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A Possible Modification of Einstein's Theory of General Relativity

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Abstract *This article suggests a new metric theory of gravitation, in which metric field is determined not only by matter and nongravitational field but also by vector graviton field, and in principle there is no need to introduce the Einstein's tensor. In order to satisfy automatically the geodesic postulate, an additional coordinate condition is needed. For the spherically symmetric static field, it leads us to quite different conclusions from those of Einstein's general relativity in the interior region of the surface of infinite redshift. Accurate to the first order of GM/r, it obtains the same results about the four experimental tests of general relativity.*

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1 Introduction

In general relativity (GR), the Einstein's field equation is known as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -8\pi G T_{\mu\nu}\,, \tag{1}$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the curvature scalar, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the energy-momentum tensor of matter (not including the gravitational field), the energy-momentum of the gravitational field is included implicitly on the left-hand side of this equation, and the Einstein's tensor $G_{\mu\nu}$ is introduced in order to satisfy automatically the geodesic postulate

$$T^{\mu\nu}_{;\nu} = 0\,. \tag{2}$$

The right-hand side (RHS) of Eq. (1) does not include explicitly the energy momentum of gravitational field, whereas the left-hand side (LHS) of Eq. (1) is not just $R_{\mu\nu}$ but an artificially introduced quantity $G_{\mu\nu}$, which is somehow unnatural and man-made.

For the spherically symmetric static field, in GR we have the famous Schwarzschild solution

$$\mathrm{d}s^2 = \Big(1 - \frac{2GM}{r}\Big)\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{(1-2GM/r)} - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\,\mathrm{d}\phi^2\,. \tag{3}$$

The Schwarzschild solution develops a "singularity" as $r \to 2GM$, and $r_s = 2GM$ is called the Schwarzschild radius of the mass M. The surface $r = r_s$ is a surface of infinite redshift (SIR) (relative to $r = \infty$). It is easily seen that when $r < r_s$, t from a timelike coordinate becomes a spacelike coordinate, whereas r from a spacelike coordinate becomes a timelike coordinate. It is quite an unsatisfactory and unnatural result.

Because of the above considerations, this paper suggests a possible modification of Einstein's field equation (1) and its consequences (3) to overcome these unnatural and man-made results. We want to modify Eq. (1) such that its RHS includes explicitly the energy-momentum of gravitational field, i.e. the vector graviton field, whereas the LHS only contains the Ricci tensor $R_{\mu\nu}$. In effect, the suggested modification is also a metric theory of gravitation, the metric field $g_{\mu\nu}$ plays the same role for gravitational field as that in the Einstein's theory of general relativity, and the vector graviton field here only plays the same role as that of matter and other fields, i.e. generating the metric field $g_{\mu\nu}$. After the suggested modification, the LHS of the new field equation is just the Ricci tensor $R_{\mu\nu}$, whereas the RHS includes explicitly the energy-momentum of the vector graviton field. In order to ensure Eq. (2) we only need to add a coordinate condition. The modified metric theory of gravitation has been verified that it satisfies all the criteria for the viability of a theory: self-consistency, completeness, and agreement with past experiments. Actually, it is a new kind of metric theory, which suggests that there exists a vector graviton field $H^{\mu\nu}$. This field $H^{\mu\nu}$ should act together with the matter and other fields to generate the metric field $g_{\mu\nu}$, which plays the same role for gravitational field as that in the Einstein's theory of general relativity. It should be emphasized that the field $H^{\mu\nu}$ does not directly influence the motion of a particle in the vector graviton field, but only acts together with the matter and other fields to generate the metric field $g_{\mu\nu}$, and that just the metric field $g_{\mu\nu}$ directly influences the motion of a particle in the vector graviton field. This modification may be called as the vector graviton metric theory (VGM), which we have found predicts almost the same results as Einstein's theory about the four famous experimental tests of general relativity: light deflection, time-delay in radar propagation, gravitational redshift, and perihelion procession of planets. But it gives a quite different and reasonable result in the neighborhood of the surface of infinite redshift and some other interesting and reasonable results. In Sec. 2 this paper will give the explicit expression of the energy-momentum tensor of the vector graviton field. In Sec. 3 this paper will give the modified field equation in VGM and the additional coordinate condition to ensure the geodesic postulate Eq. (2). In Sec. 4 this paper derives the external solution for the spherical symmetric static mass, which is slightly different from the Schwarzschild solution. In Sec. 5 this paper discusses the four experimental tests of general relativity. In the last section this paper discusses the behavior of rod and clock in the neighborhood of the surface of infinite redshift, and shows that t is always a timelike coordinate whereas r is always a spacelike coordinate in this case.

