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Quantum Standard Teleportation Based on the Generic Measurement Bases*

HAO San-Ru,[1,2,†] HOU Bo-Yu,[2] XI Xiao-Qiang,[2] and YUE Rui-Hong[2]

[1]Calculating Physics Division, Department of Computer Teaching, Hunan Normal University, Changsha 410081, China

[2]Institute of Modern Physics, Northwest University, Xi'an 710069, China

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Abstract *We study the quantum standard teleportation based on the generic measurement bases. It is shown that the quantum standard teleportation does not depend on the explicit expression of the measurement bases. We have given the correspondence relation between the measurement performed by Alice and the unitary transformation performed by Bob. We also prove that the single particle unknown states and the two-particle unknown cat-like states can be exactly transmitted by means of the generic measurement bases and the correspondence unitary transformations.*

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1 Introduction

Quantum teleportation protocols, which make use of a classical communication channel and the quantum entanglement pairs, play an important role in quantum information processing. The standard teleportation protocol[1] can be described as follows. An unknown quantum state is disassembled into purely classical information and purely nonclassical Einstein–Podolsky–Rosen (EPR) correlations. The sender traditionally named "Alice" and the receiver "Bob", which are spatially separated, prearrange the sharing of an EPR-correlated pair of particles. Then Alice makes a joint measurement on her EPR particle and the unknown quantum state system, and sends Bob the classical result of this measurement by the classical channel. Knowing this, Bob can convert the state of his EPR particle into an exact replica of the unknown state which Alice destroyed by applying a unitary local transformation suggested by Alice.

Because of the importance of quantum teleportation in quantum information theory,[2−9] it has attracted much attention of both theorists[10−12] and experimentalists.[13−16] Since the first quantum teleportation protocol was proposed by Bennett *et al.* in 1993, quantum teleportation protocols have been generalized by many authors.[17−19] These generalizations were mainly made in the following directions. (i) Generalizing the two-dimensional quantum states to d-dimensional states (i.e. generalizing qubits to qudits); (ii) Generalizing the single particle unknown states to multi-particle unknown states; (iii) Generalizing the joint von-Neumann measurement to positive operator valued measurement (POVM);[6,20,21,23] (iv) Generalizing the maximally entangled quantum channel to partially entangled quantum one.[22]

The core issue of the standard teleportation is the entanglement source shared by the communication parties and the joint measurement performed by Alice on the unknown state and her particles. So far all the joint measurements on the composite system of the unknown state and Alice's particles of the quantum teleportation for the generalization cases mentioned above are based on the Bell bases. In fact, this condition is not necessary. One can complete teleportation by means of a generic measurement bases. In this paper, we want to generalize Bell measurement bases into a generic one and consider the teleportation of single particle and two particles unknown state under the generic measurement bases.

2 Teleportation of a Single Particle Unknown State

Before we discuss the generalized teleportation case, we first review the original work of teleportation.[1] In Ref. [1], Bennett *et al.* discussed the teleportation of an unknown qubit state. In their teleportation scheme, which Alice and Bob share a Bell state $|\varphi\rangle = (1/\sqrt{2})(|00\rangle + |11\rangle)$, Alice wants to teleport the unknown state $|\alpha\rangle = a|0\rangle + b|1\rangle$ with $|a|^2 + |b|^2 = 1$. The composite system, which consists of the unknown particle (particle 1), Alice's particle (particle 2), and Bob's particle (particle 3), is expressed as

$$|\Psi_0\rangle \equiv |\alpha\rangle \otimes |\varphi\rangle = \frac{1}{\sqrt{2}}(a|000\rangle + b|100\rangle + a|011\rangle + b|111\rangle)\,. \tag{1}$$

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[†] E-mail: srhao@sparc2.hunnu.edu.cn

