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Chemical Potential Dependence of Dressed-Quark Propagator*

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Abstract *A method for obtaining the low chemical potential dependence of the dressed quark propagator from an effective quark-quark interaction model is developed. Of particular interest here is to give a general recipe to find without arbitrariness the solution representing the "Wigner" phase at non-zero chemical potential for the purpose of studying QCD phase structure.*

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It is generally believed that with increasing temperature and baryon number density the hadronic matter undergoes a phase transition to quark-gluon plasma (QGP) which is expected to appear in ultrarelativistic heavy ion collisions. Over the past years, considerable progress has been achieved in our understanding of high temperature QCD, where simulations on the lattice and universality arguments played an essential role. However, our knowledge of the density properties of strongly interaction matter is rudimentary so far. Due to the well-known difficulties to deal directly with QCD, different nonperturbative QCD models have been used to study the density properties of strongly interaction matter. However, finding suitable nonperturbative approximation scheme often relies on our knowledge about propagators or the relevant degrees of freedom (in particular, quark propagator at nonzero chemical potential plays an essential role in the study of chiral symmetry restoration and quark deconfinement). Of particular interest here is to give a general recipe to study the chemical potential dependence of the dressed-quark propagator and to find without arbitrariness solutions representing the "Nambu–Goldstone" phase (characterized by dynamical chiral symmetry breaking and confinement of the dressed quarks) and the "Wigner" phase (corresponding to the quark-gluon plasma where chiral symmetry is restored) at non-zero chemical potential in the framework of suitable nonperturbative QCD models.

Since the global color symmetry model (GCM)[1–3] provides a nonperturbative framework that admits the simultaneous study of dynamical chiral symmetry breaking and confinement which is expected to be well suited to explore the transition from hadronic matter to QGP.[4] It is the aim of this letter to study the chemical potential dependence of the dressed-quark propagator in the framework of the GCM, which provides a means of determining the behavior of the chiral and deconfinement order parameters. Up to this end let us start from the Euclidean action of the GCM at finite chemical potential μ (in the case of the chiral limit):

$$S_{\rm GCM}[\bar{q},q;\mu]=\int {\rm d}^4x\{\bar{q}(x)[\gamma\cdot\partial_x-\mu\gamma_4]q(x)\}+\int {\rm d}^4x{\rm d}^4y\Big[\frac{g_s^2}{2}j_\mu^a(x)D_{\mu\nu}^{ab}(x-y)j_\nu^b(y)\Big]\,, \tag{1}$$

where $j_\mu^a(x)=\bar{q}(x)\gamma_\mu\lambda_c^a q(x)/2$ denotes the color octet vector current and $g_s^2D_{\mu\nu}^{ab}(x-y)$ is the dressed model gluon propagator in GCM. It should be noted here that $g_s^2D_{\mu\nu}^{ab}(x-y)$ does not evolve with μ (where we have assumed that the effect of chemical potential on the gluon propagator through quark loop insertions is small in comparison to that on the quark propagator). For convenience, we will employ a model ansatz $D_{\mu\nu}^{ab}(x-y)=\delta_{\mu\nu}\delta^{ab}D(x-y)$ for the gluon propagator, which is often referred to as the so-called "Feynman-like" gauge[4] (It should be noted that the above ansatz should be regarded merely as a model form for the gluon two-point function).

Introducing an auxiliary bilocal field $B^\theta(x,y)$ and applying the standard bosonization procedure the partition function of GCM[3,4]

$$\mathcal{Z}[\mu]=\int\mathcal{D}\bar{q}\mathcal{D}q\,{\rm e}^{-S_{\rm GCM}[\bar{q},q;\mu]} \tag{2}$$

