39	0.003 850 13	4.091 69	0.030 180 7	0.003 850 13	4.091 69	0.046 091 6

Table 2 Critical temperatures for the reentrant phase transitions of BI-AdS black hole and BI-AdS black hole surrounded by quintessence. Here we set $Q = 1$, $w_q = -2/3$, $a = 0.1$.

	BI(Quit)-AdS BH			BI-AdS BH		
b	T_t	T_z	ΔT	T_t	T_z	ΔT
0.38	0.029 04	0.029 17	0.000 13	0.044 96	0.045 09	0.000 13
0.4	0.026 59	0.026 78	0.000 19	0.042 508	0.042 70	0.001 92
0.42	0.023 43	0.023 64	0.000 21	0.039 34	0.039 56	0.000 22

When $w_q \neq -2/3$, obviously, it is difficult to get the analytical solution. We appeal to numerical method for help.

We plot the P-V diagrams of BI-AdS black holes surrounded by quintessence in Figs. 1 and 2. As shown in Fig. 1, the figure is very reminiscent of corresponding behavior RN-AdS black hole when $b > 0.5$. The two upper lines correspond to the "ideal gas" phase behavior when $T > T_c$. The critical isotherm $T = T_c$ represents the coexistence state. For $T < T_c$, there exists a small-large black hole phase transition. When $0.38 < b < 0.5$, as shown in Fig. 2, there are two critical point, that means the system exists two phase transitions. For $b < 0.38$, there are no longer any critical points in this system.

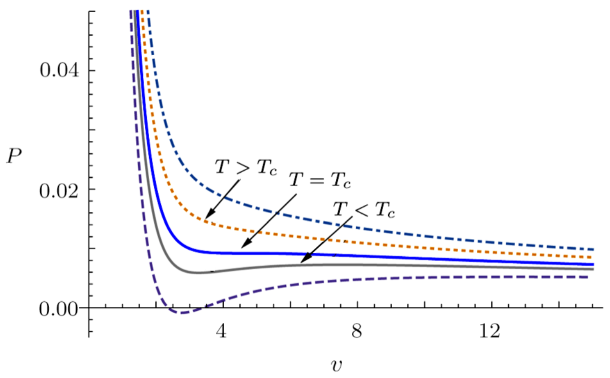

Fig. 1 (Color online) The P-v diagram for $Q = 1$, $w_q = -0.9$, $b = 1$, $a = 0.1$. The coexistence line at $T = T_c \approx 0.031\,956\,2$ is decipted by the blue solid line. The two upper lines correspond to the idea gas phase behavior for $T > T_c$. The two other lines represent the behavior of systems with $T < T_c$.

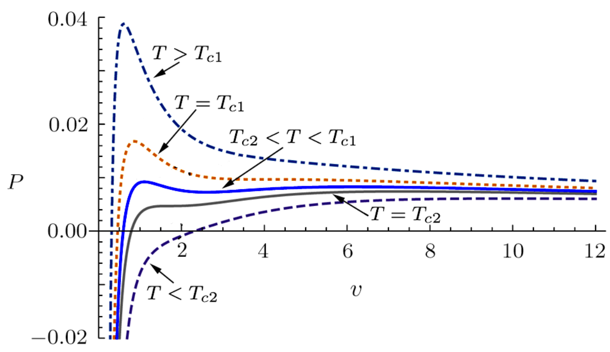

Fig. 2 (Color online) The P-v diagram for $Q = 1$, $w_q = -0.9$, $b = 0.45$, $a = 0.1$. There exist two critical points. The upper one at $T = T_{c1} \approx 0.034\,330\,1$ represented by orange dotted line occur at bigger horizon radius. The other critical isotherm occurs at $T = T_{c2} \approx 0.021\,227\,9$.

The thermodynamic phase transition can be characterized by the behavior of Gibbs free energy G in the canonical ensemble. We know that the black hole mass M can be identified with the enthalpy in extended phase space. The Gibbs free energy is $G = M - TS$, which reads

$$G = \frac{r_+}{4} - \frac{2P\pi r_+^3}{3} - \frac{b^2 {r_+}^3}{6}\left(1 - \sqrt{1 + \frac{Q^2}{b^2 {r_c}^4}}\right) - \frac{a(2+3w_q)}{4{r_+}^{3w_q}} + \frac{2Q^2}{3r_+}{}_2F_1\Big[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{Q^2}{b^2 {r_+}^4}\Big]. \tag{22}$$

With $Q = 1$, $w_q = -0.9$, and $a = 1$, the behavior of the Gibbs free energy is depicted as a function of temperature for fixed pressure.