Alice first makes a controlled-NOT (G_{12}) operation on the particles 1 and 2 with the particle 1 as the controlled qubit and the particle 2 as the target qubit, and then makes a Hadamard gate (H-matrix) operation on the particle 1 again. The total quantum system becomes

$$|\Psi_1\rangle \equiv (H \otimes 1 \otimes 1)(G_{12} \otimes 1)|\alpha\rangle \otimes |\varphi\rangle = \frac{1}{2}[|00\rangle \otimes \sigma_0|\alpha\rangle + |10\rangle \otimes \sigma_3|\alpha\rangle + |01\rangle \otimes \sigma_1|\alpha\rangle + |11\rangle \otimes (-\mathrm{i}\sigma_2)|\alpha\rangle]\,. \tag{2}$$

where σ_0, σ_1, σ_2, and σ_3 are the identity matrix $\mathbf{1}$ and the Pauli matrices σ_x, σ_y, and σ_z, respectively. If Alice measures the first two qubits based on the measurement bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$, Bob's particle state will project onto one of the four, i.e. $\sigma_0|\alpha\rangle$, $\sigma_1|\alpha\rangle$, $\sigma_2|\alpha\rangle$, and $\sigma_3|\alpha\rangle$, each with the probability of 1/4. Now Bob can transform his particle state into the original unknown state by means of acting the identity matrix σ_0 and the Pauli matrices $\sigma_1, \sigma_2, \sigma_3$ on his particle state. Because each measurement can give the desired results with probability $p = 1/4$, we can obtain the desired unknown state with certainty.

Let $U \equiv (H \otimes \mathbf{1})G_{12}$ be the unitary operator which acts on the two-qubits state. We define a group of measurement operators

$$\{\Pi_0, \Pi_1, \Pi_2, \Pi_3\} = \{U^\dagger|00\rangle\langle 00|U, U^\dagger|01\rangle\langle 01|U, U^\dagger|11\rangle\langle 11|U, U^\dagger|10\rangle\langle 10|U\}\,, \tag{3}$$

then the above-mentioned procedure can be rewritten in the reduced matrix form as

$$\rho_\alpha^{\rm out} = \sum_{k=0}^{3} \mathrm{Tr}_{\,\rm ua}\,(\Pi_k \otimes \sigma_k^{\rm T})|\Psi_0\rangle\langle\Psi_0|(\Pi_k^\dagger \otimes \sigma_k^*) = |\alpha\rangle\langle\alpha|\,, \tag{4}$$

where $\mathrm{Tr}_{\,\rm ua}$ is the partial trace over the unknown particle (particle 1) and Alice's particle (particle 2), $\sigma_k^{\rm T}$ is the transpose operation of σ_k and σ_k^* is the complex conjugate operation of σ_k. Here we have counted all possible measurement results by means of the sum operation in above equation.

In fact, equation (4) is a special case of teleportation. Here, the unitary operator U is limited to the product of the Hadamard gate and the controlled-NOT gate. We can further generalize this unitary operator to a more general form and consider the most generic measurement bases Φ_{st} (see Eq. (6)).

Now we return to the teleportation of d-level qubit unknown state. Let $\{|n\rangle, n = 0, 1, \ldots, d-1, d < \infty\}$ be an orthogonal normalized basis of a d-dimensional Hilbert space $\mathcal{H}$. We denote by $\mathcal{B}(\mathcal{H})$ the space of bounded linear operators on $\mathcal{H}$. We also denote by $\{U_{st}, s, t = 0, 1, \ldots, d-1, U_{st} \in \mathcal{B}(\mathcal{H})\}$ the unitary operators on $\mathcal{H}$. Any linear operator $\mathcal{K} \in \mathcal{B}(\mathcal{H})$: $\mathcal{H} \longrightarrow \mathcal{H}$ can be represented by a $d \times d$-matrix as

$$\mathcal{K}|n\rangle = \sum_{m=0}^{d-1} K_{nm}|m\rangle\,, \quad K_{nm} \in \mathcal{C}\,. \tag{5}$$