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can be rewritten in terms of the bilocal fields $B^\theta(x,y)$

$$\mathcal{Z}[\mu] = \int \mathcal{D}B^\theta \, \mathrm{e}^{-S_{\rm eff}[B^\theta;\mu]} \tag{3}$$

with the effective bosonic action

$$S_{\rm eff}[B^\theta;\mu] = -\ \mathrm{Tr}\, \ln \mathcal{G}^{-1}[B^\theta;\mu] + \int \mathrm{d}^4x\,\mathrm{d}^4y \frac{B^\theta(x,y)B^\theta(y,x)}{2g_s^2 D(x-y)}\,, \tag{4}$$

and the quark operator

$$\mathcal{G}^{-1}[B^\theta;\mu] = [\gamma\cdot\partial_x - \mu\gamma_4]\delta(x-y) + \Lambda^\theta B^\theta(x,y)\,. \tag{5}$$

The matrices $\Lambda^\theta = D^a \otimes C^b \otimes F^c$ are determined by Fierz transformation in Dirac, color, and flavor spaces of the current-current interaction in Eq. (1), and are given by

$$\Lambda^\theta = \frac{1}{2}\Big\{1_D, \mathrm{i}\gamma_5, \frac{\mathrm{i}}{\sqrt{2}}\gamma_\nu, \frac{\mathrm{i}}{\sqrt{2}}\gamma_\nu\gamma_5\Big\} \otimes \Big\{\frac{4}{3}1_C, \frac{\mathrm{i}}{\sqrt{3}}\lambda_C^a\Big\} \otimes \Big\{\frac{1}{\sqrt{3}}1_F, \frac{1}{\sqrt{2}}\lambda_F^a\Big\}\,. \tag{6}$$

One might expect that the complete set of the 16 Dirac matrices $\{1_D, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$ must be employed in the description. However, by limiting the gluon two-point function $g_s^2 D_{\mu\nu}^{ab}(x-y)$ to diagonal components in Lorentz indices, the tensor $\sigma_{\mu\nu}$ is excluded.

In the mean-field approximation, the fields $B^\theta(x,y)$ are substituted simply by their vacuum value $B_0^\theta(x,y)$, which is defined as $\delta S_{\rm eff}/\delta B|_{B_0} = 0$ and is given by

$$B_0^\theta[\,\mu](x,y) = g_s^2 D(x-y)\ \mathrm{tr}\,[\Lambda^\theta \mathcal{G}_0[\,\mu](x,y)]\,, \tag{7}$$

where the notation tr denotes trace over the Dirac, color, and flavor indices and $\mathcal{G}_0^{-1}(x,y)$ denotes the inverse propagator with the self-energy $\Sigma(x,y) = \Lambda^\theta B_0^\theta(x,y)$ at the finite chemical potential μ. Employing the stationary condition Eq. (7), and reversing the Fierz transformation, we have

$$\Sigma[\,\mu](y_1,y_2) = \frac{4}{3} g_s^2 D(y_1,y_2)\gamma_\nu \mathcal{G}_0[\,\mu](y_1,y_2)\gamma_\nu\,. \tag{8}$$

It should be noted that both $B_0^\theta(x,y)$ and $\mathcal{G}_0^{-1}(x,y)$ are dependent on the chemical potential μ. If the chemical potential μ is switched off, $\mathcal{G}_0[\,\mu]$ goes into the dressed vacuum quark propagator $G \equiv \mathcal{G}_0[\,\mu=0]$, which has the decomposition

$$G^{-1}(p) = \mathrm{i}\gamma\cdot p + \Sigma(p) = \mathrm{i}\gamma\cdot pA(p^2) + B(p^2) \tag{9}$$

with

$$\Sigma(p) = \int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}p\cdot x}[\Lambda^\theta B_0^\theta(x)] = \frac{4}{3}\int \frac{\mathrm{d}^4q}{(2\pi)^4} g_s^2 D(p-q)\gamma_\nu G(q)\gamma_\nu\,, \tag{10}$$

where the self-energy functions $A(p^2)$ and $B(p^2)$ are determined by the rainbow Dyson–Schwinger equation (DSE),

$$[A(\,p^2)-1]p^2 = \frac{8}{3}\int \frac{\mathrm{d}^4q}{(2\pi)^4} g_s^2 D(p-q)\frac{A(q^2)p\cdot q}{q^2A^2(q^2)+B^2(q^2)}\,,$$
$$B(\,p^2) = \frac{16}{3}\int \frac{\mathrm{d}^4q}{(2\pi)^4} g_s^2 D(p-q)\frac{B(q^2)}{q^2A^2(q^2)+B^2(q^2)}\,. \tag{11}$$