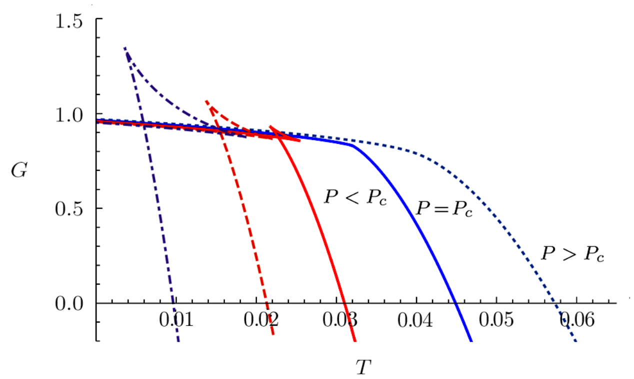

Fig. 3 (Color online) The behavior of Gibbs free energy for $Q = 1$, $w_q = -0.9$, $b = 1$, $a = 0.1$. For $b > 0.5$, there is only one critical point.The Gibbs free energy G exists the so-called "swallow tail" behavior when $P < P_c f5 \approx 0.009\,202\,554\,1$.

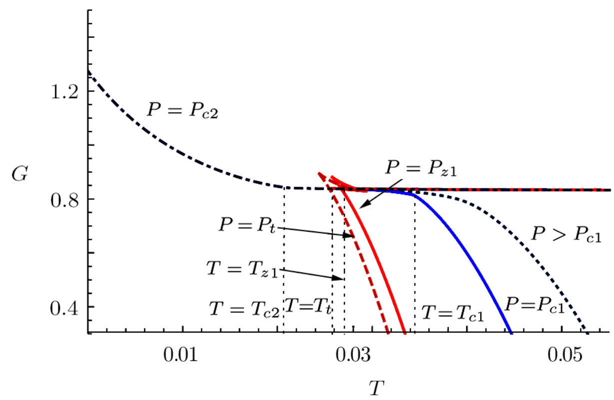

Fig. 4 (Color online) The behavior of Gibbs free energy for $Q = 1$, $w_q = -0.9$, $b = 0.45$, $a = 0.1$. In the range of $0.38 < b < 0.5$, the system exists "reentrant phase transition" when $P \in (P_t \approx 0.007\,86, P_{z1} \approx 0.008\,085)$, see more details in Fig. 5. For $P \in (P_{z1}, P_{c1})$, there only exists a first order phase transition.

When $b > 0.5$, as shown in Fig. 3, there exists a corresponding VdW-like first order phase transition between small and large black holes for $T < T_c$, the behavior of BI-AdS black hole surrounded by quintessence reminiscent of that of the RN-AdS black hole.

In the range of $0.38 < b < 0.5$, there are two phases of intermediate and small black holes from the first order phase transition called reentrant phase transition (RPT) depicted in Fig. 4. Aside from the first order phase transition, there also exist another phase transition when $T \in (T_t \approx 0.025\,445\,2, T_{z1} \approx 0.025\,710\,4)$, see Fig. 5. In this range of temperatures two separate branches of intermediate size and small size black holes co-exist, as shown as the P-T diagram in Fig. 6. They are separated by a finite jump in G. Although there is no real phase transition between them, we refer (for simplicity) to this phenomenon as zeroth-order phase transition in Ref. [5]. Note that the second critical point at T_{c2} is not a real critical point, as shown in Fig. 5. When the Hawking temperature varies in the range of $T_{c2} < T < T_t$, the Gibbs free energy G does not have any discontinuity, so there is no any phase transitions.

For $b < 0.38$, there is no first order phase transition in the system, reminiscent of the Schwarzschild-AdS case.