In this section, we shall only consider the three-tensor Hilbert space, $\mathcal{H}^{\otimes 3}$, where Alice has the first and the second Hilbert space, and the third Hilbert space belongs to Bob. For the five-tensor Hilbert space, $\mathcal{H}^{\otimes 5}$, we shall discuss in the next section. Let $|\omega\rangle$ be the maximally entangled pure state $|\omega\rangle = (1/\sqrt{d})\sum_{n=0}^{d-1}|nn\rangle$. The generic measurement bases can be expressed by

$$|\Phi_{st}\rangle = (\mathbf{1} \otimes U_{st})|\omega\rangle \quad \in \mathcal{H} \otimes \mathcal{H}\,, \quad U_{st} \in \mathcal{B}(\mathcal{H})\,, \tag{6}$$

where $\mathbf{1}$ is the $d \times d$ identity matrix, and U_{st} is any unitary operator which satisfies

$$\begin{cases} U_{st}^\dagger U_{st} = \mathbf{1}\,, \\ \mathrm{Tr}\,(U_{st}^\dagger U_{s't'}) = d\delta_{ss'}\delta_{tt'}\,. \end{cases} \tag{7}$$

One can check that the generic measurement bases $|\Phi_{st}\rangle$ are orthogonal normalized bases of a d^2-dimensional Hilbert space. It is worth while noting that the unitary operator U_{st} can be any operator satisfying condition (7). Therefore, the measurement bases $\{|\Phi_{st}\rangle\}$ are the most generalized ones, and correspond to the special measurement bases $\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ in Bennett *et al.*'s work.[1]

Because $|\omega\rangle$ is a maximal entanglement state and the unitary operator U_{st} acts locally on $|\omega\rangle$, so the base $|\Phi_{st}\rangle$ defined by Eq. (6) is also the maximal entanglement state. In order to let the generic measurement bases $\{|\Phi_{st}\rangle, s, t = 0, 1, 2, \ldots, d-1\}$ be orthogonal normalized bases, the second constraint condition in Eq. (7) is the sufficient and necessary condition. That can be seen from the following expression

$$\begin{aligned} \langle\Phi_{st}|\Phi_{s't'}\rangle &= \langle\omega|(\mathbf{1} \otimes U_{st}^\dagger)(\mathbf{1} \otimes U_{s't'})|\omega\rangle \\ &= \langle\omega|(\mathbf{1} \otimes U_{st}^\dagger U_{s't'}|\omega\rangle \\ &= \frac{1}{d}\sum_{i,j=0}^{d-1}\langle ii|(\mathbf{1} \otimes U_{st}^\dagger U_{s't'}|jj\rangle \\ &= \frac{1}{d}\sum_{i,j=0}^{d-1}\langle i|j\rangle\langle i|U_{st}^\dagger U_{s't'}|j\rangle \\ &= \frac{1}{d}\sum_{i=0}^{d-1}\langle i|U_{st}^\dagger U_{s't'}|i\rangle \end{aligned}$$

$$= \frac{1}{d}\ \mathrm{Tr}\,(U_{st}^{\dagger}U_{s't'})\,. \tag{8}$$

It is obvious from the above expression that if the second constraint condition in Eq. (7) is satisfied, the generic measurement bases (6) are orthogonal normalized bases; while if the generic measurement bases are orthogonal normalized bases, then the second constraint condition in Eq. (7) is satisfied.

We can understand the constraint condition (7) from a special set of measurement bases, e.g., the Bell bases for the d-level quantum system. We first define two $d \times d$ matrices h and g such that

$$h|j\rangle = |j \oplus 1\rangle\,, \qquad g|j\rangle = \mu^{j}|j\rangle \tag{9}$$

with $\mu = \exp\{2\pi \mathrm{i}/d\}$ and $\{|j\rangle, j = 0, 1, \ldots, d-1\}$ are the bases of the d-dimensional Hilbert space, where $\oplus$ means addition modulo d. Second, we define quantum states $|\psi_{nm}\rangle$ as

$$|\psi_{nm}\rangle \equiv (\mathbf{1} \otimes h^{m}g^{n})|\Omega\rangle\,. \tag{10}$$