Here we want to stress that $B(p^2)$ in Eq. (11) has two qualitatively distinct solutions. The "Nambu–Goldstone" solution, for which

$$B(p^2) \neq 0\,, \tag{12}$$

describes a phase in which chiral symmetry is dynamically broken because one has a nonzero quark mass function, and the dressed quarks are confined because the propagator described by these functions does not have a Lehmann representation. The alternative "Wigner" solution, for which

$$B(p^2) \equiv 0\,, \tag{13}$$

describes a phase in which chiral symmetry is not broken and the dressed-quarks are not confined. In "Wigner" phase, the Dyson–Schwinger equation (11) reduces to

$$[A'(p^2)-1]p^2 = \frac{8}{3}\int \frac{\mathrm{d}^4q}{(2\pi)^4} g_s^2 D(p-q)\frac{p\cdot q}{q^2A'(q^2)}\,,$$

where $A'(p^2)$ denotes the dressed quark vector self-energy function in "Wigner" phase. Therefore, the dressed quark propagator in "Wigner" phase can be written as $G^{(W)}(q) = -\mathrm{i}\gamma\cdot q/A'(q^2)q^2$.

Let us now study the chemical potential dependence of the dressed quark propagator. This can be obtained by numerically solving Eq. (8) directly. However, as was shown in the previous works,[5,6] there exists some arbitrariness in finding the solution representing the "Wigner" phase by dealing directly with Eq. (8) (for the purpose of studying QCD phase structure it is necessary to determine the "Wigner" solution in a definite way). In order to overcome the difficulty mentioned above, we give a general recipe to find the non-arbitrary "Wigner" solution in the case of finite chemical potential. In addition, we stress here that the model gluon propagator (11) has no explicit μ-dependence while the actual gluon propagator should be μ-dependent due to quark vacuum polarization insertions. As such it may be inadequate at large values of μ, particularly near any critical chemical potential. Therefore, here we choose to employ the method of expanding in powers of μ (to the second order) and it is appropriate for studying the low chemical potential dependence of the dressed quark propagator (Here we are only interested in the second-order dependence of $\mathcal{G}_0^{-1}[\mu]$ upon μ). One can expand $\mathcal{G}_0^{-1}[\mu]$ in powers of μ as

$$\mathcal{G}_0^{-1}[\mu] \simeq \mathcal{G}_0^{-1}[\mu]|_{\mu=0} + \frac{\partial \mathcal{G}_0^{-1}[\mu]}{\partial\mu}\Big|_{\mu=0}\mu + \frac{1}{2}\frac{\partial^2\mathcal{G}_0^{-1}[\mu]}{\partial^2\mu}\Big|_{\mu=0}\mu^2 + \mathcal{O}(\mu^3) = G^{-1} + \mu\Gamma_4 + \frac{1}{2}\mu^2\Lambda + \mathcal{O}(\mu^3)\,, \tag{14}$$

which leads to the formal expansion

$$\mathcal{G}_0[\mu] \simeq G - G\mu\Gamma_4 G - \frac{\mu^2}{2}G\Lambda G + \mu^2 G\Gamma_4 G\Gamma_4 G + \cdots \tag{15}$$

with Γ_4

$$\Gamma_4(y_1,y_2) = \frac{\partial\mathcal{G}_0^{-1}[\mu](y_1,y_2)}{\partial\mu}\Big|_{\mu=0}\,, \tag{16}$$

and Λ

$$\Lambda(y_1,y_2) = \frac{\partial^2\mathcal{G}_0^{-1}[\mu](y_1,y_2)}{\partial^2\mu}\Big|_{\mu=0}\,. \tag{17}$$

