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Non-perturbative QCD Effect on *K*-Factor of Drell–Yan Process*

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Abstract *By using a non-perturbative quark propagator with the lowest-dimensional condensate contributions from the QCD vacuum, the non-perturbative effect to K-factor of the Drell–Yan process is numerically investigated for* $^{12}_{6}C$*-*$^{12}_{6}C$ *collision at the center-of-mass energy* $\sqrt{s} = 200$ *GeV, 630 GeV respectively. Calculated results show that the non-perturbative QCD effect has just a weak influence on K-factor in the two cases.*

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1 Influence of QCD Non-perturbative Quark Condensation on Nucleon Function

The experiment of lepton-nucleon scattering has proven to be an effective approach for probing the structure of nucleons. This is due to the fact that in this process the leptonic part of the interaction can be accurately calculated with quantum electrodynamics and the results can only be interpreted in terms of the structure of the probed nucleons. Through the deep-inelastic scattering of lepton and nucleon, we can extract the structure function of the nucleon, i.e. the momentum distribution function of quark parton in the nucleon known as $F_2(x_2, Q^2)$, which parametrizes the hadronic vertex in the scattering, where x_2 is the scale variable, Q^2 the 4-momentum transfer. According to the scale Bjorken assumptions, on the condition of deep-inelastic scattering limit, the nucleon structure depends only upon x_2, which is known as the scaling violation. While this approximatively holds, the nucleon structure function is still dependent on Q^2, gradually but obviously,[1] which can be seen by the deviation of its experimental results from the predicted ones of quark-parton model.[2] We take the deep-inelastic scattering as an example to discuss the influence on nucleon structure function from QCD non-perturbative effect,

$$l + N \to l' + X\,. \tag{1}$$

The corresponding hadronic tensor can be described as

$$\begin{aligned} W_{\mu\nu} &= (2\pi)^3 \sum_X \langle P|J_\mu|X\rangle\langle X|J_\nu|P\rangle \delta^4(P_X - P - q) \\ &= \Big(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\Big)W_1 + \frac{1}{M^2}\Big(P_\mu - \frac{p\cdot q}{q^2}q_\mu\Big) \\ &\quad \times \Big(P_\nu - \frac{P\cdot q}{q^2}q_\nu\Big)W_2\,, \end{aligned} \tag{2}$$

where M is the mass of the nucleon. If $W_{\mu\nu}$ is given, the conventional method can give W_1 and W_2 through the following formulas:[3]

$$W_1 = \frac{1}{2}\Big[\frac{P^\mu P^\nu}{M^2}W_{\mu\nu} - \Big(1 - \frac{\nu^2}{q^2}\Big)W^\mu_\mu\Big]\Big(1 - \frac{\nu^2}{q^2}\Big)^{-1}\,, \tag{3}$$

$$W_2 = \frac{1}{2}\Big[3\frac{P^\mu P^\nu}{M^2}W_{\mu\nu} - \Big(1 - \frac{\nu^2}{q^2}\Big)W^\mu_\mu\Big]\Big(1 - \frac{\nu^2}{q^2}\Big)^{-2}\,. \tag{4}$$

All non-perturbative effects are entirely contained in $W_{\mu\nu}$.

According to quark-parton models assumption, the relation between the parton 4-momentum with the nuclear 4-momentum can be expressed as

$$p^\mu = yP^\mu\,, \quad (0 \le y \le 1)\,. \tag{5}$$

On the assumption of incoherence, one can further obtain the contribution of the f-quark to the nucleon structure function $F_2^{(f)} = \nu W_2$ as

$$F^{(f)}_{2\rm NP}(y) = 2Mx_2^2 e_f^2 \delta(y - x_2) R_{2\rm NP}(x_2, Q^2)\,, \tag{6}$$

where ν is the energy of the virtual photon which mediates the interaction, and

$$\begin{aligned} &R_{2\rm NP}(x_2, Q^2) = 1 + \frac{(3-4\kappa)\nu}{2MQ^2\kappa^2 x_2}\Big(\frac{4\pi\alpha_s\langle q\bar q\rangle}{3}\Big)^{2/3}\,, \\ &\kappa = 1 + \frac{Q^2}{4M^2x_2^2}\,, \\ &x_2 = \frac{Q^2}{2M\nu}\,. \end{aligned} \tag{7}$$