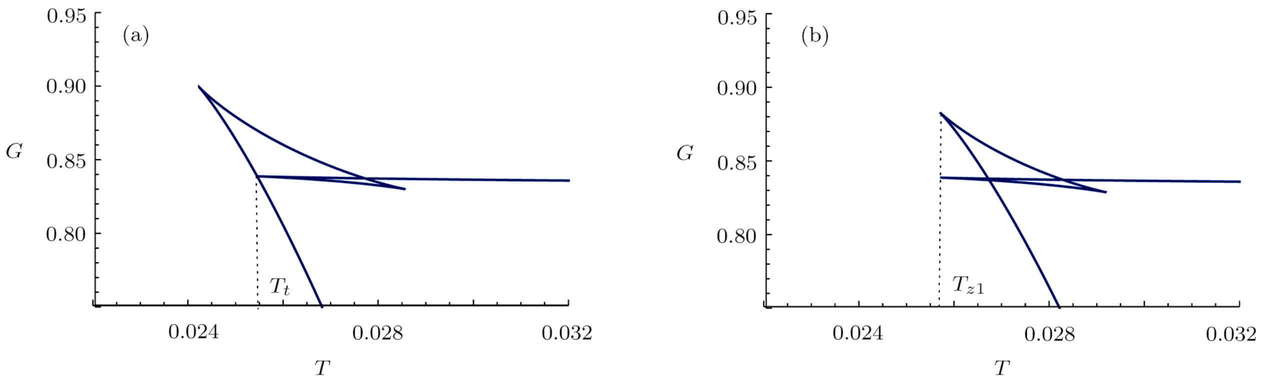

Fig. 5 (Color online) The behavior of Gibbs free energy for $Q = 1$, $w_q = -0.9$, $b = 0.45$, $a = 0.1$. There exists a finite jump of G called "zeroth-order phase transition", which begins at $T = T_t \approx 0.025\,445\,2$, and vanishes at $T = T_{z1} \approx 0.025\,710\,4$. For $T > T_{z1} \approx 0.025\,710\,4$, there are only VdW-like SBH/LBH phase transition, while there does not exist any phase transition when $T < T_t \approx 0.025\,445\,2$. (a) $P \approx 0.007\,86$; (b) $P \approx 0.008\,085$

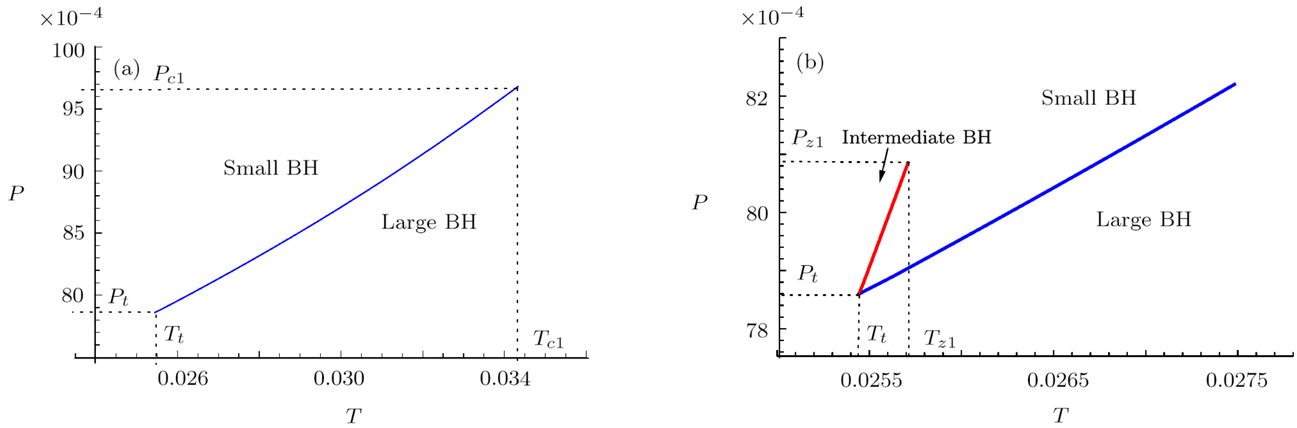

Fig. 6 (Color online) P-T diagram for $Q = 1$, $w_q = -0.9$, $b = 0.45$, $a = 0.1$. (a) The coexistence line of the VdW phase transition between small and large black holes is depicted by a blue line. The coexistence line starts from $T = T_{c1} \approx 0.034\,330\,1$ and terminates at $T = T_t \approx 0.025\,445\,2$. (b) the VdW transition occurs in the blue solid line. The red solid line shows the so called "zeroth-order phase transition" which initiates at $T = T_{z1} \approx 0.025\,710\,4$ and ends at $T = T_t \approx 0.025\,445\,2$.

4 Conclusion

In this paper, we have investigated the thermodynamic properties of BI-AdS black hole affected by the quintessence dark energy. By treating the cosmological constant Λ, Born-Infeld maximal field strength b and quintessence normalization factor a their conjugate quantities as the thermodynamic variables, we have obtained the first law of black hole thermodynamics and the Smarr relation in the extended phase space. Then we tested the critical behavior of the BI-AdS black hole in quintessence with variable maximal field strength b. We have plotted the P-V, G-T and P-T diagrams, and found the first order phase transition which resembles the Van der Waals fluid system. When $0.38 < b < 0.5$, we also found the reentrant phase transition, which was observed in four-dimensional BI-AdS black holes.