In coordinate space the dressed vertex $\Gamma_4(x,y)$ is given as the derivative of the inverse quark propagator $\mathcal{G}_0^{-1}[\mu]$ with respect to the chemical potential μ. Taking the derivative in Eq. (5) and putting it into Eq. (16), we have

$$\Gamma_4(y_1,y_2) = -\gamma_4\delta(y_1-y_2) + \frac{\delta\Sigma_0[\mu](y_1,y_2)}{\delta\mu}\Big|_{\mu=0}\,. \tag{18}$$

Substituting Eqs. (8) and (15) into Eq. (18), the inhomogeneous ladder Bethe–Salpeter equation (BSE) for Γ_4 vertex reads

$$\Gamma_4(y_1,y_2) = -\gamma_4\,\delta(y_1-y_2) - \frac{4}{3}g_s^2 D(y_1-y_2)\int \mathrm{d}^4u_1\,\mathrm{d}^4u_2\,\gamma_\nu G(y_1,u_1)\Gamma_4(u_1,u_2)G(u_2,y_2)\gamma_\nu\,. \tag{19}$$

Fourier transform of Eq. (19) leads then to the momentum space form of $\Gamma_4(p,0)$,

$$\Gamma_4(p,0) = -\gamma_4 - \frac{4}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}g_s^2 D_{\mu\nu}(p-k)\gamma_\mu G(k)\Gamma_4(k,0)G(k)\gamma_\nu\,. \tag{20}$$

Equations (9) and (20) yields

$$\Gamma_4(p,0) = \mathrm{i}\frac{\partial G^{-1}(p)}{\partial p_4} = -\gamma_4 A(p^2) - 2p_4\gamma\cdot p\frac{\partial A(p^2)}{\partial p^2} + 2\mathrm{i}p_4\frac{\partial B(p^2)}{\partial p^2}\,, \tag{21}$$

i.e., the vector "Ward identity" is satisfied.[7,8] This means that it is the nonperturbative dressing effects that modifies the fermion piece of the Euclidean action at finite chemical μ (in the case of the chiral limit): $\gamma\cdot\partial - \gamma_4\mu \to \gamma\cdot\partial - \Gamma_4\mu$.

Similarly, we have the momentum space form of $\Lambda(p,0)$,

$$\begin{aligned}\Lambda(p,0) = &-\frac{4}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}g_s^2 D(p-k)\gamma_\mu G(k)\Lambda(k,0)G(k)\gamma_\mu \\ &+\frac{8}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}g_s^2 D(p-k)\gamma_\mu G(k)\Gamma_4(k,0)G(k)\Gamma_4(k,0)G(k)\gamma_\mu\,.\end{aligned} \tag{22}$$

From Lorentz structure analysis, the most general form for the $\Lambda(p,0)$ which fulfills Eq. (22) reads

$$\Lambda(p,0) = \mathrm{i}\vec{\gamma}\cdot\vec{p}\,C(p) + \mathrm{i}\gamma_4 p_4 E(p) + F(p)\,. \tag{23}$$

By suitable projection procedure (multiplying by appropriate gamma matrices and then taking the trace) the three scalar function $C(p)$, $E(p)$, and $F(p)$ are found to satisfy the coupled Dyson–Schwinger equations,

