Commun. Theor. Phys. **70** (2018) 721–727 Vol. 70, No. 6, December 1, 2018

Testing Photons Coupled to Weyl Tensor with Gravitational Time Advancement*

Gang Li (李刚)[1,2] and Xue-Mei Deng (邓雪梅)[1,†]

[1]Purple Mountain Observatory, Chinese Academy of Sciences, Nanjing 210034, China

[2]School of Astronomy and Space Science, University of Science and Technology of China, Anhui 230026, China

(Received August 3, 2018; revised manuscript received August 17, 2018)

Abstract *Classical Solar System tests of photons coupled to Weyl tensor with two polarizations were studied in a recent work. A coupling strength parameter α in this model was firstly obtained as $|\alpha| \lesssim 4\times10^{11}$ m^2 by using available datasets in the Solar System. In this paper, a new test called by gravitational time advancement is proposed and investigated to test such the coupling. This new test, which is quite different from Shapiro time delay, depends strongly on round-trip proper time span (not coordinate time one) of flight of radio pulses between an observer on the Earth and a distant spacecraft. For ranging a spacecraft getting far away from the Sun, two special cases (the superior/inferior conjunctions) are used to analyse the observability in the advancement contributed by the Weyl coupling. We found that the situation of the inferior conjunction is more suitable for detecting the advancement caused by such the Weyl coupling. In either case, two kinds of polarizations make the advancement in the model smaller or larger than the one of general relativity. Although the observability in the advancement could be out of the reach of already existing technology, the implement of planetary laser ranging and optical clocks might provide us more insights on such the Weyl coupling in the near future.*

DOI: 10.1088/0253-6102/70/6/721

Key words: gravitation, experimental tests of gravitational theories, light propagation, astrometry

1 Introduction

In relativistic gravity, light signals propagated through the curved spacetime provide lots of primary information about the background's properties, the celestial objects, the gravitational fields, cosmic structures, gravitational theories, even tiny influence beyond the standard Einstein-Maxwell theory. In view of these unique features, it is clear that the signals for the interaction between the gravitational and electromagnetic fields will be directly imprinted on the light propagation.

In order to fully understand the fundamental interaction between the gravitational and electromagnetic fields, plenty of ideas about how a photon couples to a curved spacetime have been proposed.[1−10] Among them, there is one kind of coupling through the Weyl tensor. The coupling between the electromagnetic field and the Weyl tensor can be characterized by the coupling parameter α and has two polarizations: PPM and PPL.[11−12] This Weyl coupling was extensively studied in holographic conductivity and superconductors,[11,13−15] and the black hole's electrodynamics.[16−19] Since the Weyl tensor denotes the gravitational distortion, photons coupled to Weyl tensor provides us with an opportunity to test the interaction between the gravitational and electromagnetic fields by light propagation under the strong gravitational field and the weak gravitational field.

In the strong gravitational field, the effects of photons coupled to the Weyl tensor on strong deflection gravitational lensing[12,20] have been investigated. A more complicated strong deflection gravitational lensing has been considered in this model under a Kerr black hole spacetime.[21] Given the fact that there does not exist the high-resolution direct imaging on the vicinity of a black hole (e.g., Sgr A*) up to now, these works[12,20−21] could not give the bound on the couping imposed by the strong gravitational field. However, testing photons coupled to the Weyl tensor in the weak gravitational field, especially in the Solar System, is a totally different situation due to tracking and ranging data between terrestrial stations and spacecrafts in deep space with the high-accuracy. Since the Weyl coupling has only an impact upon the light signals, its influences on the light deflection, the time delay and the Cassini tracking experiment have been studied attentively in Ref. [22]. With considering these available datasets in the Solar System, the coupling strength parameter α was firstly obtained as $|\alpha| \lesssim 4\times10^{11}$ m^2.[22] In addition, weak deflection gravitational lensing for photons coupled to Weyl tensor has been also considered in a Schwarzschild black hole.[23]

However, in the foreseeable future, the observations on effects in the region of the strong gravitational field can not give much tighter bound on the Weyl coupling. In

*Supported by the National Natural Science Foundation of China under Grant Nos. 11773080, 11473072, and 11533004

†Corresponding author, E-mail: xmd@pmo.ac.cn

the region of the strong gravitational field, if the diameter of photon shpere on the supermassive black hole at the Galactic center measured by the Event Horizon Telescope is within the one predicted according to the general relativity by 1 micro-arcsecond (μas), it yields the bound on the coupling: $|\alpha| \lesssim 0.12 R^2_{\rm Schwarzschild} \simeq 2.03 \times 10^{19}\ {\rm m}^2$,[12] where we use $M_{\rm Sgr\ A^*} \simeq 4.4 \times 10^6 M_\odot$. It is shown that the bound imposed by the Solar System tests is still much tighter than the bound in the supermassive system (if the data in the region of the strong gravitational field exist) by at least 8 orders of magnitude. It provides compelling evidence for testing the photons coupled to Weyl tensor in the Solar System.

