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# Study of Survival Probability of Super Heavy Nuclei*

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***Abstract*** *The survival probability of super heavy nuclei produced in cold fusion reactions is studied by using the standard Fermi gas level density formula and analyzed with fission and neutron evaporation characteristics predicted in different theoretical models. The level density formula used in this letter suppresses the ratio of neutron emission width to fission width, $\Gamma_n/\Gamma_f$. The dependence of $\Gamma_n/\Gamma_f$ on the saddle point level density parameter and excitation energy is also investigated.*

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**Key words:** super-heavy nuclei, survive probability, fission width, neutron emission width

Recently, the study of SHE (Super Heavy Element) is a very hot topic in nuclear physics. Theoretical models have been put forward.[1−3] The basic physical pictures of different theoretical models are quite different, but all the models encounter the calculation of the survival probability of the excited compound nucleus, which is considered as one of the crucial factors for producing SHE. The competition between the neutron emission and the fission during the de-excitation process is the main mechanism to determine the survival probability. In this letter, the level density formula by H. Feshbach[4] is used to study the ratio of neutron emission width to fission width, $\Gamma_n/\Gamma_f$, under conditions of different fission and neutron evaporation characteristics predicted in different theoretical models. The dependence of $\Gamma_n/\Gamma_f$ on the ratio of level density at saddle point to that at ground state and on the excitation energy is also studied.

The survival probability under one neutron emission can be written as

$$W_{\rm sur}(E^*_{\rm CN}, J) = P_1(E^*_{\rm CN}, J)\frac{\Gamma_n(E^*_{\rm CN}, J)}{\Gamma_{\rm tot}(E^*_{\rm CN}, J)}\,, \qquad (1)$$

where $E^*_{\rm CN}$ and $J$ are the excitation energy and the angular momentum of compound nucleus, respectively. $P_1(E^*_{\rm CN}, J)$ is the realization probability of the $1n$ channel at given $E^*_{\rm CN}$ and $J$. $\Gamma_n(E^*_{\rm CN}, J)$ is the width of neutron emission and $\Gamma_{\rm tot}(E^*_{\rm CN}, J)$ is the total width of all the de-excitation channel widths including fission width. For $1n$ channel, $W_{\rm sur}(E^*_{\rm CN}, J)$ can be approximated[5] by $W_{\rm sur}(E^*_{\rm CN}, J=0)\exp[-J(J+1)/J_{\max}(J_{\max}+1)]$, so only $J=0$ case is considered here.

In a cold fusion reaction, the widths for the emission of a proton or $\alpha$ particle are much less than that for the emission of neutrons. And $\gamma$ ray emission is important only when the excitation energy is smaller than one neutron separation energy, thus it can be neglected. Therefore, the total width reads approximately as

$$\Gamma_{\rm tot}(E^*_{\rm CN}) = \Gamma_n(E^*_{\rm CN}) + \Gamma_f(E^*_{\rm CN})\,, \qquad (2)$$

where $\Gamma_f(E^*_{\rm CN})$ is the width of fission.

From the Bohr–Wheeler formulae[6] which is based on the instant thermal-dynamical equilibrium between the compound nuclear state and the saddle point state, the width of fission can be written as

$$\Gamma_f = \frac{1}{2\pi\rho(E^*)}\int_0^{E^*-B_f} \rho(E^* - B_f - \varepsilon)\,{\rm d}\varepsilon\,, \qquad (3)$$

and the width of neutron emission is

$$\Gamma_n = \frac{1}{2\pi\rho(E^*)}\frac{2M_nR^2}{\hbar^2}g\int_0^{E^*-B_n} \varepsilon\rho(E^* - B_n - \varepsilon)\,{\rm d}\varepsilon\,, \qquad (4)$$

where $\rho(E^*)$ is the level density, $R$, $B_f$, and $B_n$ are the radius, fission barrier, and the neutron separation energy of the compound nucleus, respectively, $M_n$ is the mass of neutron, $g$ is the spin factor of neutron, and $a$ is the level density parameter.