In addition, we further investigated the influences of quintessence dark energy on the thermodynamical phase transition of BI-AdS black hole, and found that the Van der Waals-like SBH/LBH phase transition occurs at lower critical temperature than that for BI-AdS black hole. Similar phenomenon also was recovered for reentrant phase transition of BI-AdS black hole surrounded by quintessence dark energy.

References

[1] S. W. Hawking and D. N. Page, Commun. Math. Phys. **87** (1983) 577.

[2] B. P. Dolan, Class. Quant. Grav. **28** (2011) 235017, [arXiv:1106.6260[gr-qc]].

[3] B. P. Dolan, Class. Quant. Grav. **28** (2011) 125020, [arXiv:1008.5023[gr-qc]].

[4] D. Kubiznak and R. B. Mann, J. High Energy Phys. **1207** (2012) 033, [arXiv:1205.0559[hep-th]].

[5] S. Gunasekaran, R. B. Mann, and D. Kubiznak, J. High Energy Phys. **1211** (2012) 110, [arXiv:1208.6251[hep-th]].

[6] T. Narayanan and A. Kumar, Phys. Rep. **249** (1994).

[7] N. Altamirano, D. Kubiznak, and R. B. Mann, Phys. Rev. D **88** (2013) 101502, [arXiv:1306.5756[hep-th]].

[8] D. Kubiznak and F. Simovic, Class. Quant. Grav. **33** (2016) 245001, [arXiv:1507.08630[hep-th]].

[9] R. A. Hennigar and R. B. Mann, Entropy **17** (2015) 8056, [arXiv:1509.06798[hep-th]].

[10] A. M. Frassino, D. Kubiznak, R. B. Mann, and F. Simovic, J. High Energy Phys. **1409** (2014) 080, [arXiv: 1406.7015[hep-th]].

[11] R. A. Hennigar, E. Tjoa, and R. B. Mann, J. High Energy Phys. **1702** (2017) 070, [arXiv:1612.06852[hep-th]].

[12] D. C. Zou, Y. Liu, and R. H. Yue, Eur. Phys. J. C **77** (2017) 365, [arXiv:1702.08118[gr-qc]].

[13] N. A. Bahcall, J. P. Ostriker, S. Perlmutter, and P. J. Steinhardt, Science **284** (1999) 1481, [astro-ph/9906463].

[14] S. Perlmutter, *et al.*, Astrophys. J. **517** (1999) 565, [astro-ph/9812133].

[15] V. V. Kiselev, Class. Quant. Grav. **20** (2003) 1187, [gr-qc/0210040].

[16] S. B. Chen and J. L. Jing, Class. Quant. Grav. **22** (2005) 4651, [gr-qc/0511085].

[17] C. Wu, Int. J. Mod. Phys. D **26** (2017) 1750111, [arXiv: 1606.00703[gr-qc]].

[18] R. Tharanath, N. Varghese, and V. C. Kuriakose, Mod. Phys. Lett. A **29** (2014) 1450057, [arXiv:1404.0791[gr-qc]].

[19] B. B. Thomas, M. Saleh, and T. C. Kofane, Gen. Rel. Grav. **44** (2012) 2181, [arXiv:1604.06207[gr-qc]].

[20] M. S. Ma, R. Zhao, and Y. Q. Ma, Gen. Rel. Grav. **49** (2017) 79, [arXiv:1606.06070[gr-qc]].

[21] S. G. Ghosh, M. Amir, and S. D. Maharaj, Eur. Phys. J. C **77** (2017) 530, [arXiv:1611.02936[gr-qc]].

[22] S. G. Ghosh, S. D. Maharaj, D. Baboolal, and T. H. Lee, Eur. Phys. J. C **78** (2018) 90, [arXiv:1708.03884[gr-qc]].

[23] S. G. Ghosh, Eur. Phys. J. C **76** (2016) 222, [arXiv:1512.05476[gr-qc]].

[24] G. Q. Li, Phys. Lett. B **735** (2014) 256, [arXiv:1407.0011[gr-qc]].

[25] K. Jafarzade and J. Sadeghi, *Effects of Dark Energy on P-V Criticality and Efficiency of Charged Rotational Black Hole*, arXiv:1803.04250[hep-th].

[26] D. Kastor, S. Ray, and J. Traschen, Class. Quant. Grav. **26** (2009) 195011, [arXiv:0904.2765 [hep-th]].