In this paper, a novel Solar System test of such the coupling is proposed and studied by measuring a two-way proper time interval, which is called by the gravitational time advancement. This new test, which is quite different from Shapiro time delay, depends strongly on round-trip proper time span (*not* coordinate time one) of flight of radio pulses between an observer on the Earth and a distant spacecraft. As an independent measurement, gravitational time advancement might improve the statistics on the bounds of the parameters and could be effectively complementary to the Shapiro time delay (see Introduction of Ref. [28] for details). Recently, this new test was proposed and investigated in general relativity[24] and might be able to detect dark matter and dark energy.[25] The gravitational time advancement is a way to veto a semiclassical limit of a theory of quantum gravity (gravity's rainbow).[26] This new type Solar System test was also proposed and considered under a brane world[27] and adopted for probing the quadratic $f(T)$ gravity.[28]

In this work, this new Solar System test of such a coupling based on gravitational time advancement will be proposed and investigated. And the possible observables of the advancement will also be discussed. The paper is organized as follows: In the next Section, we present the advancement under the Weyl coupling. In Sec. 3, observability of this advancement of photons coupled to the Weyl tensor is exhibited and analyzed. At last, Sec. 4 presents conclusions.

2 Gravitational Time Advancement

If a spacecraft is more distant to a body and an observer is closer to the body, then measuring a two-way proper time interval (*not* the coordinate time interval) of the observer for the light ray could find the gravitational time advancement.[24–28] It implies that the light ray takes a shorter (*not* longer) time than the one under Newton's theory owing to the existence of a body's gravitational field, which is quite different from the Shapiro time delay.[29] The gravitational time advancement of photons coupled to Weyl tensor under the Sun' gravitational field will be derived in this section.

2.1 *Metric*

The action of photons coupled to Weyl tensor is given by[13]

$$S = \int \sqrt{-g}\,{\rm d}^4 x \Big[\frac{R}{16\pi} - \frac{1}{4}\Big(F_{\alpha\beta}F^{\alpha\beta} - 4\alpha C^{\alpha\beta\gamma\delta}F_{\alpha\beta}F_{\gamma\delta}\Big)\Big]\,, \tag{1}$$

where $G = c = 1$. $C_{\alpha\beta\gamma\delta}$ is Weyl tensor, which describes a type of gravitational distortion in the curved spacetime. $F_{\alpha\beta} = A_{\beta;\alpha} - A_{\alpha;\beta}$ is the electromagnetic tensor. The photons coupled to the Weyl tensor can be characterized by the coupling parameter α.

Variation of the action (1) with respect to A^α yields

$$(F_{\alpha\beta} - 4C^{\alpha\beta\gamma\delta}F_{\gamma\delta})_{;\alpha} = 0\,. \tag{2}$$

When we consider geometric optics approximation,[1] we obtain

$$F_{\alpha\beta} = f_{\alpha\beta}\exp({\rm i}\theta)\,, \tag{3}$$

where θ is a rapidly varying variable and $f_{\alpha\beta}$ is a slowly varying one. By using the Bianchi identity of $F_{\alpha\beta}$ and letting $k_\alpha = \theta_{,\alpha}$, $f_{\alpha\beta}$ reads[1]

$$f_{\alpha\beta} = k_\alpha a_\beta - k_\beta a_\alpha\,, \tag{4}$$

where a_α is the polarization vector and $k_\alpha a^\alpha = 0$.

Substituting Eqs. (3) and (4) into Eq. (2), it yields[11–12]

$$k_\alpha k^\alpha + 8\alpha C^{\alpha\beta\gamma\delta}k_\alpha k_\delta a_\gamma = 0\,. \tag{5}$$

Considering a static and spherically symmetric metric for light propagation

$${\rm d}s^2 = -f(r)^2{\rm d}t^2 + \frac{1}{f(r)^2}{\rm d}r^2 + r^2{\rm d}\Omega^2, \tag{6}$$

where ${\rm d}\Omega^2 = {\rm d}\theta^2 + \sin^2\theta\,{\rm d}\phi^2$. And the orthonormal tetrad and the bivectors are respectively

$$e^a_\alpha = \Big(\sqrt{f}, \frac{1}{\sqrt{f}}, r, r\sin\theta\Big)\,, \tag{7}$$

$$U^{ab}_{\alpha\beta} = e^a_\alpha e^b_\beta - e^a_\beta e^b_\alpha\,. \tag{8}$$

Based on Ref. [1], we introduce three independent vectors as follows

$$l_\beta = k^\alpha U^{01}_{\alpha\beta}, \quad n_\beta = k^\alpha U^{02}_{\alpha\beta}, \quad m_\beta = k^\alpha U^{23}_{\alpha\beta}\,, \tag{9}$$

where l_β, n_β, and m_β are orthogonal to k_β.