Suppose that the nucleon state contains $f_f(y)\mathrm{d}y$ parton states of the type f in the interval $\mathrm{d}y$, then

$$F_{2\rm NP} = \sum_f \int_0^1 \mathrm{d}y f_f(y) F^{(f)}_{2\rm NP}(y)\,. \tag{8}$$

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Summing the contributions of all the quarks in the nucleon, we obtain the structure function of the nucleon,

$$F_{2\mathrm{NP}}(x_2,Q^2)=\sum_f e_f^2 x_2 q_f(x_2,Q^2)R_{2\mathrm{NP}}(Q^2)$$
$$=\sum_f e_f^2 x_2\tilde{q}_f(x_2,Q^2)\,, \tag{9}$$

where

$$\tilde{q}_f(x_2,Q^2)=q_f(x_2,Q^2)R_{2\mathrm{NP}}(x_2,Q^2)\,. \tag{10}$$

Obviously, $\tilde{q}_f(x_2,Q^2)$ is different from $q_f(x_2,Q^2)$, and does not satisfy the sum rule of quark parton model. Therefore, F_2 is no longer the sum of the probability distribution of the fractional charge square weight for quark and anti-quark as simply described in the quark parton model. After considering the non-perturbative effect, we should take into account F_2 with x_2 and Q^2 dependence, and the Q^2 dependence caused by the non-perturbative effect from QCD vacuum is different from that of scaling violation corrected by perturbative QCD impact on parton models.

From Eq. (3), We can also get $F_{1\mathrm{NP}}$,

$$F_{1\mathrm{NP}}(x_2,Q^2)=F_1(x_2,Q^2)R_{1\mathrm{NP}}(x_2,Q^2)\,, \tag{11}$$

where

$$R_{1\mathrm{NP}}(x_2,Q^2)=1+\frac{(1-4\kappa)}{\kappa Q^2}\Big(\frac{4\pi\alpha_s\langle q\bar{q}\rangle}{3}\Big)^{2/3}\,. \tag{12}$$

The ratio of longitudinal (σ_L) to transverse (σ_T) virtual photon absorpt cross sections is a sensitive quantity for revealing information about the quark gluon structure in the nucleon and for giving consistency check of theoretical models. The result of theoretical investigation on $R=\sigma_L/\sigma_T$, based on the nucleon structure with the non-perturbative $\langle q\bar{q}\rangle$ nuclear effect taken into account, seems to fit the experimental data well.[4]

2 QCD Non-perturbative Quark Condensation Effect on *K*-Factor

Let us review QCD α_s order corrections to nuclear Drell–Yan process, which come from the contribution of Compton scattering (C) and annihilation (Ann), i.e., the actual reaction cross section for the Drell–Yan cross section should be the common Drell–Yan cross section added with the two corrections due to Ann and C diagrams. Calculating to α_s order, the differential cross section for Drell–Yan process with perturbative QCD corrections is[5]

$$K(x_1,x_2)=\Big(\frac{\mathrm{d}^2\sigma^{\mathrm{DY}}}{\mathrm{d}x_1\,\mathrm{d}x_2}+\frac{\mathrm{d}^2\sigma^{\mathrm{Ann}}}{\mathrm{d}x_1\,\mathrm{d}x_2}+\frac{\mathrm{d}^2\sigma^{C}}{\mathrm{d}x_1\,\mathrm{d}x_2}\Big)\times\Big(\frac{\mathrm{d}^2\sigma^{\mathrm{DY}}}{\mathrm{d}x_1\,\mathrm{d}x_2}\Big)^{-1}. \tag{13}$$

For the Drell–Yan process of nucleus-nucleus collision, $K(x_1,x_2)$ can be calculated in terms of the expression of $Q_A^n(t_1,t_2)$ as follows:[6]

$$Q_A^{\mathrm{DY}}(t_1,t_2)=Q_A^{\mathrm{Ann}}(t_1,t_2)=\sum_f e_f^2 q_f(t_1,Q^2)\bar{q}_f(t_2,Q^2), \tag{14}$$

$$\tilde{Q}_A^{\mathrm{DY}}(t_1,t_2)=\tilde{Q}_A^{\mathrm{Ann}}(t_1,t_2)=\sum_f e_f^2\bar{q}_f(t_1,Q^2)q_f(t_2,Q^2), \tag{15}$$

$$Q_A^{C}(t_1,t_2)=\sum_f e_f^2 g^1(t_1,Q^2)[q_f(t_2,Q^2)+\bar{q}_f(t_2,Q^2)]\,, \tag{16}$$

$$\tilde{Q}_A^{C}(t_1,t_2)=\sum_f e_f^2[q_f(t_1,Q^2)+\bar{q}_f(t_1,Q^2)]g^2(t_2,Q^2)\,, \tag{17}$$

where g^1 and g^2 are the gluon distribution functions of the incident nucleus particles and the target nucleus particles, respectively.