$$
\begin{aligned}
F(p) = {} & -\frac{16}{3}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)}{[k^2A^2(k^2)+B^2(k^2)]^2}[(B^2(k^2)-k^2A^2(k^2))F(k)+2\vec{k}^{\,2}A(k^2)B(k^2)C(k) \\
& +2k_4^2A(k^2)B(k^2)E(k)] + \frac{32}{3}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)}{[k^2A^2(k^2)+B^2(k^2)]^3}\Big\{(k^2-4k_4^2)A^4(k^2)B(k^2) \\
& +4k_4^2k^2A^4(k^2)\frac{\partial B(k^2)}{\partial k^2} - 12k_4^2k^2A^3(k^2)B(k^2)\frac{\partial A(k^2)}{\partial k^2} - 12k_4^2A^2(k^2)B^2(k^2)\frac{\partial B(k^2)}{\partial k^2} \\
& +8k_4^2k^4A^3(k^2)\frac{\partial A(k^2)}{\partial k^2}\frac{\partial B(k^2)}{\partial k^2} + A^2(k^2)B^3(k^2) + 12k_4^2k^2A^2(k^2)B(k^2)\Big[\frac{\partial B(k^2)}{\partial k^2}\Big]^2 \\
& -12k_4^2k^4A^2(k^2)B(k^2)\Big[\frac{\partial A(k^2)}{\partial k^2}\Big]^2 - 24k_4^2k^2A(k^2)B^2(k^2)\frac{\partial A(k^2)}{\partial k^2}\frac{\partial B(k^2)}{\partial k^2} \\
& +4k_4^2A(k^2)B^3(k^2)\frac{\partial A(k^2)}{\partial k^2} + 4k_4^2k^2B^3(k^2)\Big[\frac{\partial A(k^2)}{\partial k^2}\Big]^2 - 4k_4^2B^3(k^2)\Big[\frac{\partial B(k^2)}{\partial k^2}\Big]^2\Big\}, \qquad (24)
\end{aligned}
$$

$$
\begin{aligned}
p_4^2E(p) = {} & -\frac{8}{3}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)k_4\cdot p_4}{[k^2A^2(k^2)+B^2(k^2)]^2}[2\left(k^2-k_4^2\right)A^2(k^2)C(k)+\left(2k_4^2-k^2\right)A^2(k^2)E(k) \\
& +2A(k^2)B(k^2)F(k)-B^2(k^2)E(k)] + \frac{16}{3}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)k_4\cdot p_4}{[k^2A^2(k^2)+B^2(k^2)]^3} \\
& \times\Big\{(3k^2-4k_4^2)A^5(k^2) - 4k^2\left(2k_4^2-k^2\right)A^4(k^2)\frac{\partial A(k^2)}{\partial k^2} - 4(4k_4^2-k^2)A^3(k^2)B(k^2)\frac{\partial B(k^2)}{\partial k^2} \\
& +3A^3(k^2)B^2(k^2) + 4(k^2+2k_4^2)A^2(k^2)B^2(k^2)\frac{\partial A(k^2)}{\partial k^2} - 24k_4^2k^2A^2(k^2)B(k^2)\frac{\partial A(k^2)}{\partial k^2}\frac{\partial B(k^2)}{\partial k^2} \\
& -4k_4^2k^4A^3(k^2)\Big[\frac{\partial A(k^2)}{\partial k^2}\Big]^2 + 4k_4^2k^2A^3(k^2)\Big[\frac{\partial B(k^2)}{\partial k^2}\Big]^2 + 12k^2k_4^2A(k^2)B^2(k^2)\Big[\frac{\partial A(k^2)}{\partial k^2}\Big]^2 \\
& -12k_4^2A(k^2)B^2(k^2)\Big[\frac{\partial B(k^2)}{\partial k^2}\Big]^2 + 4A(k^2)B^3(k^2)\frac{\partial B(k^2)}{\partial k^2} + 8k_4^2B^3(k^2)\frac{\partial A(k^2)}{\partial k^2}\frac{\partial B(k^2)}{\partial k^2}\Big\}, \qquad (25)
\end{aligned}
$$