Furthermore, with the static and spherically symmetric metric and above relationships, the modified light cone condition is[11–12]

$$(g_{00}k^0k^0 + g_{11}k^1k^1) + C(r)^2(g_{22}k^2k^2 + g_{33}k^3k^3) = 0\,, \tag{10}$$

where $C(r)^2$ relies on polarization of the photon. Considering the polarization is along the direction of l_β (which means PPL polarization), it yields

$$C(r)^2 = \frac{r^3 + 16\alpha M}{r^3 - 8\alpha M}\,, \tag{11}$$

and the polarization is along the direction of m_β (which means PPM polarization), it yields

$$C(r)^2 = \frac{r^3 - 8\alpha M}{r^3 + 16\alpha M}\,. \tag{12}$$

A light ray does not follow null geodesics of the metric under photons coupled to Weyl tensor we consider. However, the light follows null geodesics of the following effective metric given by Refs. [11–12]

$$\mathrm{d}s^2 = -f(r)^2 c^2 \mathrm{d}t^2 + \frac{1}{f(r)^2}\mathrm{d}r^2 + C(r)^2 r^2 \mathrm{d}\Omega^2, \tag{13}$$

where we restore the velocity of light and the constant of gravitation to the metric (13) and

$$f(r)^2 = 1 - \frac{2GM}{c^2 r}\,, \tag{14}$$

$$C(r)^2 = 1 + 24\epsilon\alpha\frac{GM}{c^2 r^3} + \mathcal{O}(c^{-4})\,, \tag{15}$$

where we deal with $C(r)^2$ in the form of a Taylor expansion on $1/c$. And the negative or positive sign of ϵ in Eq. (15) represents two polarizations based on Eqs. (11) and (12):

$$\epsilon = \begin{cases} -1 & \text{for PPM}\,, \\ +1 & \text{for PPL}\,. \end{cases} \tag{16}$$

When $\alpha = 0$, the metric (13) will then reduce to the Schwarzschild spacetime in general relativity. The coupling strength parameter α was firstly obtained as $|\alpha| \lesssim 4 \times 10^{11}\ \mathrm{m}^2$ by physical experiments in the Solar System,[22] including the deflection of light, the gravitational time delay, and the Cassini tracking experiment.

From Eq. (13), we obtain the motion of a photon in the spacetime

$$0 = -f(r)^2\Big(\frac{c\mathrm{d}t}{\mathrm{d}\lambda}\Big)^2 + \frac{1}{f(r)^2}\Big(\frac{\mathrm{d}r}{\mathrm{d}\lambda}\Big)^2 + C(r)^2 r^2\Big(\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}\Big)^2, \tag{17}$$

where λ is the affine parameter. With $\theta = \pi/2$, we obtain two following conserved quantities:

$$E = f(r)^2\frac{c\mathrm{d}t}{\mathrm{d}\lambda}, \quad L = C(r)^2 r^2\frac{\mathrm{d}\phi}{\mathrm{d}\lambda}\,. \tag{18}$$

Using Eq. (17), E and L, the expression between r and t is obtained as

$$\frac{c\mathrm{d}t}{\mathrm{d}r} = \pm\frac{1}{b}\frac{1}{f(r)^2}\Big[\frac{1}{b^2} - \frac{f(r)^2}{C(r)^2 r^2}\Big]^{-1/2}, \tag{19}$$

which leads to

$$\begin{aligned} t(r,d) &= \frac{1}{c}\int_d^r \frac{1}{b}\frac{1}{f(r)^2}\Big[\frac{1}{b^2} - \frac{f(r)^2}{C(r)^2 r^2}\Big]^{-1/2}\mathrm{d}r \\ &= \frac{1}{c}\sqrt{r^2-d^2} + \frac{GM}{c^3}\sqrt{\frac{r-d}{r+d}} \\ &\quad + 2\frac{GM}{c^3}\ln\Big(\frac{r+\sqrt{r^2-d^2}}{d}\Big) \\ &\quad + 12\epsilon\alpha\frac{GM}{c^3 d^2 r}\sqrt{\frac{r-d}{r+d}}(d+2r) + \mathcal{O}(c^{-5})\,, \end{aligned} \tag{20}$$

in which $b \equiv L/E$ and is derived from $0 = \mathrm{d}r/\mathrm{d}\phi$ at the closest approach d.

Additionally, we adopt r_A and r_B are the radial distances of a spacecraft and an observer to the Sun respectively. Two special configurations are considered in this work, where $r_A > r_B \gg d$ (see Fig. 1). One case is the superior conjunction, which occurs when the observer and the spacecraft lie in a line on the opposite side of the Sun (see Fig. 1(a)). Another one is the inferior conjunction, which occurs when the observer and the spacecraft lie in a line on the same side of the Sun (see Fig. 1(b)).

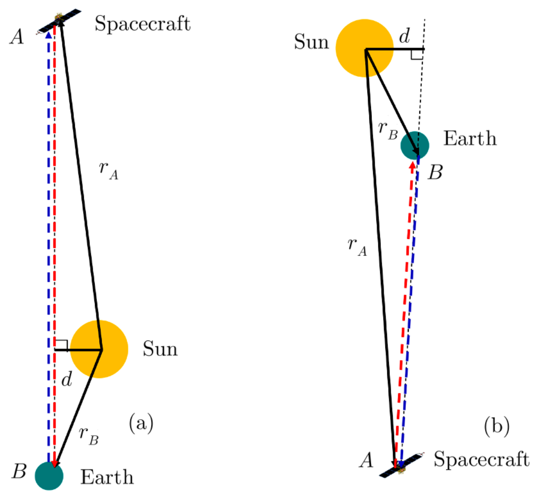

Fig. 1 Two special configurations of the distant spacecraft at point A, the Sun and the observer (at point B) on the Earth. (a) The superior conjunction case; (b) The inferior conjunction case.