It is straightforward to calculate and obtain the following expression,

$$|\psi_{nm}\rangle \equiv (\mathbf{1} \otimes h^{m}g^{n})|\Omega\rangle = \frac{1}{\sqrt{d}}\sum_{k=0}^{d-1} \mathrm{e}^{2\pi kn/d}|k\rangle|k+m\rangle\,. \tag{11}$$

This just the Bell bases for the d-level quantum system. We introduce a matrix operator $U_{nm} = h^{m}g^{n}$. For the Bell bases (11), one can be straightforward to prove that U_{nm} is the unitary matrix and satisfies the trace orthogonal relation, i.e.,

$$U_{nm}^{\dagger}U_{nm} = \mathbf{1}\,, \tag{12}$$

$$\mathrm{Tr}\,(U_{nm}^{\dagger}U_{n'm'}) = d\delta_{nn'}\delta_{mm'}\,. \tag{13}$$

This means that for the orthogonal normalized bases (e.g., Bell bases), the second constraint condition in Eq. (7) for the unitary operator U_{st} is satisfied. However, for other bases, we must assume that the constraint condition (7) is satisfied if we desire these bases to be orthogonal normalized bases.

Now we consider the teleportation of a single particle unknown state. We denote by $|\phi\rangle$ the single particle unknown state to be teleported from Alice to Bob. From Eq. (6), we define the measurement operators Π_{st} based on the generic measurement basis Φ_{st} as

$$\Pi_{st} = |\Phi_{st}\rangle\langle\Phi_{st}| = (\mathbf{1} \otimes U_{st})\Gamma(\mathbf{1} \otimes U_{st}^{\dagger})\,, \tag{14}$$

where $\Gamma = |\omega\rangle\langle\omega|$. It is obvious that the measurement operators Π_{st} satisfy

$$\Pi_{st}^{\dagger}\Pi_{s't'} = \Pi_{st}\delta_{ss'}\delta_{tt'}\,. \tag{15}$$

We assume that Alice and Bob share the maximally entanglement pair Ψ_{+} (Bell state, $\Psi_{+} = (1/d)\sum_{i=0}^{d-1}|i\rangle \otimes |i\rangle$), then, for the present case, the composite system is expressed as

$$\rho = |\phi\rangle\langle\phi| \otimes P_{+}\,, \tag{16}$$

where $P_{+} = |\Psi_{+}\rangle\langle\Psi_{+}|$.

From the following theorem 1, we can know that Bob can convert exactly his particle state into the unknown state $|\phi\rangle$ if he makes a unitary operation U_{st}^{T} (where T denotes the transpose of U_{st}) on his particle when Alice makes a propriety measurement Π_{st} on the composite system of the unknown state and her particle state.

Theorem 1 Bob can convert exactly his particle state into the unknown state if he makes a unitary transformation U_{st}^{T} on his particle provided that Alice makes a measurement Π_{st} on her particle and the unknown particle of the composite system and projects these two particles into the state $|\Phi_{st}\rangle$.

Proof For the composite system (16), Alice must make a joint measurement Π_{st} on the first particle and the second particle and send Bob her measurement results. Conditioned on the measurement results, Bob makes a unitary transformation on his particle (the third particle), and gets the desired final result. Considering each measurement result that is produced with a finite probability, this teleportation procedure can be represented by the following mathematical formula

$$\rho_{\mathrm{Bob}}^{\mathrm{out}} = \sum_{s,t=0}^{d-1} \mathrm{Tr}_{\,\mathrm{ua}}((\Pi_{st} \otimes U_{st}^{\mathrm{T}}) \times [|\phi\rangle\langle\phi| \otimes |\Psi_{+}\rangle\langle\Psi_{+}|](\Pi_{st}^{\dagger} \otimes U_{st}^{*}))\,. \tag{17}$$

Here $\rho_{\mathrm{Bob}}^{\mathrm{out}}$ is the state of Bob's particle, $\mathrm{Tr}_{\mathrm{ua}}$ is a partial trace over the unknown state and Alice's particle state. U_{st}^{T} is the transpose of the U_{st} unitary transformation, which will be performed by Bob on his particle. The asterisk "*" denotes complex conjugate operation. Because each joint operation (Alice's measurement operation and Bob's unitary transformation operation) produces desired result with finite probability, the sum operation is needed when we count all the measurement results.

