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Bare-State Time-Evolving Operator Solution to Raman Model in Λ Configuration*

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Abstract *We derive exact analytical expressions of time-evolving bare-state operators of level occupation numbers and the photon numbers for a composite system consisting of a three-level atom interacting with two modes of a quantized electromagnetic field in Λ configuration. These results demonstrate the oscillations with three-family frequencies for a nonzero detuning, which dramatically differ from the previous results showing only single-family Rabi oscillations.*

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In the past two decades, there has been considerable interest in investigating the interactions of electromagnetic wave(s) with two-level[1−10] and three-level[11−30] as well as multi-level atom(s).[31−34] An efficient standardized approach has been developed to deal with the energy eigenvalues, eigenstates and the corresponding dynamics of various two-level models in a unitized way.[7] In the three-level situation, Wu and Yang have obtained successfully the exact results for the energy eigenvalues and eigenstates for the Raman models in the Λ, ladder and V configurations.[17−19] However, there still exist some unsolved problems in obtaining the exact analytical results for the corresponding dynamics for the nonzero detuning case although exact results have already been obtained in the resonance (i.e. zero detuning) case.[15,16] The approximated dynamics in the large detuning limit has been investigated analytically by either adiabatic elimination[11−13] or perturbatively calculating unitary transformation.[14] Subsequently, Wu[17] has obtained exact analytical results valid for any values of detuning (from zero to large detuning) when the dynamics is governed by the transformed Hamiltonian, and has shown that these exact results reduce to the previous zero-detuning results[15,16] and the previous approximated results in the large detuning case.[11−14] However, it is emphasized that the dynamics governed by the transformed Hamiltonian may differ from the original bare-state dynamics when the unitary transformation operator depends on time which is so far the problem at hand.

In this paper, we shall derive exact analytical expressions of time-evolving bare-state operators of level occupation numbers and the photon numbers for a composite system consisting of a three-level atom interacting with two modes of a quantized electromagnetic field in Λ configuration. These novel exact results are valid for any values of detuning from zero to very large detuning, and they demonstrate the oscillations with three-family frequencies for a nonzero detuning, which dramatically differ from the previous results showing only single-family Rabi oscillations. The dramatic difference between the dynamics governed by the transformed Hamiltonian and the dynamics governed by the bare-state Hamiltonian implies that the adiabatic approximation may fail to capture some important effects even in its seemingly suitable range (i.e. the large detuning limit).

We consider a three-level atom of energies E_1, E_2, and E_3 interacting with a pump ω_1, and a Stokes mode ω_2 in the Λ configuration under the two-photon resonance condition with detuning Δ as shown in Fig. 1. The corresponding Hamiltonian is[17]

$$\hat{H} = \sum_{i=1}^{3} E_i \hat{\sigma}_{ii} + \hbar \sum_{j=1}^{2} [\omega_j \hat{a}_j^\dagger \hat{a}_j + g_j(\hat{a}_j \hat{\sigma}_{3j} + \hat{a}_j^\dagger \hat{\sigma}_{j3})] \,, \quad (1)$$

where $\hat{a}_j$ represents the annihilation operators of field mode j, $\hat{\sigma}_{ii}$ and $\hat{\sigma}_{ij}$ $(i \neq j)$ are the level occupation numbers and transition operators respectively, and g_j are dipole-coupling constants. The Hamiltonian (1) can be rewritten as[17]

$$\hat{H} = \hbar \sum_{j=1}^{2} \Big[\omega_j \hat{N}_j + \frac{1}{2}\Delta(1 - 2\hat{\sigma}_{jj}) + \bar{g}_j \hat{q}_j^{(+)} \Big] + E_0 \,, \quad (2)$$

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where

$$E_0 = (E_1 + E_2 - \hbar\omega_1 - \hbar\omega_2)/2$$

is a constant, $\hat{N}_j = \hat{a}_j^\dagger \hat{a}_j + 1 - \hat{\sigma}_{jj}$ are two conserved operators,[17]

$$\bar{g}_j = g_j\sqrt{\hat{N}_j}\,,$$

and

$$\hat{q}_j^{(+)} = (\hat{a}_j\hat{\sigma}_{3j} + \hat{a}_j^\dagger\hat{\sigma}_{j3}) \Big/ \sqrt{\hat{N}_j}$$

with $j = 1, 2$.