2.2 *Superior Conjunction Case*

From Eq. (20) plus $r_A > r_B \gg d$, the coordinate time interval of the light ray emitting from the observer to the distant spacecraft and returning to this observer yields

$$\begin{aligned} \Delta t^{\mathrm{WT}}_{\mathrm{SC}} &= 2t(r_A,d) + 2t(r_B,d) \\ &= \frac{2}{c}(r_A + r_B) + \frac{4GM}{c^3} - \frac{2GM}{c^3}\Big(\frac{d}{r_A} + \frac{d}{r_B}\Big) \\ &\quad + 4\frac{GM}{c^3}\ln\frac{4r_A r_B}{d^2} + 96\epsilon\alpha\frac{GM}{c^3 d^2} - 24\epsilon\alpha\frac{GM}{c^3 d r_A} \\ &\quad - 24\epsilon\alpha\frac{GM}{c^3 d r_B} + \mathcal{O}\Big(c^{-5}, \frac{d^2}{r_A^2}, \frac{d^2}{r_B^2}\Big)\,, \end{aligned} \tag{21}$$

within the precision $\mathcal{O}(c^{-5}, d^2/r_A^2, d^2/r_B^2)$. Equation (21) will return to the result given by Ref. [22] within the precision $\mathcal{O}(c^{-5}, d/r_A, d/r_B)$. When $\alpha = 0$, Eq. (21) will be identical with the time delay of the general relativity in the first post-Newtonian (1PN) order of the superior conjunction case.[30] It is worth emphasizing that the above time delay we consider belongs to 1PN order rather than to 1.5PN order. The speed of light c appears in the denominator of the time delay in the framework of classical Newton mechanics (see the first term in Eq. (21)) and the terms at 1PN order are generally $1/c^2$ of the Newtonian term. It results in the time delay expression at 1PN order is the $1/c^3$ terms, which is quite different from the equation of motion (EOM) for a two-body system. In the EOM of a binary pulsar, the $1/c^3$ terms belong to 1.5PN order. In general, there exits differences between Lagrangian and Hamiltonian approaches at 1.5PN order. These differences are studied with analytical and numerical methods at the same post-Newtonian order.[31] It is shown that theses differences do not affect qualitative and quantitative results of the two approaches for a weak gravitational system such as the Solar System.

Then, the two-way proper time span measured by the observer in this case is

$$\begin{aligned}\Delta\tau_{\rm SC}^{\rm WT} &= \Big(1-\frac{GM}{c^2 r_B}\Big)\Delta t_{\rm SC}\\ &= \frac{2}{c}(r_A+r_B)+\frac{2GM}{c^3}-\frac{2GM}{c^3}\Big(\frac{d}{r_A}+\frac{d}{r_B}\Big)\\ &\quad + 4\frac{GM}{c^3}\ln\frac{4r_Ar_B}{d^2}+96\epsilon\alpha\frac{GM}{c^3d^2}-24\epsilon\alpha\frac{GM}{c^3dr_A}\\ &\quad -24\epsilon\alpha\frac{GM}{c^3dr_B}-\frac{2GM}{c^3}\frac{r_A}{r_B}\\ &\quad +\mathcal{O}\Big(c^{-5},\frac{d^2}{r_A^2},\frac{d^2}{r_B^2}\Big)\,. \end{aligned} \tag{22}$$

Under general relativity ($\alpha=0$), the last term of Eq. (22) gives rise to the gravitational time advancement.[24] The key is that the radial distance of the spacecraft to the Sun must be adequately bigger than the radial distance of the observer to the Sun. From Eq. (22), it is found that two polarizations (PPM and PPL) have different effects on the advancement in this case. The coupling of PPM can make the advancement larger than the one of GR, but the case of PPL is opposite.

2.3 *Inferior Conjunction Case*

For this case, the coordinate time interval of the light ray emitting from the observer to the reflector of the distant spacecraft and back to this observer is

$$\begin{aligned}\Delta t_{\rm IC}^{\rm WT} &= 2t(r_A,d)-2t(r_B,d)\\ &= \frac{2}{c}(r_A-r_B)+\frac{2GM}{c^3}\Big(\frac{d}{r_B}-\frac{d}{r_A}\Big)\\ &\quad +4\frac{GM}{c^3}\ln\frac{r_A}{r_B}+24\epsilon\alpha\frac{GM}{c^3dr_B}\\ &\quad -24\epsilon\alpha\frac{GM}{c^3dr_A}+\mathcal{O}\Big(c^{-5},\frac{d^2}{r_A^2},\frac{d^2}{r_B^2}\Big)\,, \end{aligned} \tag{23}$$

within the precision $\mathcal{O}(c^{-5},d^2/r_A^2,d^2/r_B^2)$. The first term in Eq. (23) belongs to the Newtonian order. The others in Eq. (23) belong to 1PN order. Equation (23) will return to the result given by Ref. [22] within the precision $\mathcal{O}(c^{-5},d/r_A,d/r_B)$. Equation (23) matches the one given by Ref. [32] in 1PN order of GR in this case. And the two-way proper time interval of the light recorded by the observer is