Having made use of Eq. (15), substituting the expression of $|\Psi_{+}\rangle$ into Eq. (17) and completing the trace over the first two subsystems, one can obtain

$$\rho_{\mathrm{Bob}}^{\mathrm{out}} = \frac{1}{d}\sum_{s,t,i,j=0}^{d-1} \langle\phi| \otimes \langle j|\Pi_{st}|\phi\rangle \otimes |i\rangle U_{st}^{\mathrm{T}}|i\rangle\langle j|U_{st}^{*}\,. \tag{18}$$

Suppose that the unknown state $|\phi\rangle$ which Alice wants to teleport is in the form of superposition states

$$|\phi\rangle = \sum_{k=0}^{d-1} a_{k}|k\rangle\,. \tag{19}$$

By making use of the measurement operator Π_{st}, Bob's state can be further written as

$$\begin{aligned}\rho_{\rm Bob}^{\rm out} &= \frac{1}{d^2}\sum_{k,l,s,t,i,j=0}^{d-1} a_k^* a_l \langle j|U_{st}|k\rangle\langle l|U_{st}^\dagger|i\rangle U_{st}^{\rm T}|i\rangle\langle j|U_{st}^* = \frac{1}{d^2}\sum_{k,l,s,t,i,j=0}^{d-1} a_k^* a_l \langle k|U_{st}^{\rm T}|j\rangle\langle i|U_{st}^*|l\rangle U_{st}^{\rm T}|i\rangle\langle j|U_{st}^* \\ &= \frac{1}{d^2}\sum_{k,l,s,t,i,j=0}^{d-1} a_k^* a_l U_{st}^{\rm T}|i\rangle\langle i|U_{st}^*|l\rangle\langle k|U_{st}^{\rm T}|j\rangle\langle j|U_{st}^* = \frac{1}{d^2}\sum_{k,l,s,t=0}^{d-1} a_k^* a_l U_{st}^{\rm T}U_{st}^*|l\rangle\langle k|U_{st}^{\rm T}U_{st}^* \\ &= \frac{1}{d^2}\sum_{k,l,s,t=0}^{d-1} a_k^* a_l (U_{st}^\dagger U_{st})^*|l\rangle\langle k|(U_{st}^\dagger U_{st})^* = |\phi\rangle\langle\phi|\,. \qquad \square \qquad (20)\end{aligned}$$

It is obvious that one can teleport any unknown single particle state with the fidelity $f = 1$ by means of the maximally entanglement state $|\Psi_+\rangle$ and the generic measurement bases $|\Phi_{st}\rangle$. It is worth while noting that during the calculation of the above equation, we only require that U_{st} be a unitary operator and satisfy the orthogonal relationship (7). That is to say, we need not ask $|\Phi_{st}\rangle$ to be a Bell bases. Therefore, one can teleport any unknown state based on the generic measurement bases. Then we generalize the standard teleportation to the case of the generic measurement bases.

Making use of the generic measurement bases $|\Phi_{st}\rangle$, one can complete the teleportation of unknown state by means of not only the maximally entanglement pure state $|\Psi_+\rangle$ quantum channel but also the partial entanglement mixed states quantum channel.