When studying nuclear Drell–Yan process, investigations are carried on in the framework of the perturbative QCD by usually regarding partons in nucleon as "free" particles approximatively, namely based on the nature of "asymptotic freedom". The most important nature of "asymptotic freedom" is that the interaction between quarks or gluons is very weak in the small length of about 10^{-14} cm, however, in the length of about 10^{-13} cm, there is very strong interaction and QCD is non-perturbative, so, the non-perturbative effect from QCD vacuum should be taken into account.

According to the above, the substantive influence on the nucleon structure function due to $\langle q\bar{q}\rangle$ condensation is reflected by the corrections to the functions of quark momentum distribution. In the Drell–Yan process, $Q_A^n(t_1,t_2)$ is the joint distribution of quarks (anti-quarks) and gluons in the colliding hadrons for the three subprocesses. Considering the effect on the nucleon structure function from $\langle q\bar{q}\rangle$ condensation, the formulas of $Q_A^n(t_1,t_2)$ including $\langle q\bar{q}\rangle$ condensation should be rewritten as

$$Q_A^{\mathrm{DY}}(t_1,t_2)=Q_A^{\mathrm{Ann}}(t_1,t_2)=\sum_f e_f^2 q_f(t_1,Q^2)\bar{q}_f(t_2,Q^2)R_{1NP}(x_2,Q^2)R_{2NP}(x_2,Q^2)\,, \tag{18}$$

$$\tilde{Q}_A^{\mathrm{DY}}(t_1,t_2)=\tilde{Q}_A^{\mathrm{Ann}}(t_1,t_2)=\sum_f e_f^2\bar{q}_f(t_1,Q^2)q_f(t_2,Q^2)R_{1NP}(x_2,Q^2)R_{2NP}(x_2,Q^2)\,, \tag{19}$$

$$Q_A^{C}(t_1,t_2)=\sum_f e_f^2 g^1(t_1,Q^2)[q_f(t_2,Q^2)+\bar{q}_f(t_2,Q^2)]R_{2NP}(x_2,Q^2)\,, \tag{20}$$

$$\tilde{Q}_A^{C}(t_1,t_2)=\sum_f e_f^2[q_f(t_1,Q^2)+\bar{q}_f(t_1,Q^2)]g^2(t_2,Q^2)R_{1NP}(x_2,Q^2)\,. \tag{21}$$

3 Results and Discussion

Considering QCD α_s order corrections, and taking

$$\alpha_s(Q^2) = \frac{12\pi}{(33-2f)\ln(Q^2/\Lambda^2)}\Big\{1 - \frac{6(153-19f)}{(33-2f)^2}\frac{\ln[\ln(Q^2/\Lambda^2)]}{\ln(Q^2/\Lambda^2)}\Big\}, \quad (22)$$

where $f = 3$, $\Lambda = 232$ MeV. We have calculated and discussed the K-factor in the following two situations at the center-of-mass energy $\sqrt{s} = 200$ GeV and $\sqrt{s} = 630$ GeV for the Drell–Yan process in ${}^{12}_{6}$C-${}^{12}_{6}$C collision by using the quark-parton models. We take $x_1 = 0.5$ and the standard phenomenological value of the quark condensation $\langle q\bar{q}\rangle = -(0.25\ \text{GeV})^3$.[7]

K_R is the K-factor value with the non-perturbative correction, while the K-factor value without considering it is expressed as K_0; $\bar{K}_R$, $\bar{K}_0$ are the average value of K_R and K_0 respectively in interval x_2, $(\bar{K}_R - \bar{K}_0)/\bar{K}_0$ is the value of average variation for $\bar{K}_R$ to $\bar{K}_0$. The calculated results are given in Tables 1 and 2, and their corresponding plotted curves are presented in Fig. 1.