$$\begin{aligned}\Delta\tau_{\rm IC}^{\rm WT} &= \Big(1-\frac{GM}{c^2 r_B}\Big)\Delta t_{\rm IC}\\ &= \frac{2}{c}(r_A-r_B)+\frac{2GM}{c^3}+\frac{2GM}{c^3}\Big(\frac{d}{r_B}-\frac{d}{r_A}\Big)\\ &\quad +4\frac{GM}{c^3}\ln\frac{r_A}{r_B}+24\epsilon\alpha\frac{GM}{c^3dr_B}-24\epsilon\alpha\frac{GM}{c^3dr_A}\\ &\quad -\frac{2GM}{c^3}\frac{r_A}{r_B}+\mathcal{O}\Big(c^{-5},\frac{d^2}{r_A^2},\frac{d^2}{r_B^2}\Big)\,. \end{aligned} \tag{24}$$

It suggests that the advancement at the inferior conjunction depends on α.

3 Observability in Time Advancement

After deriving the gravitational time advancement caused by such the coupling, this section will be focused on its observability and its difference from the delay. In order to discuss the observability in two special configurations, the value of the coupling parameter α is taken as $|\alpha| = 4\times10^{11}$ m^2 based on the result of Ref. [22].

According to Eqs. (21) and (22), in the superior conjunction case, the deviations of the gravitational time delay and advancement from the Euclidean geometric delay are respectively

$$\delta t_{\rm SC}^{\rm WT} \equiv \Delta t_{\rm SC}^{\rm WT} - \Delta t_{\rm SC}^{\rm WT}|_{M=0}\,, \tag{25}$$

$$\delta \tau_{\rm SC}^{\rm WT} \equiv \Delta \tau_{\rm SC}^{\rm WT} - \Delta \tau_{\rm SC}^{\rm WT}|_{M=0}\,. \tag{26}$$

Then, we plot $\delta t_{\rm SC}^{\rm WT}$ and $\delta\tau_{\rm SC}^{\rm WT}$ with two polarizations as a function of ratio between r_A and r_B on the rang from 30 astronomical unit (au) to 100 au (see Fig. 2 for details). Here we assume the closest approach d is $1.5R_\odot$. From Fig. 2, it is shown that the delay and the advancement can reach serval sub-millseconds ($\sim 10^{-1}$ ms). And the time delay is positive and the time advancement is negative. It indicates that the gravitational time advancement is quite different from the gravitational time delay. The point is that the radial distance of the spacecraft to the Sun must be adequately bigger than the radial distance of the observer to the Sun. A two-way proper time span recorded by the observer is another key factor.

The gravitational time delay and the advancement of Einstein's general relativity at the post-Newtonian order in the same configuration are

$$\delta t_{\rm SC}^{\rm GR} \equiv \Delta t_{\rm SC}^{\rm WT}\big|_{\alpha=0} - \Delta t_{\rm SC}^{\rm WT}\big|_{M=0}\,, \tag{27}$$

$$\delta \tau_{\rm SC}^{\rm GR} \equiv \Delta \tau_{\rm SC}^{\rm WT}\big|_{\alpha=0} - \Delta \tau_{\rm SC}^{\rm WT}\big|_{M=0}\,. \tag{28}$$

Furthermore, we assume that the observer is on the Earth, who is conducting a two-way ranging to the spacecraft at 40 au from the Sun. The configuration of the superior conjunction is quite suitable for detecting the advancement of the coupling owing to the largeness of r_A and the smallness of d. The estimated observability of the superior conjunction is outlined in Table 1. It is shown that, in the superior conjunction, the delay and the advancement contributed by the Weyl coupling and general relativity are respectively ~ 315 and ~ -88 micro-second (μs). The signal of the time delay is larger than the one of the advancement. For the polarization of PPM, it is found that the advancement caused by the coupling is larger than the one in the general relativity; and the case of PPL is opposite.