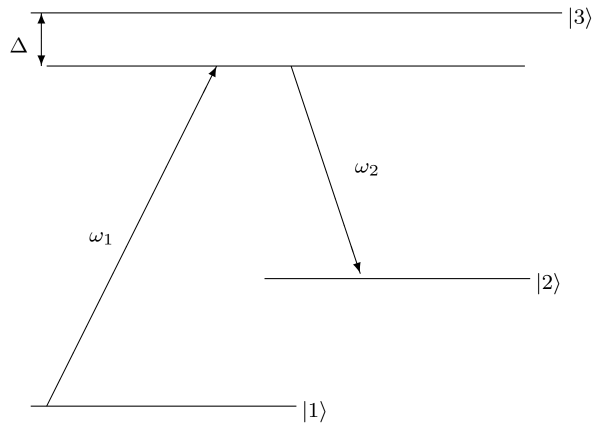

Fig. 1 Three-level atom interacting with two quantized fields in Λ configuration with the detuning Δ.

We describe the dynamics in the Heisenberg picture by the Heisenberg equation $\mathrm{i}\hbar\dot{\hat{x}} = [\hat{x}, \hat{H}]$ for the operator $\hat{x}(t)$. It is readily seen that to determine the time evolution for the operators of level occupation numbers $\hat{\sigma}_{ii}(t)(i = 1, 2, 3)$ and photon numbers $\hat{n}_j(t) \equiv \hat{a}_j^\dagger(t)\hat{a}_j(t)(j = 1, 2)$, one has to solve first the following eight coupled first-order differential equations

$$\mathrm{i}\dot{\hat{q}}_1^{(-)}(t) = -\Delta\hat{q}_1^{(+)}(t) - \bar{g}_2\hat{q}^{(+)}(t) - 2\bar{g}_1[\hat{\sigma}_{11}(t) - \hat{\sigma}_{33}(t)]\,, \tag{3a}$$

$$\mathrm{i}\dot{\hat{q}}_2^{(-)}(t) = -\Delta\hat{q}_2^{(+)}(t) - \bar{g}_1\hat{q}^{(+)}(t) - 2\bar{g}_2[\hat{\sigma}_{22}(t) - \hat{\sigma}_{33}(t)]\,, \tag{3b}$$

$$\mathrm{i}\dot{\hat{q}}_1^{(+)}(t) = -\Delta\hat{q}_1^{(-)}(t) + \bar{g}_2\hat{q}^{(-)}(t)\,, \tag{3c}$$

$$\mathrm{i}\dot{\hat{q}}_2^{(+)}(t) = -\Delta\hat{q}_2^{(-)}(t) - \bar{g}_1\hat{q}^{(-)}(t)\,, \tag{3d}$$

$$\mathrm{i}\dot{\hat{\sigma}}_{11}(t) = -\bar{g}_1\hat{q}_1^{(-)}(t)\,, \qquad \mathrm{i}\dot{\hat{\sigma}}_{22}(t) = -\bar{g}_2\hat{q}_2^{(-)}(t)\,, \tag{3e}$$

$$\mathrm{i}\dot{\hat{q}}^{(\pm)}(t) = -\bar{g}_1\hat{q}_2^{(\mp)}(t) \mp \bar{g}_2\hat{q}_1^{(\mp)}(t)\,, \tag{3f}$$

where $\bar{g}_j = g_j\sqrt{\hat{N}_j}$, and

$$\hat{q}_j^{(\pm)}(t) = \frac{\hat{a}_j(t)\hat{\sigma}_{3j}(t) \pm \hat{a}_j^\dagger(t)\hat{\sigma}_{j3}(t)}{\sqrt{\hat{N}_j}}\,, \tag{4a}$$

$$\hat{q}^{(\pm)}(t) = \frac{\hat{a}_1^\dagger(t)\hat{a}_2(t)\hat{\sigma}_{12}(t) \pm \hat{a}_1(t)\hat{a}_2^\dagger(t)\hat{\sigma}_{21}(t)}{\sqrt{\hat{N}_1\hat{N}_2}}\,, \tag{4b}$$

$$\hat{\sigma}_{33}(t) = 1 - \hat{\sigma}_{11}(t) - \hat{\sigma}_{22}(t)\,, \tag{4c}$$