2 Graviton Field Tensor and Graviton Energy-Momentum Tensor

Schwartz[1] has proved that Maxwell's equations could be derived from the Coulomb's law and the requirement of Lorentz invariance. Since Newton's gravitation law and Coulomb's law are completely similar in their form, hence we recognize that there exists a vector graviton field described by an antisymmetric tensor $H^{\mu\nu}$. For an observer in the flat space

$$H^{\mu\nu} = \begin{pmatrix} 0 & F_x & F_y & F_z \\ -F_x & 0 & \Omega_z & -\Omega_y \\ -F_y & -\Omega_z & 0 & \Omega_x \\ -F_z & \Omega_y & -\Omega_x & 0 \end{pmatrix}, \tag{4}$$

where (F_x, F_y, F_z) is the ordinary gravitational field strength, $(\Omega_x, \Omega_y, \Omega_z)$ is the gravitational vortex strength introduced by Carstoiu.[2] In flat space, $H^{\mu\nu}$ satisfies Maxwell-like equations

$$H^{\mu\nu}_{,\nu} = -4\pi G k^\mu\,, \qquad \{H_{\mu\nu,\lambda}\} = 0\,, \tag{5}$$

where k^μ is the four-dimensional mass current density vector, G is the gravitation constant, speed of light c is taken as 1, and $\{\}$ designates antisymmetrized sum over all permutations of the indices μ, ν, λ. It has to be emphasized that $H^{\mu\nu}$ is not the metric field $g^{\mu\nu}$ which is equivalent to the gravitational field. $H^{\mu\nu}$ plays the same role as other matters, which only influences metric field, not directly determines the motion of a particle, whereas under the action of metric field, particle moves along a geodesic line in the Riemannian space.

For photon field, energy-momentum tensor is[3]

$$T^{\mu\nu}_{(\rm em)} = F^\mu_\alpha F^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}\,, \tag{6}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor. The vector graviton field of a body is similar to the electromagnetic field of a negative charge, and its energy-momentum tensor can be analogously written as

$$T^{\mu\nu}_{(\rm gr)} = \frac{1}{4\pi G}\Big(H^\mu_\alpha H^{\alpha\nu} + \frac{1}{4} g^{\mu\nu} H^{\alpha\beta} H_{\alpha\beta}\Big)\,. \tag{7}$$

Its covariant divergence obeys

$$T^{\mu\nu}_{(\rm gr);\nu} = -H^\mu_\alpha k^\alpha\,. \tag{8}$$

When $k^\alpha = 0$, we have $T^{\mu\nu}_{(\rm gr);\nu} = 0$. In curved space, $H^{\mu\nu}$ satisfies

$$H^{\mu\nu}_{;\nu} = -4\pi G k^\mu\,, \qquad \{H_{\mu\nu,\lambda}\} = 0\,. \tag{9}$$

3 Field Equations in VGM

In our new metric theory of gravitation VGM, $T^{\mu\nu}_{(\rm gr)}$ plays the same role as $T^{\mu\nu}$, hence in RHS of Einstein's field equations we use $T_{\mu\nu} + T_{\mu\nu(\rm gr)}$ to replace $T_{\mu\nu}$, furthermore in LHS of Eq. (1) we use $R_{\mu\nu}$ to replace the man-made quantity $G_{\mu\nu}$, finally we suggest the following field equations

$$R_{\mu\nu} = -8\pi G(T_{\mu\nu} + T_{\mu\nu(\rm gr)})\,. \tag{10}$$

In order to satisfy Eq. (2), from Eqs. (9), (10), and the Bianchi identity

$$G^{\mu\nu}_{;\nu} = 0\,, \tag{11}$$

we obtain

$$\Big(\frac{1}{2} g^{\mu\nu} R\Big)_{;\nu} = -8\pi G T^{\mu\nu}_{(\rm gr);\nu} = 8\pi G H^\mu_\alpha k^\alpha\,. \tag{12}$$

Equation (12) can be considered as an additional condition imposed on $g^{\mu\nu}$, which plays a similar role as the coordinate condition. In summary, the field equation of VGM is Eq. (10), where $T^{\mu\nu}_{(\rm gr)}$ is given by Eq. (7), and there is an additional equation imposed on the metric tensor $g^{\mu\nu}$, which is Eq. (12). Equation (2) is automatically satisfied by using the additional coordinate condition Eq. (12).