As an example, one can consider the one-parameter state,[24] given by

$$\rho_p = pP_+ + (1-p)\frac{\mathbf{1}\otimes\mathbf{1}}{d^2}\,, \quad 0 \le p \le 1\,, \qquad (21)$$

which is called the noisy singlet by some authors. Obviously it is a mixed entanglement state. From Theorem 1 or Eq. (17), we can get the final state of Bob's particle, which is also expressed by

$$\rho_{\rm Bob}^{\rm out} = \sum_{s,t=0}^{d-1} {\rm Tr}_{\rm ua}((\Pi_{st}\otimes U_{st}^{\rm T})[|\phi\rangle\langle\phi|\otimes\rho_p](\Pi_{st}^\dagger\otimes U_{st}^*))\,. \qquad (22)$$

Making use of the same process, one can get the teleportation result of the unknown state by means of the partial entanglement mixed state (21), i.e.

$$\rho_{\rm Bob}^{\rm out} = p|\phi\rangle\langle\phi| + (1-p)\frac{\mathbf{1}}{d}\,, \qquad (23)$$

where $|\phi\rangle$ is the unknown state that Alice wishes to teleport. The fidelity of this teleportation is given by $f = p + (1-p)/d$.

3 Teleportation of a Two-Particle Cat-like Unknown State

The teleportation of two-particles unknown states for the $d = 2$ case through a GHZ quantum channel has been proposed by Gorbachev *et al.*[11] Zeng[17] *et al.* have gone further to discuss the same problem for the high-dimensional quantum system ($d > 2$). However, they only realized the teleportation for the case of Bell measurement bases. Here we want to prove that we can realize the same teleportation by means of the generic measurement bases.

Considering the d-dimensional GHZ quantum channel states $|\Lambda\rangle = (1/\sqrt{d})\sum_{l=0}^{d-1}|l\rangle\otimes|l\rangle\otimes|l\rangle$ and three-particle state $|\Omega\rangle = (1/d)\sum_{k,l=0}^{d-1}|k\rangle\otimes|l\rangle\otimes|l\rangle$, we can define the generic measurement bases as

$$\begin{aligned}&|\Phi_{rst}\rangle = (\Gamma_r\otimes\mathbf{1}\otimes U_{st})|\Omega\rangle \in \mathcal{H}\otimes\mathcal{H}\otimes\mathcal{H}\,, \\ &\Gamma_r, U_{st} \in \mathcal{B}(\mathcal{H})\,, \qquad (24)\end{aligned}$$

where Γ_r and U_{st} are $d\times d$ unitary operators and they satisfy the following relations

$$\begin{aligned}{\rm Tr}\,(\Gamma_r^\dagger\Gamma_s) &= d\delta_{rs}\,, \qquad (25) \\ {\rm Tr}\,(U_{st}^\dagger U_{s't'}) &= d\delta_{ss'}\delta_{tt'}\,. \qquad (26)\end{aligned}$$

It is easy to check that the generic measurement bases defined by Eq. (24) are orthogonal, i.e.

$$\langle\Phi_{rst}|\Phi_{r's't'}\rangle = \delta_{rr'}\delta_{ss'}\delta_{tt'}\,, \qquad (27)$$

which can serve as a set of orthogonal measurement bases for the d^3-dimensional Hilbert space.

In order to teleport a two-particle unknown state to Bob, Alice needs to define a set of orthogonal measurement operators measuring on the unknown state system (particles 1 and 2) and her particle state system (particle 3) of the composite system, and Bob needs a group of unitary transformations acting on his particles (particles 4 and 5). We denote by Π_{rst} and B_{rst} the measurement operators and the unitary transformations, respectively. Let Π_{rst} be $|\Phi_{rst}\rangle\langle\Phi_{rst}|$ and the unitary transformation B_{rst} satisfy the following relations,

$$B_{rst}A = A\Gamma_r\otimes U_{st}^{\rm T}\,, \qquad (28)$$

where A is a $d^2\times d^2$ matrix with elements $A_{ij,kl} = \delta_{ij}\delta_{jl}$. Then we have the following theorem.