$$\hat{N}_j = \hat{n}_j(t) + 1 - \hat{\sigma}_{jj}(t) \equiv \hat{n}_j(0) + 1 - \hat{\sigma}_{jj}(0)\,, \tag{4d}$$

where $j = 1, 2$. It is pointed out that the operators $\hat{N}_j \equiv \hat{N}_j(t) \equiv \hat{N}_j(0)$ are not only conserved quantities (i.e., they do not vary with respect to time t) since they commute with the Hamiltonian (1) but also they (as well as the operators $\bar{g}_j = g_j\sqrt{\hat{N}_j}$) can be regarded as c-numbers because they commute with all the other operators involved in Eqs. (3) and (4) such as $\hat{\sigma}_{ii}$ and those in the nominators of the right-hand side of Eqs. (4a) and (4b). Consequently, they can be placed either before or behind the nominators on the right-hand side of Eqs. (4a) and (4b). Besides, the ambiguity in Eqs. (4a) and (4b) when one of the two quantities $\hat{N}_{1,2}$ equals zero does not exist at all because the corresponding nominators must be zero simultaneously. For instance, it can readily show from Eqs. (3) and (4) that both $\hat{q}_1^{(\pm)}(t) \equiv 0$ and $\hat{q}^{(\pm)}(t) \equiv 0$ when $\hat{N}_1 = 0$ while both $\hat{q}_2^{(\pm)}(t) \equiv 0$ and $\hat{q}^{(\pm)}(t) \equiv 0$ as $\hat{N}_2 = 0$.

We are now ready to solve Eq. (3), which can be written as the following matrix form

$$\mathrm{i}\dot{\Psi}^{(+)}(t) = -Q\Psi^{(-)}(t)\,, \qquad \mathrm{i}\dot{\Phi}(t) = P\Psi^{(-)}(t)\,, \quad \mathrm{i}\dot{\Psi}^{(-)}(t) = -Q^{\mathrm{T}}\Psi^{(+)}(t) + \Phi(t)\,, \tag{5}$$

with the time-dependent operator-type column vectors

$$\Psi^{(\pm)}(t) = (\hat{q}_1^{(\pm)}(t), \hat{q}_2^{(\pm)}(t), \hat{q}^{(\pm)}(t))^{\mathrm{T}}\,, \qquad \Phi(t) = 2(\bar{g}_1[\hat{\sigma}_{33}(t) - \hat{\sigma}_{11}(t)], \bar{g}_2[\hat{\sigma}_{33}(t) - \hat{\sigma}_{22}(t)], 0)^{\mathrm{T}}\,,$$

and the time-independent operator-type (but can be considered as c-number type) matrices

$$Q = \begin{pmatrix} \Delta & 0 & -\bar{g}_2 \\ 0 & \Delta & \bar{g}_1 \\ \bar{g}_2 & \bar{g}_1 & 0 \end{pmatrix}, \qquad P = 2\begin{pmatrix} 2\bar{g}_1^2 & \bar{g}_1\bar{g}_2 & 0 \\ \bar{g}_1\bar{g}_2 & 2\bar{g}_2^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{6}$$

From Eq. (5), we obtain

$$\ddot{\Psi}^{(-)}(t) = \begin{pmatrix} \alpha_1 & 3\bar{g}_1\bar{g}_2 & -\bar{g}_2\Delta \\ 3\bar{g}_1\bar{g}_2 & \alpha_2 & \bar{g}_1\Delta \\ -\bar{g}_2\Delta & \bar{g}_1\Delta & g^2 \end{pmatrix}\Psi^{(-)}(t) = G\begin{pmatrix} \Omega^2 & 0 & 0 \\ 0 & (\Omega+\Delta)^2 & 0 \\ 0 & 0 & (2\Omega+\Delta)^2 \end{pmatrix}G^{-1}\Psi^{(-)}(t)\,, \tag{7}$$

where $g=\sqrt{\bar{g}_1^2+\bar{g}_2^2}$, $\alpha_j=\Delta^2+g^2+3\bar{g}_j^2$ $(j=1,2)$ and

$$\Omega\equiv\hat{\Omega}(\hat{N}_1,\hat{N}_2)=\sqrt{\Big(\frac{\Delta}{2}\Big)^2+g_1^2\hat{N}_1+g_2^2\hat{N}_2}-\frac{\Delta}{2}\,, \tag{8a}$$