$$
\begin{aligned}
\vec{p}^{\,2}C(p) = {} & -\frac{8}{3}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)\vec{k}\cdot\vec{p}}{[k^2A^2(k^2)+B^2(k^2)]^2}[(k^2-2k_4^2)A^2(k^2)C(k)+2k_4^2A^2(k^2)E(k) \\
& +2A(k^2)B(k^2)F(k)-B^2(k^2)C(k)] + \frac{16}{3}\int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)\vec{k}\cdot\vec{p}}{[k^2A^2(k^2)+B^2(k^2)]^3} \\
& \times\Big\{(k^2-4k_4^2)A^5(k^2) - 8k^2k_4^2A^4(k^2)\frac{\partial A(k^2)}{\partial k^2} - 16k_4^2A^3(k^2)B(k^2)\frac{\partial B(k^2)}{\partial k^2} \\
& +A^3(k^2)B^2(k^2) + 8k_4^2A^2(k^2)B^2(k^2)\frac{\partial A(k^2)}{\partial k^2} - 24k_4^2k^2A^2(k^2)B(k^2)\frac{\partial A(k^2)}{\partial k^2}\frac{\partial B(k^2)}{\partial k^2} \\
& -4k_4^2k^4A^3(k^2)\Big[\frac{\partial A(k^2)}{\partial k^2}\Big]^2 + 4k_4^2k^2A^3(k^2)\Big[\frac{\partial B(k^2)}{\partial k^2}\Big]^2 + 12k^2k_4^2A(k^2)B^2(k^2)\Big[\frac{\partial A(k^2)}{\partial k^2}\Big]^2 \\
& -12k_4^2A(k^2)B^2(k^2)\Big[\frac{\partial B(k^2)}{\partial k^2}\Big]^2 + 8k_4^2B^3(k^2)\frac{\partial A(k^2)}{\partial k^2}\frac{\partial B(k^2)}{\partial k^2}\Big\}. \qquad (26)
\end{aligned}
$$

For a given model gluon propagator $g_s^2D(p)$, we can solve consistently Eq. (11) and Eqs. (24) $\sim$ (26) to obtain the five scalar functions $A(p^2)$, $B(p^2)$, $C(p)$, $E(p)$, and $F(p)$.

As was shown above in Eq. (11), there exist two solutions, i.e. the "Nambu–Goldstone" solution ($B(p^2)\neq 0$) and the "Wigner" solution ($B(p^2)\equiv 0$). Substituting these two solutions into Eq. (21) and Eqs. (24) $\sim$ (26), we can obtain two solutions for $\Gamma_4(p,0)$, $C(p)$, $E(p)$, and $F(p)$ which give the "Nambu–Goldstone" solution and the "Wigner" solution for the quark propagator at small but finite chemical potential. For example, substituting $B(p^2)\equiv 0$ into

Eq. (21) and Eqs. (24) $\sim$ (26), we have the "Wigner" solution for $\Gamma_4'(p,0)$

$$\Gamma_4'(p,0) = -\gamma_4 A'(p^2) - 2p_4\gamma\cdot p\frac{\partial A'(p^2)}{\partial p^2}\,, \tag{27}$$

and the scalar functions ($C'(p)$, $E'(p)$, and $F'(p)$)

$$F'(p) = \frac{16}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2 D(p-k)}{k^2A'^2(k^2)}F'(k)\,, \tag{28}$$

$$\begin{aligned} p_4^2E'(p) = {} & -\frac{8}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2D(p-k)k_4\cdot p_4}{k^4A'^2(k^2)}[2(k^2-k_4^2)C'(k)+(2k_4^2-k^2)E'(k)] \\ & + \frac{16}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2D(p-k)k_4\cdot p_4}{k^6A'^3(k^2)}\Big\{(3k^2-4k_4^2)A'^2(k^2) \\ & - 4k^2(2k_4^2-k^2)A'(k^2)\frac{\partial A'(k^2)}{\partial k^2} - 4k_4^2k^4\Big[\frac{\partial A'(k^2)}{\partial k^2}\Big]^2\Big\}\,, \end{aligned} \tag{29}$$

$$\begin{aligned} \vec{p}^{\,2}C'(p) = {} & -\frac{8}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2D(p-k)\vec{k}\cdot\vec{p}}{k^4A'^2(k^2)}[(k^2-2k_4^2)C'(k)+2k_4^2E'(k)] \\ & + \frac{16}{3}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{g_s^2D(p-k)\vec{k}\cdot\vec{p}}{k^6A'^3(k^2)}\Big\{(k^2-4k_4^2)A'^2(k^2) \\ & - 8k_4^2k^2A'(k^2)\frac{\partial A'(k^2)}{\partial k^2} - 4k_4^2k^4\Big[\frac{\partial A'(k^2)}{\partial k^2}\Big]^2\Big\}\,. \end{aligned} \tag{30}$$