Theorem 2 The unknown two-particle cat-like state can be teleported from Alice to Bob by means of the generic measurement bases $|\Phi_{rst}\rangle$ provided that

(i) Alice and Bob share the d^3-dimensional maximally entanglement GHZ state $|\Lambda\rangle$;

(ii) Alice projects the unknown cat-like state and her particle state of the composite system into the state $|\Phi_{rst}\rangle$; and

(iii) Bob makes a unitary transformation B_{rst} on his two-particle system.

Proof For the unknown two-particle cat-like state $|\phi\rangle$, it can be expressed as

$$|\phi\rangle = \sum_{l}^{d-1} a_l |l\rangle \otimes |l\rangle\,, \tag{29}$$

and the process of the teleportation of this unknown state can be represented by the following mathematical formula,

$$\rho_{\mathrm{Bob}}^{\mathrm{out}} = \sum_{r,s,t=0}^{d-1} \mathrm{Tr}_{uua}((\Pi_{rst} \otimes B_{rst})(|\phi\rangle\langle\phi| \otimes |\Lambda\rangle\langle\Lambda|)(\Pi_{rst}^{\dagger} \otimes B_{rst}^{\dagger}))\,, \tag{30}$$

where $\rho_{\mathrm{Bob}}^{\mathrm{out}}$ is Bob's two-particle state after Alice has projected her composite system onto $|\Phi_{rst}\rangle$ and Bob has made a unitary transformation B_{rst} on his two particles. Here the measurement operator Π_{rst} acts on the first three particles and the unitary transformation B_{rst} acts on the last two particles belonging to Bob, and Tr_{uua} denotes the partial trace over the two-particle unknown state and Alice's particle states.

Applying the quantum channel state $|\Lambda\rangle$ and completing the partial trace, equation (30) becomes

$$\rho_{\mathrm{Bob}}^{\mathrm{out}} = \frac{1}{d} \sum_{r,s,t,m,n=0}^{d-1} \langle\phi| \otimes \langle n|\Pi_{rst}^{\dagger}\Pi_{rst}|\phi\rangle \otimes |m\rangle B_{rst}|mm\rangle\langle nn|B_{rst}^{\dagger}\,. \tag{31}$$

In terms of the orthogonal properties $\Pi_{srt}^{\dagger}\Pi_{r's't'} = \Pi_{rst}\delta_{rr'}\delta_{ss'}\delta_{tt'}$ of the measurement operators Π_{rst}, considering Eqs. (24) and (29), the state of Bob's particles can be further rewritten as

$$\rho_{\mathrm{Bob}}^{\mathrm{out}} = \frac{1}{d^3} \sum_{r,s,t,m,n,i,j,k,k'=0}^{d-1} a_i^* a_j \langle i|\Gamma_r|k\rangle\langle n|U_{st}|i\rangle\langle k'|\Gamma_r^{\dagger}|j\rangle\langle j|U_{st}^{\dagger}|m\rangle B_{rst}|mm\rangle\langle nn|B_{rst}^{\dagger}\,. \tag{32}$$

Considering the transpose relations of the matrix elements, $\langle n|U_{st}|i\rangle = \langle i|U_{st}^{\mathrm{T}}|n\rangle$ and $\langle j|U_{st}^{\dagger}|m\rangle = \langle m|U_{st}^{*}|j\rangle$, and applying the equality $(A_1B_1C_1) \otimes (A_2B_2C_2) = (A_1 \otimes A_2)(B_1 \otimes B_2)(C_1 \otimes C_2)$, equation (32) becomes

$$\rho_{\mathrm{Bob}}^{\mathrm{out}} = \frac{1}{d^3} \sum_{r,s,t,i,j=0}^{d-1} a_i^* a_j \biggl(B_{rst}\biggl(\sum_{m,k'=0}^{d-1} |mm\rangle\langle k'm|\biggr)\Gamma_r^{\dagger} \otimes U_{st}^{*}|jj\rangle\langle ii|\Gamma_r \otimes U_{st}^{\mathrm{T}}\biggl(\sum_{n,k=0}^{d-1} |kn\rangle\langle nn|\biggr)B_{rst}^{\dagger}\biggr). \tag{33}$$