$$G=\begin{pmatrix}\bar{g}_2\eta^{-1} & \bar{g}_2\tau^{-1} & \bar{g}_1 g^{-1}\\ -\bar{g}_1\eta^{-1} & -\bar{g}_1\tau^{-1} & \bar{g}_2 g^{-1}\\ (\Omega+\Delta)\eta^{-1} & -\Omega\tau^{-1} & 0\end{pmatrix}, \tag{8b}$$

$$G^{-1}=\begin{pmatrix}\bar{g}_2\eta^{-1} & -\bar{g}_1\eta^{-1} & (\Omega+\Delta)\eta^{-1}\\ \bar{g}_2\tau^{-1} & -\bar{g}_1\tau^{-1} & -\Omega\tau^{-1}\\ \bar{g}_1 g^{-1} & \bar{g}_2 g^{-1} & 0\end{pmatrix}, \tag{8c}$$

where

$$g=\sqrt{\bar{g}_1^2+\bar{g}_2^2}\,,\qquad \eta=\sqrt{\bar{g}_1^2+\bar{g}_2^2+(\Delta+\Omega)^2}\,,\qquad \tau=\sqrt{\bar{g}_1^2+\bar{g}_2^2+\Omega^2}\,.$$

Equation (7) is now readily solved to obtain the solution

$$\begin{aligned}\begin{pmatrix}\hat{q}_1^{(-)}(t)\\ \hat{q}_2^{(-)}(t)\\ \hat{q}^{(-)}(t)\end{pmatrix}&=G\begin{pmatrix}\cos(\Omega t) & 0 & 0\\ 0 & \cos[(\Delta+\Omega)t] & 0\\ 0 & 0 & \cos[(\Delta+2\Omega)t]\end{pmatrix}G^{-1}\begin{pmatrix}\hat{q}_1^{(-)}(0)\\ \hat{q}_2^{(-)}(0)\\ \hat{q}^{(-)}(0)\end{pmatrix}\\ &+G\begin{pmatrix}\dfrac{\sin(\Omega t)}{\Omega} & 0 & 0\\ 0 & \dfrac{\sin[(\Delta+\Omega)t]}{(\Delta+\Omega)} & 0\\ 0 & 0 & \dfrac{\sin[(\Delta+2\Omega)t]}{(\Delta+2\Omega)}\end{pmatrix}G^{-1}\dot{\Psi}^{(-)}(0)\,,\end{aligned} \tag{9}$$

where $\dot{\Psi}^{(-)}(0)$ [according to Eq. (5)] has the form

$$\dot{\Psi}^{(-)}(0)=\mathrm{i}Q^{\mathrm{T}}\Psi^{(+)}(0)-\mathrm{i}\Phi(0)\,. \tag{10}$$

The solution (9) together with the first equation in Eq. (5) immediately leads to the result

$$\begin{aligned}\begin{pmatrix}\hat{q}_1^{(+)}(t)\\ \hat{q}_2^{(+)}(t)\\ \hat{q}^{(+)}(t)\end{pmatrix}&=\begin{pmatrix}\hat{q}_1^{(+)}(0)\\ \hat{q}_2^{(+)}(0)\\ \hat{q}^{(+)}(0)\end{pmatrix}+G\begin{pmatrix}\dfrac{\sin(\Omega t)}{\Omega} & 0 & 0\\ 0 & \dfrac{\sin[(\Delta+\Omega)t]}{\Delta+\Omega} & 0\\ 0 & 0 & \dfrac{\sin[(\Delta+2\Omega)t]}{\Delta+2\Omega}\end{pmatrix}G^{-1}\begin{pmatrix}\hat{q}_1^{(-)}(0)\\ \hat{q}_2^{(-)}(0)\\ \hat{q}^{(-)}(0)\end{pmatrix}\\ &+G\begin{pmatrix}\dfrac{1-\cos(\Omega t)}{\Omega^2} & 0 & 0\\ 0 & \dfrac{1-\cos[(\Delta+\Omega)t]}{(\Delta+\Omega)^2} & 0\\ 0 & 0 & \dfrac{1-\cos[(\Delta+2\Omega)t]}{(\Delta+2\Omega)^2}\end{pmatrix}G^{-1}\dot{\Psi}^{(-)}(0)\,.\end{aligned} \tag{11}$$