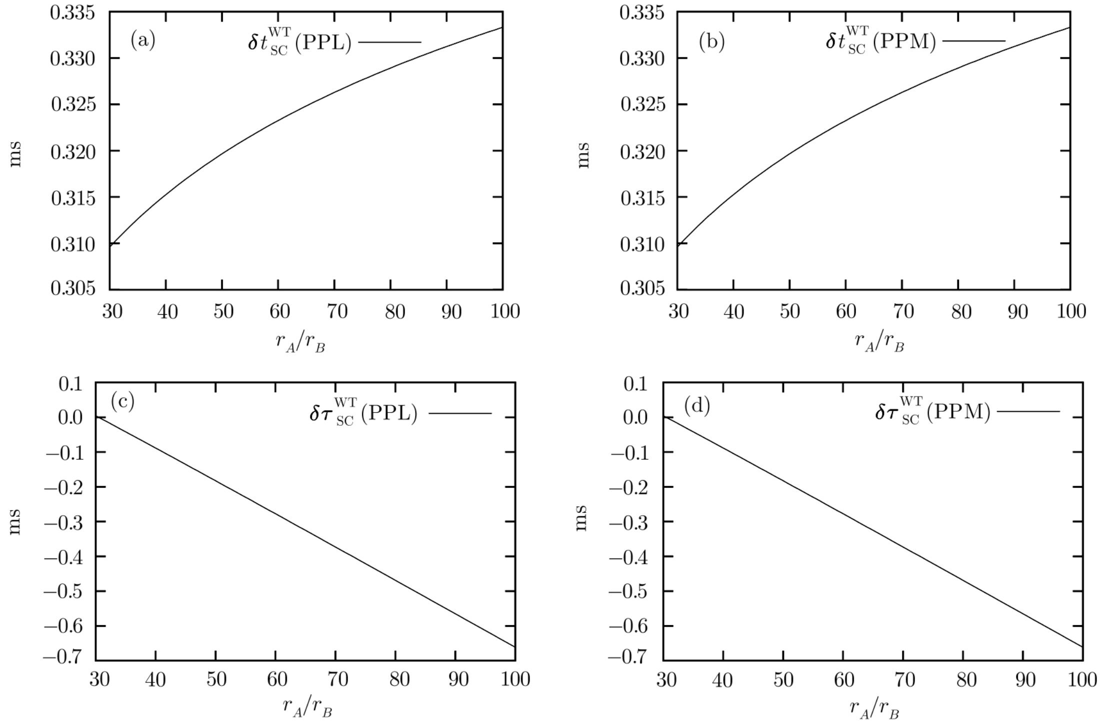

Fig. 2 In the same configurations of the superior conjunction, gravitational time delay (positive) $\delta t_{\rm SC}^{\rm WT}$ and time advancement (negative) $\delta\tau_{\rm SC}^{\rm WT}$ contributed by photons coupled to Weyl tensor with two polarizations as a function of ratio between r_A and r_B. Here we assume that the observer on the Earth is conducting as two-way measurements in the radio band to range a distant spacecraft. $|\alpha| = 4\times 10^{11}$ m^2[22] and the closest approach d is $1.5R_\odot$.

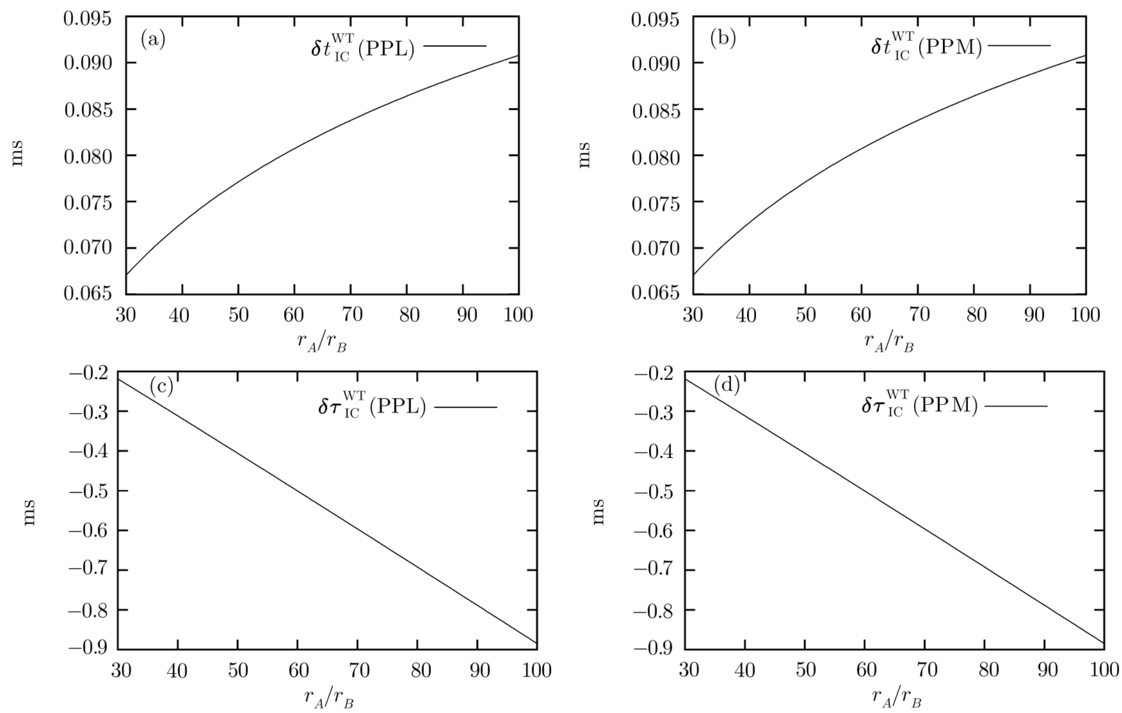

Fig. 3 In the same configurations of the inferior conjunction, gravitational time delay (positive) $\delta t_{\rm IC}^{\rm WT}$ and time advancement (negative) $\delta\tau_{\rm IC}^{\rm WT}$ contributed by photons coupled to Weyl tensor with two polarizations as a function of ratio between r_A and r_B. Here we assume that the observer on the Earth is conducting as two-way measurements in the radio band to range a distant spacecraft with $|\alpha| = 4\times 10^{11}$ m^2[22] and the closest approach d is $1.5R_\odot$.