For metric field in the vacuum, $T_{\mu\nu} = 0$, from Eq. (10) we have

$$R_{\mu\nu} = -8\pi G T_{\mu\nu(\rm gr)}\,. \tag{13}$$

Considering that in the vacuum $k^\alpha = 0$, from Eqs. (8) and (13) we get $R^{\mu\nu}_{;\nu} = 0$, then from Eq. (11) we have $R_{;\nu} = R_{,\nu} = 0$. Hence $R = \text{const}$. The simplest choice for R is $R = 0$, hence in the vacuum there exist only vector graviton field and the required metric field. The constant curvature scalar R is taken to be zero and there is no difference between $G_{\mu\nu}$ and $R_{\mu\nu}$ for the vacuum field, thence equation (13) can be written as

$$G_{\mu\nu} = R_{\mu\nu} = -8\pi G T_{\mu\nu(\rm gr)}\,. \tag{14}$$

The coordinate condition Eq. (12) is automatically satisfied in this case. As to equation of motion of a particle in the gravitational field, from Eq. (2) we get the geodesic equation

$$\frac{{\rm d}^2 x^\mu}{{\rm d}\tau^2} + \Gamma^\mu_{\nu\lambda} \frac{{\rm d}x^\nu}{{\rm d}\tau} \frac{{\rm d}x^\lambda}{{\rm d}\tau} = 0\,, \tag{15}$$

where $\Gamma^\mu_{\nu\lambda}$ are Christoffel symbols and ${\rm d}\tau$ is proper time interval.

4 External Solution for the Spherical Symmetric Static Mass

Considering the static character and the spherical symmetry of the external field, the space-time interval can be written as[4]

$$\mathrm{d}s^2 = \mathrm{e}^{N(r)}\mathrm{d}t^2 - \mathrm{e}^{L(r)}\mathrm{d}r^2 - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2\,. \tag{16}$$

Similar to the argument in Sec. 13.1 of Ref. [3] for the field of a charged mass point, from Eqs. (4) and (5) we obtain the nonzero component of $H_{\mu\nu}$,

$$H_{10} = -H_{01} = \frac{GM}{r^2}\exp\Big[\frac{1}{2}(N+L)\Big], \tag{17}$$

thence from Eq. (7) we find

$$T_{\mu\nu(\mathrm{gr})} = \frac{1}{8\pi G}\Big(\frac{GM}{r}\Big)^2\begin{pmatrix} \mathrm{e}^{N(r)} & 0 & 0 & 0 \\ 0 & -\mathrm{e}^{L(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}. \tag{18}$$

Solving Eq. (14) we finally obtain

$$\mathrm{d}s^2 = \Big(1-\frac{GM}{r}\Big)^2\mathrm{d}t^2 - \frac{\mathrm{d}r^2}{(1-GM/r)^2} - r^2\mathrm{d}\theta^2 - r^2\sin^2\theta\mathrm{d}\phi^2\,. \tag{19}$$

5 Four Experimental Tests of General Relativity

Comparing Eq. (19) with the Schwarzschild line element, we readily find that to the precision of the first order of GM/r, two expressions are completely coincident. Hence, for two experimental tests: light deflection and the time-delay in radar propagation, we get the same results as GR. As to gravitational redshift, from Eq. (19) we get

$$\frac{\nu_2}{\nu_1} = \frac{1-GM/r_1}{1-GM/r_2}\,, \tag{20}$$

whereas Schwarzchild's metric gives

$$\Big(\frac{\nu_2}{\nu_1}\Big)_{\mathrm{Sch}} = \sqrt{\frac{1-2GM/r_1}{1-2GM/r_2}}\,. \tag{21}$$

For weak field equations (20) and (21) give almost the same results, which in the strong field region they should give apparently different results, i.e. the surface of infinite redshift (SIR) is different for two theories. For VGM, it situates at $r = r_c = GM$, while for GR it situates at $r = r_s = 2GM = 2r_c$. In consideration of perihelion procession of Mercury, since it needs to consider the terms involved $(GM/r)^2$, there is apparent difference between VGM and GR: for GR the value is 43".03/century, while for VGM the value is 35".86/century. The recent observed value is 41".4/century (it needs further observation), which is not only due to relativistic gravity but also due to solar oblateness[5] and anomalous perihelion shifts[6] (it needs further consideration). The reasonable perihelion shifts due to relativistic gravity is perhaps around 39"/century, between the value predicted by VGM and the value predicted by GR. From the perihelion shifts itself, we cannot say at this time which theory is better, GR or VGM.