Equations (9) and (11) explicitly give the expressions of the operators $\hat{q}_j^{(\pm)}(t)$ and $\hat{q}^{(\pm)}(t)$ in Eqs. (4a) and (4b) in terms of the initial values $\hat{q}_j^{(\pm)}(0)$, $\hat{q}^{(\pm)}(0)$, and $\hat{\sigma}_{ii}(0)$, $(i=1,2,3;\ j=1,2)$. The expressions of $\hat{\sigma}_{ii}(t)$ $(i=1,2,3)$ can be obtained by Eq. (4c) and the integrations [see Eq. (3e)]

$$\hat{\sigma}_{jj}(t)=\hat{\sigma}_{jj}(0)+\mathrm{i}\bar{g}_j\int_0^t\hat{q}_j^{(-)}(t)\,\mathrm{d}t\,,\quad j=1,2\,.$$

Then using Eqs. (9) and (4d), and after some tedious but straightforward manipulation, we obtain

$$\begin{aligned}\hat{a}_j^{\dagger}(t)\hat{a}_j(t)-\hat{a}_j^{\dagger}(0)\hat{a}_j(0)=\hat{\sigma}_{jj}(t)-\hat{\sigma}_{11}(0)&=2(1-2\delta_{j,2})\bar{g}_1^2\bar{g}_2^2[f(\Omega,t)+f(\Delta+\Omega,t)][\hat{\sigma}_{22}(0)-\hat{\sigma}_{11}(0)]\\ &-2\bar{g}_1^2 f(\Delta+2\Omega,t)\sum_{k=1}^{2}\bar{g}_k^2[\hat{\sigma}_{kk}(0)-\hat{\sigma}_{33}(0)]+\hat{I}_j(t)\,,\quad j=1,2\,,\end{aligned} \tag{12}$$

where $\hat{I}_2(t)$ is obtained by interchanging subscripts 1 and 2 in the expression of $\hat{I}_1(t)$ and

$$\hat{I}_1(t) = (R(\Omega,t), R(\Delta+\Omega,t), R(\Delta+2\Omega,t)) \begin{pmatrix} \bar{g}_2^2 & -\bar{g}_1^2 & \Delta+\Omega \\ \bar{g}_2^2 & -\bar{g}_1^2 & -\Omega \\ \bar{g}_1^2 & \bar{g}_1^2 & 0 \end{pmatrix} \begin{pmatrix} \mathrm{i}\bar{g}_1\hat{q}_1^{(-)}(0) \\ \mathrm{i}\bar{g}_2\hat{q}_2^{(-)}(0) \\ \mathrm{i}\bar{g}_1\bar{g}_2\hat{q}^{(-)}(0) \end{pmatrix}$$
$$- (f(\Omega,t), f(\Delta+\Omega,t), f(\Delta+2\Omega,t)) \begin{pmatrix} -\Omega\bar{g}_2^2 & \Omega\bar{g}_1^2 & \bar{g}_2^2-\bar{g}_1^2 \\ (\Delta+\Omega)\bar{g}_2^2 & -(\Delta+\Omega)\bar{g}_1^2 & \bar{g}_2^2-\bar{g}_1^2 \\ \Delta\bar{g}_1^2 & \Delta\bar{g}_1^2 & 2\bar{g}_1^2 \end{pmatrix} \begin{pmatrix} \bar{g}_1\hat{q}_1^{(+)}(0) \\ \bar{g}_2\hat{q}_2^{(+)}(0) \\ \bar{g}_1\bar{g}_2\hat{q}^{(+)}(0) \end{pmatrix} \tag{13}$$

with the operator function $R(x,t) = \partial f(x,t)/\partial t$ and

$$f(x,t) = \frac{1}{\bar{g}_1^2+\bar{g}_2^2+(x-\Delta-2\Omega)^2}\,\frac{1-\cos(xt)}{x^2}\,, \tag{14}$$

where $\bar{g}_j = g_j\sqrt{\hat{N}_j}$ and $\hat{N}_j = \hat{a}_j^\dagger(0)\hat{a}_j(0) + 1 - \hat{\sigma}_{jj}(0)$, $(j=1,2)$ are time-independent operators, and the time-independent Rabi frequency operator Ω is given by Eq. (8a).