So we have the chemical potential dependence of the dressed quark propagator in "Nambu–Goldstone" and "Wigner" phase separately (here we only consider the second order dependence of $\mathcal{G}_0^{-1}[\mu]$ upon μ at the mean field level),

$$\begin{aligned} \mathcal{G}_0^{(NG)^{-1}}[\mu] \simeq {} & \mathrm{i}\vec{\gamma}\cdot\vec{p}\Big[A(p^2)+2\mathrm{i}\mu p_4\frac{\partial A(p^2)}{\partial p^2}+\frac{1}{2}\mu^2C(p)\Big]+B(p^2)+\frac{1}{2}\mu^2F(p)+2\mathrm{i}\mu p_4\frac{\partial B(p^2)}{\partial p^2} \\ & + \mathrm{i}\gamma_4\Big[p_4A(p^2)+2\mathrm{i}\mu p_4^2\frac{\partial A(p^2)}{\partial p^2}+\mathrm{i}\mu A(p^2)+\frac{1}{2}\mu^2p_4E(p)\Big]+\mathcal{O}(\mu^3)\,, \end{aligned} \tag{31}$$

$$\begin{aligned} \mathcal{G}_0^{(W)^{-1}}[\mu] \simeq {} & \mathrm{i}\vec{\gamma}\cdot\vec{p}\Big[A'(p^2)+2\mathrm{i}\mu p_4\frac{\partial A'(p^2)}{\partial p^2}+\frac{1}{2}\mu^2C'(p)\Big]+\frac{1}{2}\mu^2F'(p) \\ & + \mathrm{i}\gamma_4\Big[p_4A'(p^2)+2\mathrm{i}\mu p_4^2\frac{\partial A'(p^2)}{\partial p^2}+\mathrm{i}\mu A'(p^2)+\frac{1}{2}\mu^2p_4E'(p)\Big]+\mathcal{O}(\mu^3)\,. \end{aligned} \tag{32}$$

As is shown in Eqs. (31) and (32), for $\mu\neq 0$ the dressed-quark self-energies in general acquire an imaginary part driven by the chemical potential μ. Just as pointed out in Ref. [4], this effect is not observed in the study of the Nambu–Jona-Lasinio model[9−12] and the instanton-induced four-fermion interaction model in which the interaction is energy-independent,[13,14] i.e., instantaneous. It is now apparent that the above approach for getting the nonperturbative vertex $\Gamma_4(p,0)$ and $\Lambda(p,0)$ has the advantage that the effects of dynamical and explicit chiral symmetry breaking driven by chemical potential μ can be analyzed separately. Just due to this merit, we obtained the "Wigner" solution at finite chemical potential without any arbitrariness. In addition, we want to stress that the above approach for getting the nonperturbative vertex has been proven to be very useful for the studies of nonperturbative vector,[7,8] axial vector,[15] and scalar vertex[16] at mean field level.

With the two "phases" characterized by Eqs. (31) and (32), one can study the bag constant and the measure of dynamical chiral breaking[17] in the presence of finite chemical potential. These question will be further discussed elsewhere.

To summarize, in the present paper, we provide a general recipe to calculate the low chemical potential dependence of the dressed quark propagator at the mean field level in the framework of GCM. This employs a consistent treatment of dressed quark propagator $G(p)$, the dressed vertex $\Gamma_4(p,0)$, and $\Lambda(p,0)$, which are all determined from the effective quark-quark interaction in GCM. From this, of particular interest here is to give a general recipe to find without arbitrariness the solution representing the "Wigner" phase at non-zero chemical potential, which is very important for the study of QCD phase structure.

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