Since the unitary transformation B_{rst} that Bob performs on his particles satisfies relation (28) with $A_{ij,kl} = \delta_{ij}\delta_{jl}$, and the matrix A and unitary matrix U_{st} have properties: $A|jj\rangle = |jj\rangle$ and $U_{st}^{\mathrm{T}}U_{st}^{*} = (U_{st}^{\dagger}U_{st})^{\mathrm{T}} = \mathbf{1}$, the final state of Bob's particles becomes

$$\begin{aligned}
\rho_{\mathrm{Bob}}^{\mathrm{out}} &= \frac{1}{d^3} \sum_{r,s,t,i,j=0}^{d-1} a_i^* a_j (B_{rst}A(\Gamma_r^{\dagger} \otimes U_{st}^{*})|jj\rangle\langle ii|(\Gamma_r \otimes U_{st}^{\mathrm{T}})A^{\mathrm{T}}B_{rst}^{\dagger}) \\
&= \frac{1}{d^3} \sum_{r,s,t,i,j=0}^{d-1} a_i^* a_j (A(\Gamma_r \otimes U_{st}^{\mathrm{T}})(\Gamma_r^{\dagger} \otimes U_{st}^{*})|jj\rangle\langle ii|(\Gamma_r \otimes U_{st}^{\mathrm{T}})(\Gamma_r^{\dagger} \otimes U_{st}^{*})A^{\mathrm{T}}) \\
&= \frac{1}{d^3} \sum_{r,s,t,i,j=0}^{d-1} a_i^* a_j A(\Gamma_r\Gamma_r^{\dagger} \otimes U_{st}^{\mathrm{T}}U_{st}^{*})|jj\rangle\langle ii|(\Gamma_r\Gamma_r^{\dagger} \otimes U_{st}^{\mathrm{T}}U_{st}^{*})A^{\mathrm{T}} \\
&= \frac{1}{d^3} \sum_{r,s,t,i,j=0}^{d-1} a_i^* a_j A|jj\rangle\langle ii|A^{\mathrm{T}} = |\phi\rangle\langle\phi|\,. \qquad \square
\end{aligned} \tag{34}$$

This result tells us that one can transmit a cat-like state (29) by means of the generic measurement bases $|\Phi_{rst}\rangle$, based on a quantum channel state $|\Lambda\rangle$. Our result agrees with the one in Ref. [17]. However, the results in Ref. [17] are obtained by the explicit Bell measurement bases, and our result is obtained by means of the generic measurement bases. We generalize the measurement bases to the generic orthogonal states.

4 Concluding Remarks

We have generalized the measurement bases of the standard teleportation to the generic measurement bases $|\Phi_{st}\rangle = (\mathbf{1} \otimes U_{st})|\omega\rangle$ or $|\Phi_{rst}\rangle = (\Gamma_r \otimes \mathbf{1} \otimes U_{st})|\Omega\rangle$. This case is different from the result of Ref. [25]. In Ref. [25] the unitary transformation U_{st} is constrained to be a like-Hadamard matrix, however, in the present case, we only require that the operators U_{st} and Γ_r be unitary and orthogonal operators, and satisfy the conditions (28), so we give the most general case for the measurement bases.

In our paper, we have also given the mathematical expression of the process of the standard teleportation of the unknown particle states, and have given the explicit correspondence relations of the measurement bases that Alice performs on her particles states and the unitary transformation operators that Bob performs on his particles states. It is shown that the process of the standard teleportation does not depend on the explicit forms of the measurement bases, and the unknown states can be exactly transmitted to Bob with fidelity $f = 1$. In fact, this process allows us to transmit an unknown mixed state, which can be seen from Eqs. (22) and (23). It is worth while pointing out that the process of the standard teleportation hides rich (Lie) algebra structures (Weyl pairs). It is also an interesting topic and we will give the results in a future paper.

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