Equation (12) together with Eq. (4c) is the central results of the present paper. They explicitly demonstrate that time-evolving operators of the level occupation numbers $\hat{\sigma}_{ii}(t)$, $i=1,2,3$ and the photon numbers $\hat{a}_j^\dagger(t)\hat{a}_j(t)$, $j=1,2$ oscillate with three-family frequencies characterized by the three frequency operators $\Omega, (\Delta+\Omega)$, and $(\Delta+2\Omega)$ via their eigenvalues, which differs remarkably from the single-family Rabi frequency operator of the previous results on the same model.[11,12,14–16] Specifically, let us illustrate this point by calculating the atomic inversion $W(t) = \mathrm{Tr}\,[(\hat{\sigma}_{22}(t) - \hat{\sigma}_{11}(t))\hat{\rho}]$ between levels 1 and 2 by using the same density operator as before[11–16] i.e. the density operator $\hat{\rho} = \hat{\rho}^{\mathrm{A}}\otimes\hat{\rho}^{\mathrm{F}}$. Here the atomic part $\hat{\rho}^{\mathrm{A}} = |1\rangle\langle 1|$ and the field part $\hat{\rho}^{\mathrm{F}} = |\alpha_1,\alpha_2\rangle\langle\alpha_1,\alpha_2|$ with $|\alpha_1,\alpha_2\rangle = \sum_{n_1,n_2=0}^{\infty} C_{n_1n_2}|n_1,n_2\rangle$ denote the two-mode coherent states (Note that we have made use of the relations between the Heisenberg and Schrödinger pictures). Using the fact $\mathrm{Tr}\,[\hat{I}_j(t)\hat{\rho}] = 0$, $j=1,2$ which result from $\mathrm{Tr}\,[\hat{q}_j(0)\hat{\rho}] = \mathrm{Tr}\,[\hat{q}_j^{(\pm)}(0)\hat{\rho}] = 0$, $j=1,2$, we easily obtain from Eq. (12)

$$W(t) = -1 + 2\sum_{n_1,n_2=0}^{\infty} n_1 g_1^2 |C_{n_1n_2}|^2 \{2(n_2+1)g_2^2[f(\Omega_{n_1n_2},t) + f(\Delta+\Omega_{n_1n_2},t)]$$
$$+ [n_1g_1^2 - (n_2+1)g_2^2] f(\Delta+2\Omega_{n_1n_2},t)\}\,, \tag{15}$$

where f is the c-number function given by Eq. (14) but with its operators $\hat{N}_1$, $\hat{N}_2$, and Ω replaced respectively by n_1, (n_2+1), and $\Omega_{n_1n_2}$. Here $\Omega_{n_1n_2}$ is

$$\Omega_{n_1n_2} = \sqrt{\Big(\frac{\Delta}{2}\Big)^2 + g_1^2 n_1 + g_2^2(n_2+1)} - \frac{\Delta}{2}\,, \qquad n_1,n_2 = 0,1,2,\ldots\,, \tag{16}$$

which is the eigenvalues of the Rabi frequency operator Eq. (8a) and hence can be called as Rabi frequencies. It is clear from Eq. (15) that the atomic inversion oscillates with three families of characteristic frequencies $\Omega_{n_1n_2}$, $(\Delta+\Omega_{n_1n_2})$, and $(\Delta+2\Omega_{n_1n_2})$ $(n_1,n_2=0,1,2,\ldots)$. Obviously, the mean photon numbers of the two modes and other average quantities also display the same dynamical behaviors. In the zero detuning case $\Delta=0$, the three-family characteristic frequencies merge into the single-family frequency

$$\Omega_{n_1n_2} = \sqrt{g_1^2 n_1 + g_2^2(n_2+1)}\,,$$

which is identical to the previous results under the zero detuning (or resonance) case.[13,15,16] However, the feature of three-family frequencies persists in nonzero detuning case and it becomes more pronounced for the large detuning limit, which dramatically differs from the previous results under the large detuning condition.[11–14]