6 Neighborhood of SIR

Firstly we consider the proper time interval $\mathrm{d}\tau$, for VGM a clock placed (at rest) near SIR ($r = r_c = GM$) shows a proper time

$$\mathrm{d}\tau^2 = \Big(1-\frac{GM}{r}\Big)^2\mathrm{d}t^2\,. \tag{22}$$

Hence the clock runs slower and slower when it approaches SIR, $\mathrm{d}\tau$ tends to zero as $r \to r_c$, i.e. the clock runs infinitely slow compared to a clock at infinity, when the clock crosses SIR it runs faster and faster when r becomes smaller and smaller. For GR

$$(\mathrm{d}\tau^2)_{\mathrm{Sch}} = \Big(1-\frac{2GM}{r}\Big)\mathrm{d}t^2\,. \tag{23}$$

When $r < r_s = 2GM$, the RHS of Eq. (23) is negative, and equation (23) is not usable, for t becomes spacelike coordinate, so it is very unnatural, whereas in VGM t keeps timelike coordinate when $r < r_c$.

Secondly we consider space interval $\mathrm{d}l$. From Ref. [7] we know

$$\mathrm{d}l^2 = \Big(\frac{g_{0\alpha}g_{0\beta}}{g_{00}} - g_{\alpha\beta}\Big)\mathrm{d}x^\alpha\mathrm{d}x^\beta\,, \tag{24}$$

where the repeated appeared Greek letters mean summation from 1 to 3. From Eq. (19) we get

$$\mathrm{d}l^2 = \frac{\mathrm{d}r^2}{(1-GM/r)^2} + r^2\mathrm{d}\theta^2 + r^2\sin^2\theta\mathrm{d}\phi^2\,. \tag{25}$$

For a rod placed along the radial direction,

$$\mathrm{d}l^2 = \frac{\mathrm{d}r^2}{(1-GM/r)^2}\,, \tag{26}$$

hence the rod contracts shorter and shorter when it approaches SIR, and $\mathrm{d}l$ tends to zero as $r \to r_c$, i.e. the rod contracts infinitely short compared to a rod at infinity. When the rod crosses SIR it elongates longer and longer when r becomes smaller and smaller. For GR

$$(\mathrm{d}l^2)_{\mathrm{Sch}} = \frac{\mathrm{d}r^2}{(1-2GM/r)}\,. \tag{27}$$

When $r < r_s = 2GM$, the RHS of Eq. (27) is negative, and equation (27) is not usable, for r becomes timelike coordinate, and it is very unnatural, whereas in VGM r keeps spacelike coordinate when $r < r_c$.

From Eq. (19), we readily know that the radial speed of the light signal is

$$\Big(\frac{\mathrm{d}r}{\mathrm{d}t}\Big)_{\mathrm{light}} = \Big(1-\frac{r_c}{r}\Big)^2\,, \tag{28}$$

when $r \to r_c$ it tends to zero, hence SIR is the surface at which radial speed of light equals zero. In the interior region of SIR, $(\mathrm{d}r/\mathrm{d}t)_{\mathrm{light}}$ increases as r decreases, when $r = r_c/2$, it increases to 1, when r further decreases, it will be greater than 1, i.e. in the region of strong gravitational field the speed of light can surpass c, the speed of light in the vacuum.

It is very important that in Eq. (19) when $r < r_c$, $g_{00} = (1 - r_c/r)^2$ is still positive, $g_{11} = -(1 - r_c/r)^{-2}$ is still negative, g_{00} and g_{11} do not change their sign, hence t is always a timelike coordinate, and r is always a spacelike coordinate. They do not change their roles when they cross SIR. Whereas in GR, they do change their roles when they cross SIR $r = r_s$. In GR, when $r > r_s$, future light cone directs towards the direction of increasing t, but when $r < r_s$ future light cone directs abruptly towards the direction of decreasing r (r becomes timelike coordinate). In the interior region of SIR $r = r_s$, anybody can only move in the direction of decreasing r, the reverse direction is impossible, and SIR acts as a one-way membrane. In VGM, future light cone always directs towards the direction of increasing t, and SIR never acts as a one-way membrane.

Thus it can be seen that when the radius of the spherical mass is less than $r_c = GM$, in the neighborhood of r_c, the results of VGM is quite different from those of GR. There is no one-way membrane, t is always a timelike coordinate whereas r is always a spacelike coordinate, i.e. the conclusions from VGM are quite different from those of Schwarzchild black hole.

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