Table 1 Estimation of observability on the delay and the advancement caused by the coupling in the superior conjunction case, where $r_B = 1$ au, $r_A = 40$ au, $d = 1.5R_\odot$ and $|\alpha| = 4 \times 10^{11}$ m^2.[22]

Delay (Coordinate time span)	Model	$\delta t_{\rm SC}^{\rm WT}$ (μs)	$\delta t_{\rm SC}^{\rm GR}$ (μs)	$(\delta t_{\rm SC}^{\rm WT} - \delta t_{\rm SC}^{\rm GR})/\Delta t_{\rm SC}^{\rm WT}$
	PPL ($\epsilon = +1$)	315.235 472 637	315.235 299 156	$4.239\ 668\ 688 \times 10^{-15}$
	PPM ($\epsilon = -1$)	315.235 125 676	ibid.	$-4.239\ 668\ 688 \times 10^{-15}$
Advancement (Proper time span)	**Model**	$\delta \tau_{\rm SC}^{\rm WT}$ (μs)	$\delta \tau_{\rm SC}^{\rm GR}$ (μs)	$(\delta \tau_{\rm SC}^{\rm WT} - \delta \tau_{\rm SC}^{\rm GR})/\Delta \tau_{\rm SC}^{\rm WT}$
	PPL ($\epsilon = +1$)	−88.534 664 089	−88.534 837 569	$4.239\ 668\ 730 \times 10^{-15}$
	PPM ($\epsilon = -1$)	−88.535 011 050	ibid.	$-4.239\ 668\ 730 \times 10^{-15}$

Table 2 Estimation of observability on the delay and the advancement caused by the coupling in the inferior conjunction case, where $r_B = 1$ au, $r_A = 40$ au, $d = 1.5R_\odot$ and $|\alpha| = 4 \times 10^{11}$ m^2.[22]

Delay (Coordinate time span)	Model	$\delta t_{\rm IC}^{\rm WT}$ (μs)	$\delta t_{\rm IC}^{\rm GR}$ (μs)	$(\delta t_{\rm IC}^{\rm WT} - \delta t_{\rm IC}^{\rm GR})/\Delta t_{\rm IC}^{\rm WT}$
	PPL ($\epsilon = +1$)	72.723 563 026	72.723 562 731	$7.581\ 773\ 356 \times 10^{-15}$
	PPM ($\epsilon = -1$)	72.723 562 436	ibid.	$-7.581\ 773\ 356 \times 10^{-15}$
Advancement (Proper time span)	**Model**	$\delta \tau_{\rm IC}^{\rm WT}$ (μs)	$\delta \tau_{\rm IC}^{\rm GR}$ (μs)	$(\delta \tau_{\rm IC}^{\rm WT} - \delta \tau_{\rm IC}^{\rm GR})/\Delta \tau_{\rm IC}^{\rm WT}$
	PPL ($\epsilon = +1$)	−311.350 469 469	−311.350 469 764	$7.581\ 773\ 431 \times 10^{-15}$
	PPM ($\epsilon = -1$)	−311.350 470 059	ibid.	$-7.581\ 773\ 431 \times 10^{-15}$

In the inferior conjunction case, from Eqs. (23) and (24), the deviations of the gravitational time delay and advancement from the Euclidean geometric delay are

$$\delta t_{\rm IC}^{\rm WT} \equiv \Delta t_{\rm IC}^{\rm WT} - \Delta t_{\rm IC}^{\rm WT}|_{M=0}\,, \tag{29}$$

$$\delta \tau_{\rm IC}^{\rm WT} \equiv \Delta \tau_{\rm IC}^{\rm WT} - \Delta \tau_{\rm IC}^{\rm WT}|_{M=0}\,. \tag{30}$$

Then, we plot $\delta t_{\rm IC}^{\rm WT}$ and $\delta \tau_{\rm IC}^{\rm WT}$ with two polarizations as a function of ratio between r_A and r_B (see Fig. 3 for details). From Fig. 3, it is shown that the advancement can reach serval sub-millseconds ($\sim 10^{-1}$ ms), which is about 10 times larger than the results of the delay.

The gravitational time delay and the advancement of Einstein's general relativity at the post-Newtonian order in the inferior conjunction configuration are

$$\delta t_{\rm IC}^{\rm GR} \equiv \Delta t_{\rm IC}^{\rm WT}|_{\alpha=0} - \Delta t_{\rm IC}^{\rm WT}|_{M=0}\,, \tag{31}$$

$$\delta \tau_{\rm IC}^{\rm GR} \equiv \Delta \tau_{\rm IC}^{\rm WT}|_{\alpha=0} - \Delta \tau_{\rm IC}^{\rm WT}|_{M=0}\,. \tag{32}$$

The estimated observability of this case is outlined in Table 2. It is shown that, in the inferior conjunction, the delay and the advancement contributed by the Weyl coupling and general relativity are respectively ~ 72 and ~ -311 micro-second (μs). The signal of the time advancement is larger than the one of the delay. Thus, the inferior conjunction is more fit for detecting the gravity than the superior conjunction.