Let us now explain why previous approximated results[11–14] fail to capture the feature of the oscillations with three-family frequencies. First, it has been shown[14,17] that the approximated results based on the adiabatic elimination under the large detuning condition[11–13] are identical or equivalent to those results obtained by unitary transformation with large detuning limit.[14,17]. Second, we shall show that the dynamics considered in the previous literature[14,17] and hence the dynamics obtained by the adiabatic elimination under the large detuning condition[11–13] are different from the dynamics considered here. Specifically, let the bare-state and dressed-state operators be denoted by $\hat{x}$ and $\hat{x}' = \hat{U}\hat{x}\hat{U}^{-1}$, respectively.[14,17] Here the unitary operator $\hat{U} = \exp(\hat{S})$ with

$$\hat{S} = \sum_{j=1,2} \beta_j \sqrt{\hat{N}_j}\hat{q}_j^{(-)} ,$$

and β_j are two (time-independent) transformation parameters.[17] The dynamics considered in the previous literature[14,17] (which is identical to the dynamics obtained by the adiabatic elimination under the large detuning condition[11−13] just as shown before[14,17]) is readily shown to be equivalent to the Heisenberg description $\mathrm{i}\hbar\dot{\hat{x}}' = [\hat{x}', \hat{H}']$, which obviously differs from the Heisenberg description $\mathrm{i}\hbar\dot{\hat{x}} = [\hat{x}, \hat{H}]$. The latter is identical to

$$\mathrm{i}\hbar\dot{\hat{x}}' = [\hat{x}', \hat{H}'] + \mathrm{i}\hbar(\dot{\hat{U}}\hat{x}\hat{U}^{-1} + \hat{U}\hat{x}\dot{\hat{U}}^{-1}) .$$

It should be pointed out that $(\dot{\hat{U}}\hat{x}\hat{U}^{-1} + \hat{U}\hat{x}\dot{\hat{U}}^{-1})$ is generally nonzero because

$$\hat{U} = \exp(\hat{S})$$

is a time-dependent quantity, noting the fact that $\hat{q}_{1,2}^{(-)}$ in $\hat{S}$ vary with time t according to Eq. (9).

This three-frequency characteristic can easily be understood by considering the energy-level structure of the composite system (a three-level atom and two quantized fields). We have the following exact energy eigenvalues for this composite system,[17]

$$E^{(3)}_{n_1n_2} = E^{(2)}_{n_1n_2} + \hbar(\Omega_{n_1n_2} + \Delta) , \tag{17a}$$

$$E^{(2)}_{n_1n_2} = E_1 + n_1\hbar\omega_1 + n_2\hbar\omega_2 , \tag{17b}$$

$$E^{(1)}_{n_1n_2} = E^{(2)}_{n_1n_2} - \hbar\Omega_{n_1n_2} . \tag{17c}$$

One sees that the level structure consists of infinite triplets with each of them corresponding to a given set of photon numbers n_1 and n_2. The energy differences of a typical triplet are

$$(E^{(3)}_{n_1n_2} - E^{(2)}_{n_1n_2})/\hbar = \Omega_{n_1n_2} ,$$

$$(E^{(2)}_{n_1n_2} - E^{(1)}_{n_1n_2})/\hbar = \Omega_{n_1n_2} + \Delta ,$$

$$(E^{(3)}_{n_1n_2} - E^{(1)}_{n_1n_2})/\hbar = 2\Omega_{n_1n_2} + \Delta ,$$

which are nothing but the three characteristic frequencies in the atomic inversion expression (15). This is similar to the well-known situation in the usual JC model (describing a two-level atom and one quantized field), where the energy difference of a typical level doublet gives the characteristic frequency (Rabi frequency) of atomic inversion.

In summary, we have obtained the exact analytical time-evolution expressions of the level occupation operators and the photon number operators of the two modes in the composite system of a three-level atom interacting with two quantized fields in Λ configuration. Within the framework of the model, they are valid for any values of coupling constants and field intensities no matter whether the level 3 is far off resonance or not, and can be used to investigate the dynamical and statistical properties of the model without resorting any approximation. They also bridge the gap between the two limiting cases of very large detuning[11−14] and zero-detuning[15,16] situations. We have found a novel and intriguing fact that the atomic inversions and the mean photon numbers oscillate with three-family of frequencies remarkably different from the single-family Rabi oscillations of the previous studies on the same model. It should have imparts on the collapse and revivals and may also manifest itself in other phenomena involving quantum-mechanical interference effects. It is worth while mentioning that the fact that three-frequency characteristic persists and even more pronounced when level 3 is far off resonance (i.e. large detuning) illustrates that perturbed approximations even within their seemingly suitable range may sometimes result in both quantitative and qualitative differences from the corresponding exact results.

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