In the statistical code GROGIF used in Ref. [5], the level density $\rho(E^*) \sim \exp[2\sqrt{(aE^*_{\rm CN})}]$ is adopted. However, according to the Fermi-gas model,[4] the level density at the excitation energy $E_{\rm CN}$ is expressed as

$$\rho(E^*) = \frac{1}{\sqrt{48}E^*_{\rm CN}}\exp\left[\,2\sqrt{aE^*_{\rm CN}}\,\right]. \qquad (5)$$

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In cold fusion reactions, the compound nuclei are usually formed at the excitation energy around 10 MeV. The factor $1/E^*_{\rm CN}$ is of importance. The existence of SHE is mainly attributed to the shell closure beyond $Z = 82$ and $N = 126$ whose structure is different from that of nuclei we are familiar with. The fission barrier for SHE can be divided into the macroscopic part $B_f^{\rm LD}$ determined by liquid-drop and microscopic part $B_f^{\rm Mic}$ determined by shell correction. The microscopic energy will be damped due to the dependence of shell effect on the nuclear excitation. Thus, the fission barrier can be written as

$$B_f = B_f^{\rm LD} + B_f^{\rm Mic}\exp\Big[-\frac{E^*_{\rm CN}}{E_D}\Big]\,, \qquad (6)$$

where $E_D$ is a damping factor, in this letter, we take the expression[7]

$$E_D = 0.4A^{4/3}/a\,, \qquad (7)$$

where $A$ is the mass number of the nucleus. From the liquid drop model, the macroscopic fission barrier decreases as the element charge number increases. In Ref. [8], for the nuclei $^{256}$No, $^{258}$Rf, and $^{262}$Sg, the $B_f^{\rm LD}$ values are estimated as 1.1, 0.5, 0.2 MeV, respectively. For $Z > 106$, $B_f^{\rm LD} = 0$. Because the shell correction energy at the saddle point is usually disregarded,[9,10] the microscopic barrier $B_f^{\rm Mic}$ equals the shell correction energy of the nucleus at its ground state. The shell correction energy and the neutron separation energy values are taken from Refs. [11] and [12]. The level density parameter at saddle point $a_f = 1.1a$ is used in our calculations.

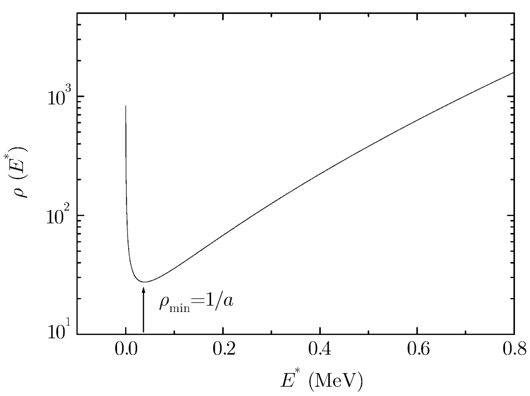


**Fig. 1** Level density as a function of the excitation energy.

The level density shown in Eq. (5) has an abnormal behavior in the excitation energy range from zero to $1/a$ as shown in Fig. 1, which is unphysical. We avoid this region by simply disregarding it since it is very small. Then equations (3) and (4) can be written as

$$\Gamma_f = \frac{1}{2\pi}E^*\,{\rm e}^{-2\sqrt{aE^*}}\int_{1/a}^{E^*-B_f}\frac{{\rm e}^{2\sqrt{a\varepsilon}}}{\varepsilon}\,{\rm d}\varepsilon\,, \qquad (8)$$

and

$$\Gamma_n = \frac{M_nR^2g}{\pi h^2}E^*\,{\rm e}^{-2\sqrt{aE^*}}\Big[(E^*-B_n)\int_{1/a}^{E^*-B_n}\frac{{\rm e}^{-2\sqrt{a\varepsilon}}}{\varepsilon}\,{\rm d}\varepsilon - \Big[\frac{{\rm e}^{-2\sqrt{a(E^*-B_n)}}}{a}\big(\sqrt{a(E^*-B_n)}-1/2\big)-\frac{{\rm e}^2}{2a}\Big]\Big]\,. \qquad (9)$$

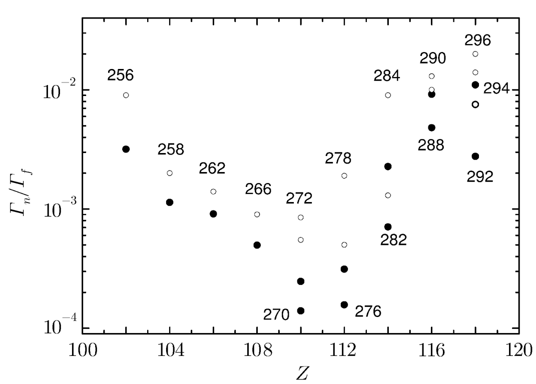


**Fig. 2** The ratio of neutron emission width to fission width as a function of the charge number. Open circles are for the results from Fig. 3 in Ref. [5] and solid circles are for our results. The neutron separation energies and fission barriers are taken from Refs. [11] and [12]. The numbers in the figure represent the mass numbers of the nuclei.