The last columns in Table 1 and Table 2 denote the resolution of time measurement, which could tell the Weyl coupling from general relativity in the delay and advancement. The time resolutions are all about 10^{-15}. Currently, the accuracy and stability of laser-cooled atomic clocks have achieved at about $10^{-15} \sim 10^{-16}$ level. Although these effects of the coupling on the advancement at two cases seem to be marginally within the current capability of atomic clock, the combination of the noises of the radio link and the uncertainty and instability of the clock can make a definite detection unreachable. Now, optical clocks on the ground have achieved the accuracy and stability at the 10^{-18} level and beyond.[33−35] In the near future, the implement of planetary laser ranging and optical clocks might provide us more insights on such a Weyl coupling.

4 Conclusions and Discussion

In this work, a new test of testing photons coupled to the Weyl tensor in the Solar System, which is called as the time advancement, is proposed and studied. If an observer records a two-way proper time interval (*not* coordinate time interval) of a light ray, the advancement appears as a natural result of the gravitation in the spacetime when the light ray passes through a weaker field and the observer is in the stronger gravitational field. This is quite different from the Shapiro time delay and the difference between the delay and the advancement can be found from Fig. 2 and Fig. 3.

Taking ranging a spacecraft far from the Earth into account, we discuss the observability of such the coupling with two polarizations on the advancement under two special configurations: the superior conjunction and the inferior conjunction. The observability on the time advancement and its difference from the delay under two special configurations are analyzed and estimated by adopting the distant spacecraft at 40 au from the Sun (see Table 1 and Table 2). We found that the situation of the inferior conjunction is more suitable for detecting the advancement caused by such the Weyl coupling. In either case, two kinds of polarizations make the advancement in the model smaller or larger than the one of general relativity. It is marginally within the current capability of technology, but a clean detection is still unreachable. It is looking forward to testing photons coupled to the Weyl tensor by using optical clocks and the planetary laser ranging in the future.

References

[1] I. T. Drummond and S. J. Hathrell, Phys. Rev. D **22** (1980) 343.

[2] L. Mankiewicz and M.Misiak, Phys. Rev. D **40** (1989) 2134.

[3] I. B. Khriplovich, Phys. Lett. B **346** (1995) 251.

[4] R. D. Daniels and G. M. Shore, Nucl. Phys. B **425** (1994) 634.

[5] G. M. Shore, Nucl. Phys. B **460** (1996) 379.

[6] M. S. Turner and L. M. Widrow, Phys. Rev. D **37** (1988) 2743.

[7] F. D. Mazzitelli and F. M. Spedalieri, Phys. Rev. D **52** (1995) 6694.

[8] G. Lambiase and A. R. Prasanna, Phys. Rev. D **70** (2004) 063502.

[9] R. Lafrance and R. C. Myers, Phys. Rev. D **51** (1995) 2584.

[10] M. Novello, L. A. R. Oliveira, and J. M. Salim, Class. Quantum Gravity **13** (1996) 1089.

[11] J. Jing, S. Chen, and Q. Pan, Ann. Phys. **367** (2016) 219.

[12] S. Chen and J. Jing, J. Cosmol. Astropart. Phys. **10** (2015) 002.

[13] A. Ritz and J. Ward, Phys. Rev. D **79** (2009) 066003.

[14] D. Z. Ma, Y. Cao, and J. P. Wu, Phys. Lett. B **704** (2011) 604.

[15] J. P. Wu, Y. Cao, X. M. Kuang, and W. J. Li, Phys. Lett. B **697** (2011) 153.

[16] S. Chen and J. Jing, Phys. Rev. D **88** (2013) 064058.

[17] S. Chen and J. Jing, Phys. Rev. D **89** (2014) 104014.

[18] S. Chen and J. Jing, Phys. Rev. D **90** (2014) 124059.

[19] H. Liao, S. Chen, and J. Jing, Phys. Lett. B **728** (2014) 457.

[20] X. Lu, F. W. Yang, and Y. Xie, Eur. Phys. J. C **76** (2016) 357.

[21] S. Chen, S. Wang, Y. Huang, *et al.*, Phys. Rev. D **95** (2017) 104017.

[22] G. Li and X. M. Deng, Ann. Phys. **382** (2017) 136.

[23] W. G. Cao and Y. Xie, Eur. Phys. J. C **78** (2018) 191.

[24] A. Bhadra and K. K. Nandi, Gen. Relativ. Gravit. **42** (2010) 293.

[25] S. Ghosh and A. Bhadra, Eur. Phys. J. C **75** (2015) 494.

[26] X. M. Deng and Y. Xie, Phys. Lett. B **772** (2017) 152.

[27] X. M. Deng, Mod. Phys. Lett. A **33** (2018) 1850110.

[28] X. M. Deng, Class. Quantum Gravity **35** (2018) 175013.

[29] I. I. Shapiro, Phys. Rev. Lett. **13** (1964) 789.

[30] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity*, Wiley, New York (1972).

[31] X. Wu, L. Mei, G.Huang, and S. Liu, Phys. Rev. D **91** (2015) 024042.

[32] R. A. Nelson, Metrologia **48** (2011) S171.

[33] C. W. Chou, D. B. Hume, J. C. J. Koelemeij, *et al.*, Phys. Rev. Lett. **104** (2010) 070802.

[34] C. W. Chou, D. B. Hume, T. Rosenband, and D. J. Wineland, Science **329** (2010) 1630.

[35] B. J. Bloom, T. L. Nicholson, J. R. Williams, *et al.*, Nature (London) **506** (2014) 71.