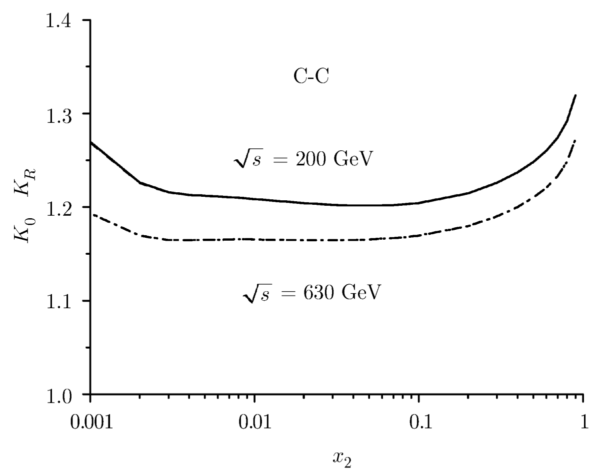

Fig. 1 K_0-factor corrected by perturbative QCD and K_R-factor corrected by the non-perturbative QCD at the different energies $\sqrt{s} = 200$ GeV and $\sqrt{s} = 630$ GeV for ${}^{12}_{6}$C-${}^{12}_{6}$C collision for Drell–Yan process.

Table 1 The average for K-factor and the values of average variation for $\bar{K}_R$ to $\bar{K}_0$ in the case of the perturbative QCD and the non-perturbative QCD in ${}^{12}_{6}$C-${}^{12}_{6}$C collision for Drell–Yan process at the center-of-mass system energy $\sqrt{s} = 200$ GeV.

x_2	$3\times10^{-5} \sim 6\times10^{-5}$	$7\times10^{-5} \sim -5\times10^{-4}$	$0.0006 \sim 0.01$	$0.02 \sim 0.1$	$0.2 \sim 0.9$
$\bar{K}_R$	1.762 926 57	1.460 588 66	1.240 400 22	1.202 767 61	1.258 916 59
$\bar{K}_0$	1.795 261 90	1.466 215 76	1.240 804 84	1.202 769 45	1.258 916 58
$(\bar{K}_R-\bar{K}_0)/\bar{K}_0$	−1.801 148 3%	−0.383 783%	−0.032 609%	−0.000 153%	−0.000 000%

Table 2 The average for K-factor and the values of average variation for $\bar{K}_R$ to $\bar{K}_0$ in the case of the perturbative QCD and the non-perturbative QCD in ${}^{12}_{6}$C-${}^{12}_{6}$C collision for Drell–Yan process at the center-of-mass system energy $\sqrt{s} = 630$ GeV.

x_2	$3\times10^{-5} \sim 6\times10^{-5}$	$7\times10^{-5} \sim -5\times10^{-4}$	$0.0006 \sim 0.01$	$0.02 \sim 0.1$	$0.2 \sim 0.9$
$\bar{K}_R$	1.290 197 86	1.260 155 88	1.179 598 26	1.166 472 82	1.219 275 77
$\bar{K}_0$	1.290 576 08	1.260 349 13	1.179 620 76	1.166 472 94	1.219 275 78
$(\bar{K}_R-\bar{K}_0)/\bar{K}_0$	−0.029 307%	−0.015 333%	−0.001 876%	−0.000 010%	−0.000 000%

From Fig. 1, we get that the curves for K_R and K_0 approximatively overlap, which have almost no difference for the collision of ${}^{12}_{6}$C-${}^{12}_{6}$C for the Drell–Yan process at the same energies. But, from Tables 1 and 2, we find that the QCD non-perturbative effects exist in K-factor. On one hand, when the system momentum parameter x_2 is smaller, the K-factor changes a lot relatively, and with x_2 increasing, the K-factor becomes smaller and smaller until K_R keeps the same as K_0 when x_2 exceeds a specific value, then the QCD non-perturbative effect stops playing any role. On the other hand, comparison of both situations above indicates that the QCD non-perturbative effect is associated with the center-of-mass energy. The smaller the center-of-mass energy, the bigger the QCD non-perturbative effect.

In a word the calculated results for the present cases show that the QCD non-perturbative effects exist in K-factor, but considerably small. The reason would be that the non-perturbative effect plays an important role as in the function of parton distribution for the simple Drell–Yan process, while the annihilation term and Compton's scattering term contribute only a small part in the QCD corrections. The ratio of the simple Drell–Yan cross section added with the item of the QCD to the simple Drell–Yan cross section weakens the non-perturbative effect, and makes it cause a trivial influence on K-factor. Such an influence has something to do with the center-of-mass system energy, the higher the energy, and the smaller the influence.

Further theoretical investigations should be required for the physical reasons for the different influences on K-factor from the above two corrections.

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