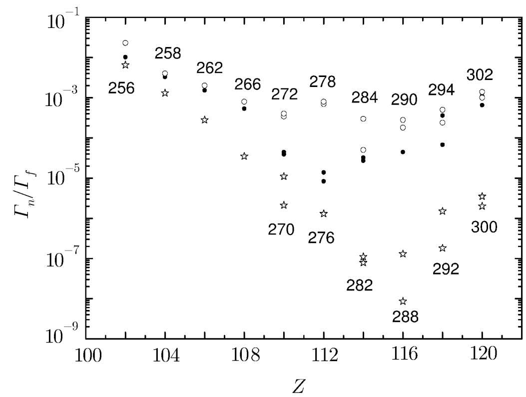


**Fig. 3** The ratio of neutron emission width to fission width as a function of the charge number. Open circles are for the results from Fig. 4 in Ref. [5] and solid circles are for our results. The star symbols are for the results from Table 1 of Ref. [2]. The neutron separation energies and fission barriers are taken from Refs. [2], [14], and [6]. The numbers in the figure represent the mass numbers of the nuclei.

The excitation energies of synthesized super heavy nuclei for different combinations of the projectile and target are taken from Refs. [5] and [13]. The $\Gamma_n/\Gamma_f$ values calculated with the level density (5) and with the neutron separation energy and fission barrier for the $1n$ evaporation channel taken from Refs. [11] and [12] are shown with solid circles in Fig. 2 for nuclei with different charge number $Z$. The results in Fig. 3 of Ref. [5] are represented with open circles for comparison. It is found that the same tendencies are obtained. The values of $\Gamma_n/\Gamma_f$ decrease with nuclear charge number $Z$ for nuclei with $Z \leq 112$ and increase for those with $Z > 112$. The increase of the $\Gamma_n/\Gamma_f$ at $Z > 112$ may be caused by the magic number predicted by the theoretical models. However, generally our calculated results are smaller than the results in Ref. [5] (about three times or even more, especially for $Z = 110, 112$ ). It seems that the influence of factor $1/E^*_{\mathrm{CN}}$ is rather important.

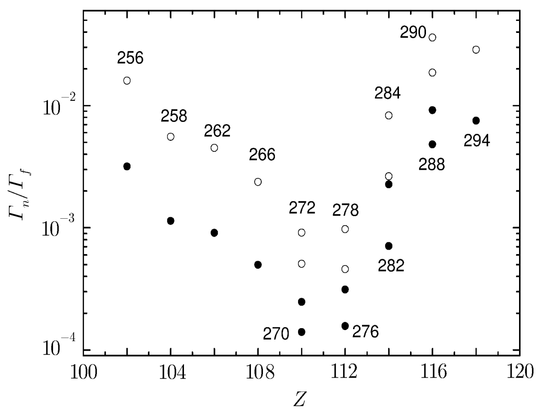


**Fig. 4** The ratio of neutron emission width to fission width as a function of the charge number. Open circles are for the results calculated with $a_f = a$ and solid circles are for the results calculated with $a_f = 1.1a$. The numbers in the figure represent the mass numbers of the nuclei.

The calculated results with the neutron separation energy and fission barrier data taken from Refs. [2], [14], and [15] are shown with solid circles in Fig. 3. It is observed that the $\Gamma_n/\Gamma_f$ increases with $Z$ from $Z = 112$, but the $\Gamma_n/\Gamma_f$ values for $Z > 112$ in Fig. 3 are much smaller than those in Fig. 2 due to the difference between the fission barriers in Refs. [2], [14], and [15] and those in Refs. [11] and [12]. In Ref. [5], the results of $\Gamma_n/\Gamma_f$ with neutron separation energies and fission barriers data from Refs. [2], [14], and [15] are shown with open circles in Fig. 4 of Ref. [5]. In order to make comparisons, we also show these results with open circles in Fig. 3, and similar tendencies are found for the two sets of results. For $Z = 110, 112, 114, 116$ nuclei, the values of $\Gamma_n/\Gamma_f$ in solid circles are 4 to 10 times smaller than those in open circles due to the different forms of level density formulae. The values of $\Gamma_n/\Gamma_f$ in Ref. [2] are studied with the expression $\Gamma_n/\Gamma_f = \exp(2\sqrt{a_n E^*_n} - 2\sqrt{a_f E^*_f})$ in which some physical conception has been over simplified. The calculated results listed in Table 1 of Ref. [2] are shown with star symbols also in Fig. 3. Great differences can be found between the results in star symbols and the results in solid circles. Generally, the results in solid circles are larger than those in stars, especially for the relatively heavier elements. For $Z > 110$ nuclei, our results are about 2 to 3 orders of magnitude greater than those in stars.

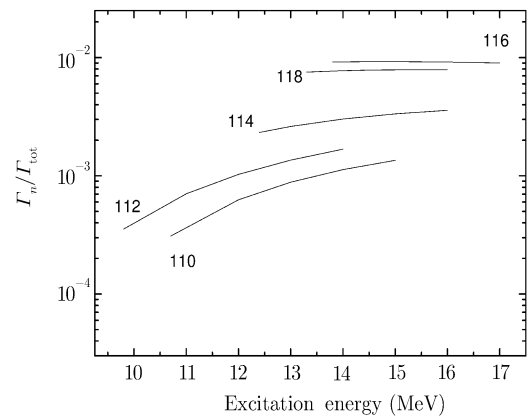


**Fig. 5** The ratio of neutron emission width to fission width as a function of the excitation energy for some super heavy nuclei. The numbers in the figure represent the mass numbers of the nuclei.

In the above calculations, the level density parameter at saddle point $a_f$ equals $1.1a$ with $a$ being the level density parameter at ground state. The influence of $a_f/a$ on $\Gamma_n/\Gamma_f$ is studied as follows. The results calculated with $a_f = a$ and $a_f = 1.1a$ are shown as open circles and solid circles in Fig. 4, respectively. The results in solid circles are generally smaller than those in open circles (about $3 \sim 5$ times). It seems that $\Gamma_n/\Gamma_f$ is rather sensitive to the variance of the level density parameter $a_f$ at saddle point. The increase of the level density parameter $a_f$ will lead to the decrease of the $\Gamma_n/\Gamma_f$ because the larger level density at saddle point can make fission probability larger.

The dependence of $\Gamma_n/\Gamma_f$ on excitation energy for some compound nuclei is shown in Fig. 5. For the relatively light nuclei, $\Gamma_n/\Gamma_f$ increases slowly with excitation energy while for the heavier nuclei, the dependence of $\Gamma_n/\Gamma_f$ on excitation energy is very weak. The dependence of $\Gamma_n/\Gamma_f$ and that of survive probability on excitation energy of nucleus $^{258}104$ are shown in Figs. 6(a) and 6(b), respectively. The $\Gamma_n/\Gamma_f$ increases by an order of magnitude as the excitation energy increases from 10 MeV to 18 MeV. However, the survival probability $W_{\mathrm{sur}}$ for $^{258}104$ increases at first, then reaches a peak with the excitation energy at about 14 MeV, after that, it decreases with excitation energy.

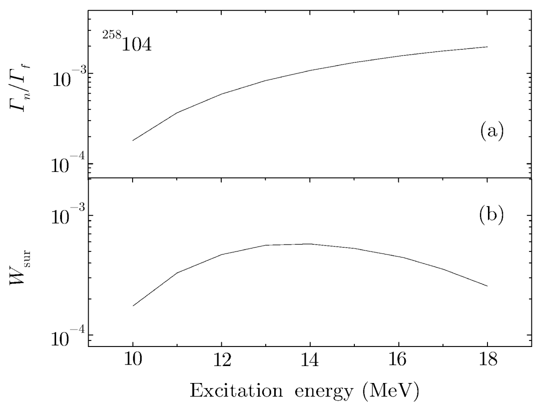


**Fig. 6** The ratio of neutron emission width to fission width (a) and survive probability (b) as a function of the excitation energy for $^{258}$Rf.

In summary, the level density form (5) is used to study the $\Gamma_n/\Gamma_f$ in the de-excitation process for the super heavy nuclei produced in cold fusion reactions. Our results show that $\Gamma_n/\Gamma_f$ decreases with element charge number at $Z \leq 112$ and increases at $Z > 112$. The systematics is coincident with those in Figs. 2 and 3 of Ref. [5]. The effects of different fission and neutron evaporation characteristics predicted in different theoretical models to $\Gamma_n/\Gamma_f$ are analyzed. Unfortunately, $\Gamma_n/\Gamma_f$ is not a measurable quantity, thus the correct calculation must be tested by the final evaporation residue cross section. The level density used in this letter is a standard form of Fermi-gas model, however, the factor $1/E^*_{\rm CN}$ is omitted in Ref. [5]. Our results show that its correction to $\Gamma_n/\Gamma_f$ is noticeable. Furthermore the ratio $\Gamma_n/\Gamma_f$ is sensitive to the single particle level density parameter $a_f$. And $\Gamma_n/\Gamma_f$ increases with the excitation energy of the compound nucleus with different tendencies for different compound nuclei, while the corresponding survival probability $W_{\rm sur}$ usually has a maximum value at certain excitation energy.

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